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A markovian queueing model with catastrophe, unreliable and backup server

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***Abstract*---**In this article, we considered a finite size Markovian queue with single server. When the server breaks down, in order to facilitate the customer, backup server is provided. When system happened to undergo catastrophe, the customers are being removed and by restoration time the system get back to its normal state. Here, we have analysed the number of times the system reached its capacity. By utilising matrix geometric method, model has been solved and measures of effectiveness are done. Also numerical examples and graphical representation are given.

***Keywords*---**Breakdown, repair, catastrophe, restoration, backup server, matrix geometric method.

I. Introduction

The scope and diverse applications of queueing theory is countless. It's been often used in business to make decisions etc. the finite Markovian queue is considered here with catastrophe, restoration, breakdown, repair and backup server. The term catastrophe is referred to a sudden ruin happened to the system and as its effect, all customers are removed from the system. The system takes its own time to get ready to accept the new customer. This time is nothing but restoration. Decrescendo et al(2003) analyzed onthe M/M/1 queue with catastrophes and its continuous approximation. Kumar et al (2007) derived the transient analysis of a

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single server queue with catastrophes, failures and repairs. The number of times, a system reaches its capacity is been analysed and derived by Danesh garg(2013) in Performance Analysis of number of times a system reaches its capacity with catatrophe and restoration. Jain and Kumar (2007) have explored the catastrophe in Transient Solution of a Catastrophic-Cum-Restorative Queueing Problem with Correlated Arrivals and Variable Service Capacity .Rakesh kumar investigated both catastrophe and restoration in a catastrophic-cum-restorative queueing model with correlated input for the cell traffic generated by new broadband services . X. Chao (1995) incorporated the catastrophe in A Queueing Network Model with Catastrophes and Product from Solution.

The breakdown of a server happens when there is a failure and therefore the server undergoes repair. During the repair time, the server is not available to offer service . In order to accord a continual service even in repair time , a back up server is set. The back up server will continue to offer service until the server returns from the breakdown. Wartenhosrt obtained the solution for breakdown and repairs in N parallel Queueing systems with server breakdowns and repair". Kumar et al (2007) procured transient analysis of a single server queue with catastrophes, failures and repairs . Neuts &Lucantoni (1979) indagated on breakdowns and repairs in a Markovian queue with N servers subject to breakdowns and repairs. Wang & Chang made a analysis in Cost analysis of a finite M/M/R queueing system with balking, reneging and server breakdown. Shoukry et al derivedMatrix geometric method for M/M/1 queueing model with and without breakdown ATM machines. Sridharan &Jayashree incorporated , "Some characteristics on a finite queue with normal partial and total failures" . Kimet al explored the brekdown in analysis of unreliable BMAP/PH/n type queue with Markovian flow of breakdowns. Neuts made remarkable research inMatrix-Geometric solutions in stochastic models. Walrand explored queues in an Introduction to Queueing Networks. This paper has totally three sections.the second section has model description, numerical study is done in third section and the fourth section has the performance measures.

II. Model Description

We considered a Markovian queue with finite size. The customers arrived at the mean rate of λ and are being served by a single server. The server rendered the service at a mean rate of μ . When the system experiences catastrophe at the rate of ξ, all the cutomers are deleted from the system . The restoration time is taken by the sytem to restore its customers back . The restoration time is distributed by the parameter γ. The server is subject to breakdown and so the backup server is provided for uninterrupted service.After the repair , the server will be back to the service. The breakdown occurs at the rate of α_0 , when the system is in the process of reaching its capacity first time and at α_1 when the system is in he process of reaching its capacity second time. Their corresponding repair times are β_0 and β_1. The backup server will service at recduceable rate of μ_1.

