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The Chaos in g-Non Autonomous discrete systems

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***Abstract*---**In this work, we generalized some chaotic propositions in space more general than non-autonomous discrete systems for new system we call it a g-non autonomous discrete systems.

***Keywords*---**g-non autonomous discrete systems, sensitive dependent on initial condition, Touhey Property, strongly blending, locally eventually onto, transitive set and weakly mixing set.

Introduction

In [4] Kolyada.S and Snoha introduced space called non autonomous discrete systems , this space define by $h_n: Y \longrightarrow Y$, where Y is compact topological space , $n \in N$, so $h_{1,\infty}$ is the symbol to sequences begin from 1 to an infinite with $(h_1, h_2, \ldots, h_n, \ldots)$ such that $h_1^n(y) = h_n \circ h_{n-1} \circ \ldots \circ h_1$. Lei Liu and Yuejuan Sun [5] discussed transitive set and weakly mixing set in non-autonomous discrete systems. In particular case, when $h_{1,\infty}$ be uniformly convergent to h (i.e $h_n = (h, h, h.,,,)$ then we get system called autonomous discrete system (Dynamical system) [6]. Non autonomous discrete systems are suitable for modeling some phenomena in applied Sciences, including biology [2],[7], physics [3] and economy [8], they are also useful for solving mathematical puzzles [16]. In this work, we generalized this systems such that the sequences became begin from any $n \in N$ to an infinite, with $h_{n,\infty}$=($h_n, h_{n+1}, \ldots, h_{m,} \ldots$) define by $h_n^m(y) = h_m \circ h_{m-1} \circ \ldots \circ h_n(y)\ \forall\, n < m \in N$, and discussed some chaotic concept and chaotic properties in space more general than non-autonomous discrete systems called general-non autonomous discrete systems or g-non autonomous discrete systems.

Preliminaries

In this section, we will give generalized some definition for g-non autonomous discrete systems, so we define some chaotic concept: $h_n: Y \longrightarrow Y$ is called strongly blending on g- non autonomous discrete systems if for any open sets $W, M\ \exists\, n < m \in N\ \ni\ h_n^m(W) \cap h_n^m(M)$ contain an open set. also we need another definition: $h_n: Y \longrightarrow Y$ is called (Locally eventually onto on g- non autonomous discrete systems if for any non-empty open subset W of $Y \exists n < m \in N \ni\ h_n^m(W) = Y$. And we will generalize sensitive dependent on initial condition on g- non autonomous discrete systems by: $h_n: Y \longrightarrow Y$ is called sensitive dependent on initial condition if $\exists \sigma > 0$ and any $y \in Y$ and W any neighborhood of y, there is x$\in Y$ and $n < m \in N\ \ni\ d(h_n^m(y), h_n^m(x)) > \sigma$. In [1], Ali. B and Al-Shara'a. I introduced definition of transitive set and weakly mixing set in g-non autonomous discrete systems, and they studied some result on this definition. Now, we introduced another definition for transitive set and weakly mixing set in same spaces: A subset $\emptyset \neq B$ is called transitive set if for any $W, M \subseteq Y$ with $W \cap B \neq \emptyset$ and $M \cap B \neq \emptyset$, then $\mathcal{G}(\boldsymbol{B} \cap \boldsymbol{M}, \boldsymbol{W}) = \{\boldsymbol{n} < \boldsymbol{m} \in \mathbb{N} : \boldsymbol{h_n^m}(\boldsymbol{B} \cap \boldsymbol{M}) \cap \boldsymbol{W} \neq \emptyset\}$ is an infinite. also we generalized weakly mixing set for g- non autonomous discrete systems by: Then a non-empty subset B is called Weakly mixing set with at least two element if for any open sets $W_1, W_2, \dots W_n$ and $M_1, M_2, \dots M_n \exists n < m \in N\ \ni\ h_n^m(W_i \cap B) \cap M_i \neq \emptyset$. For any $i = 1, \dots, n$. in next section, we prove that srongly blending maps satisfy Touhey property, so we generalized Touhey property for g- non autonomous discrete systems by : $h_n$satisfy Touhey property if there exists two open sets W, M and $\exists y \in W$ and $n < m \in N\ \ni\ h_n^m(y) \in M$.

Main Theorems

In this section, we discussed some result that need above concept on g-non autonomous discrete systems.

Theorem 3.1

Let $(Y, h_{n,\infty})$ be for g- non autonomous discrete systems system and $\{Y_i\}_{i=1,\dots,n}$ be family of transitive set where $Y_i \subseteq Y_j$, if $i \leq j$, Then $\cup_{i=1}^n \{Y_i\}$ is transitive set.

proof :

Let $B \subseteq Y$. Then by(Definition 2.4) , there exist W and M are non-empty open sets such that

$\mathcal{G}(\cup_{i=1}^n Y_i \cap M, W) = \{n < m \in \mathbb{N} : h_n^m(\cup_{i=1}^n Y_i \cap M) \cap W \neq \emptyset\}$

$\mathcal{G}(\cup_{i=1}^n Y_i \cap M, W) = \{n < m \in \mathbb{N} :\ h_n^m\big((Y_1 \cap M) \cup (Y_2 \cap M) \cup \dots \cup (Y_n \cap M)\big) \cap W \neq \emptyset\}$

$\mathcal{G}(\cup_{i=1}^n Y_i \cap M, W) = \{n < m \in \mathbb{N}: h_n^m(Y_1 \cap M) \cup h_n^m(Y_2 \cap M) \cup \dots \cup h_n^m(Y_n \cap M)) \cap W \neq \emptyset\}$

$\mathcal{G}(\cup_{i=1}^n Y_i \cap M, W) = \{n < m \in \mathbb{N}, h_n^m(Y_1 \cap M) \cap W \cup h_n^m(Y_2 \cap M) \cap W \cup \dots \cup h_n^m(Y_n \cap M) \cap W \neq \emptyset\}$

Since Y_i are transitive set, for all $i = 1, \dots, n$, then $\mathcal{G}(Y_i \cap M, W)$ is an infinite . Then $\mathcal{G}(\cup_{i=1}^n Y_i \cap M, W)$ is an infinite so $\cup_{i=1}^n \{Y_i\}$ is transitive set .■

Proposition 3.2

Let $(Y, h_{n,\infty})$ be g- non autonomous discrete systems and B is transitive set of Y . Then $A \subseteq B$ also transitive set .
proof :
Let $\emptyset \neq W,\ M$ be open subset of B , then U and V are open subset of W and M respectively . So by (definition)
$\mathcal{G}(\boldsymbol{A} \cap \boldsymbol{U}, \boldsymbol{V}) = \{\boldsymbol{n} < \boldsymbol{m} \in \mathbb{N} : \boldsymbol{h}_{\boldsymbol{n}}^{\boldsymbol{m}}(\boldsymbol{A} \cap \boldsymbol{U}) \cap \boldsymbol{M} \neq \emptyset\}$Since $A \subseteq B$, then

$$\mathcal{G}(\boldsymbol{A} \cap \boldsymbol{U}, \boldsymbol{V}) \subseteq \mathcal{G}(\boldsymbol{B} \cap \boldsymbol{W}, \boldsymbol{M})$$

Since B is T.S , then $\mathcal{G}(B \cap W, M)$ is an infinite , so $\mathcal{G}(A \cap U, V)$is an infinite . Then A is transitive set.■
So in spatial case , the intersection of family of transitive set also transitive set .

Theorem 3.3

Let $(Y, h_{n,\infty})$ and $(Z, g_{n,\infty})$ be two g-NADS with $\pi : (Y, h_{n,\infty}) \longrightarrow (Z, g_{n,\infty})$ be factor map and B be Weakly mixing set subset of Y . Then $\pi(B)$ is Weakly mixing subset of Z.
Proof :
In this proof , we have two cases :
Case I : if $\pi\ (B)$ is singleton set . then the result hold and trivial .
Case II : Let $\pi\ (B)$ has contain at least two element and $B\ \subseteq\ Y$ is W.M of Y . Then $\exists$ i,k$\in$ $\mathbb{N}$ $\ni$ M_1^i, M_2^i , … , M_K^i are open subsets of $\pi\ (B)$ and W_1^i, W_2^i , … , W_K^i are open subsets of Z with $W_j^i \cap\ \pi\ (B) \neq \emptyset$, j = 1 ,2 ,…, k .Then $B\ \cap\ \pi^{-1}(M_j)$ are open subset of B and $\pi^{-1}(W_j^i)$ is
subset of Y with $\pi^{-1}(W_j^i) \cap B \neq \emptyset$, where j = 1 , … , k . Since B is W.M.S of Y then $\exists\ n < m \in \mathbb{N}$

$$\ni\ h_n^m(B \cap \pi_i^{-1}(M_j^i) \cap \pi_i^{-1}(W_j^i) \neq \emptyset\ ,$$

for each j = 1,…, k, so $g_n^m(M_j^i) \cap W_j^i \neq \emptyset$, for i = 1,…, k .
Then $\pi(B)$ is Weakly mixing subset of Z .■

Proposition 3.4

Let $(Y, h_{n,\infty})$ be g- non autonomous discrete systems . If $cl(B)$ is transitive set , then B is transitive set.
Proof :
Assume that $cl(B)$ is transitive set , then by(Definition) there exists two non-empty open subsets W, M of $Y$with $W\ \cap B \neq \emptyset$ and $M\ \cap B \neq \emptyset$ such that
$\mathcal{G}(M\ \cap cl(B), W)\ =\ \{n < m \in \mathbb{N}, h_n^m(M\ \cap cl(B)) \cap W \neq \emptyset\}$is an infinite . Since $B\ \subseteq\ cl(B)$,which impels that
$\mathcal{G}(M\ \cap B, W)\ =\ \{n < m \in \mathbb{N}, h_n^m(M\ \cap B) \cap W \neq \emptyset\}$ is an infinite so $\mathcal{G}(M\ \cap B, W) \neq \emptyset$. Then B is transitive set .■

Theorem 3.5

Let $(Y, h_{n,\infty})$ be g- non autonomous discrete systems. If $h_n : Y \longrightarrow Y$ are Locally eventually onto . Then $h_{n,\infty}$ is sensitive dependent on initial conditions .

Proof :
Let $\mathcal{P}\ ,q \in Y$ such that $d(\mathcal{P}\ ,q) > 3r$, where r positive integer number and let $\mathcal{B}_r(\mathcal{P})$ be ball with radius r and center $\mathcal{P}$ and $\mathcal{B}_r(q)$ ball with radius r and center q . Let $y \in Y$ and $S_\epsilon(y)$ open neighborhood of y for some$\epsilon > 0$. Since $h_{n,\infty}$ is locally eventually onto, there is $c_1 \in \mathbb{N}$ such that $(h_n^m)^{c_1}(\mathcal{B}_r(\mathcal{P})) = Y$, so there exists $d_1 < c_1 \in \mathbb{N}$ such that

$$(h_n^m)^{d_1}(\mathcal{B}_r(q) \cap S_\epsilon(y) \neq \emptyset$$

And there is $d_2 < c_2$ such that

$$(h_n^m)^{d_2}(\mathcal{B}_r(q) \cap S_\epsilon(y) \neq \emptyset$$

We take $d = max\{d_1, d_2\}$, then there exists $x_1, x_2 \in S_\epsilon(y)$ such that $(h_n^m)^d(x_1) \in \mathcal{B}_r(\mathcal{P})$ and $(h_n^m)^d(x_2) \in \mathcal{B}_r(q)$. Then we have

$$d((h_n^m)^d(x_1)\ , h_n^m)^d(x_2)) \geq \delta$$

By triangular inequality , we get

$((h_n^m)^d(x_1)\ , h_n^m)^d(y)) \geq \delta$ or $(h_n^m)^d(x_2)\ , h_n^m)^d(y) \geq \delta$Then $h_{n,\infty}$ is sensitive dependent on initial conditions . ■

Theorem 3.6

Let $(Y, h_{n,\infty})$ be with $h_n: Y \longrightarrow Y$ two g- non autonomous discrete systems with continuous maps. Therefore if h_n is strongly blending and has dense periodic points , then h_n is satisfy Touhey property .

proof :
Let $\emptyset \neq W, M$ be open subsets of Y ,there is $n < m \in \mathbb{N}$:
$h_n^m(W) \cap h_n^m(M)$ contain open set say U ,$U \subset Y: U \subset h_n^m(W) \cap h_n^m(M)$ .Now. Let $M_1 = h_n^{-m}(U) \cap M$, since U open set and h_n continuous maps,
then $h_n^{-m}(U)$ is open set, so M_1 is open set. Since The periodic points are dense then there is periodic point y of period m such that $h_n^m(y) \in U$ and
$y \in M$, so there is $p \in W : h_n^m(y) = h_n^m(p)$, therefor

$$h_n^m(p) = h_n^{m-k}(h_n^k(p)) = h_n^{m-k}(h_n^k(y)) = h_n^m(y) = y$$

So $y \in h_n^m(W) \cap M$, that mean $h_n^m(W) \cap M \neq \emptyset$,then $y \in (W)$. Then we obtained $h_{n,\infty}$ satisfy Touhey property. ■

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