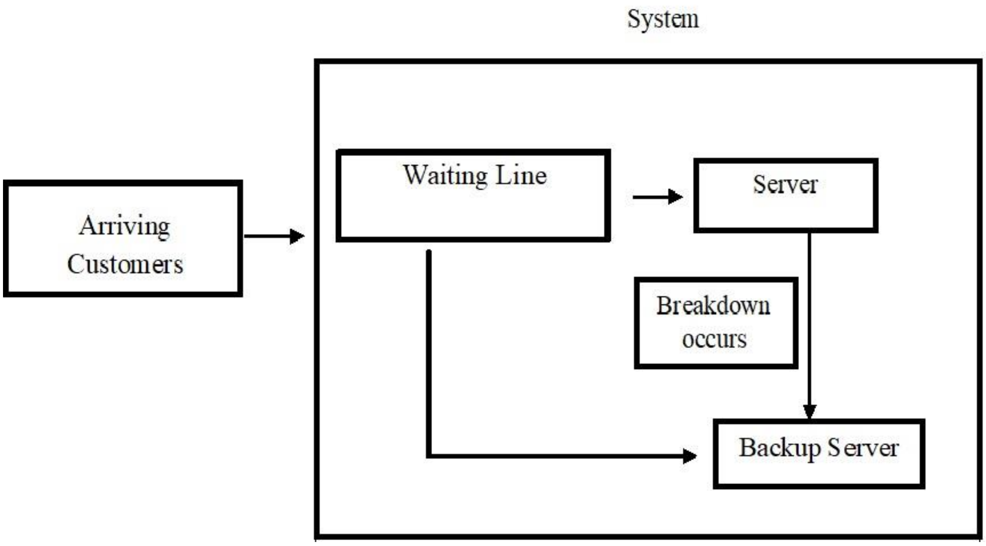

Fig.1 Queue Structure

Let us define $P_{u,v}(t)$ = Prob[U(t) = u,V(t) = v], 0≤v≤N

Where $U(t)=\begin{cases} 0, \textit{when the system reaches its capacity first time} \\ 1, \textit{when the system reaches its capacity first time} \\ 2, \textit{when the server breaks down} \\ 3, \textit{when the backup server services} \end{cases}$

V(t) denotes the number of customer in the system at time t

The QBD process along with state space Ω as follows $\Omega=(0,0)U(1,0)U(i,j);i=0,1,2,3\&j=1,2,\ldots,n\geq 1$

The infinitesimal generator matrix Q is presented by,

$$Q=\begin{pmatrix} B_{00} & B_{01} & 0 & \ldots & \ldots & \ldots \\ A_{20} & A_1 & A_2 & 0 & \ldots & \ldots \\ A_{10} & A_0 & A_1 & A & 0 & \ldots \\ A_{10} & 0 & A_0 & A_1 & A_2 & 0 \\ A_{10} & 0 & 0 & A_0 & A_1 & A_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix}$$

Where

$$B_{00}=-4(\lambda+\gamma)$$

$$B_{01}=\begin{pmatrix}\lambda+\gamma & \lambda+\gamma & \lambda+\gamma & \lambda+\gamma\end{pmatrix}$$

$$A_{20}=\begin{pmatrix}\mu+\xi\\ \mu+\xi\\ \xi\\ \mu_1\end{pmatrix}\quad A_0=\begin{pmatrix}\mu&0&0&0\\0&\mu&0&0\\0&0&0&0\\0&0&0&\mu_1\end{pmatrix}\quad A_2=\begin{pmatrix}\lambda&0&0&0\\0&\lambda&0&0\\0&0&\lambda&\lambda\\0&0&0&\lambda\end{pmatrix}$$

$$A_1=\begin{pmatrix}-(\lambda+\mu+\xi+\alpha_0)&0&\alpha_0&0\\0&-(\lambda+\mu+\xi+\alpha_1)&\alpha_1&0\\\beta_0&\beta_1&-(2\lambda+\xi+\beta_0+\beta_1)&0\\0&0&0&-(\lambda+\mu_1+\xi)\end{pmatrix}$$

The static row vectors are given by P= (P_0,P_1,P_2,...) where P_0= P_{00} and P_i= (P_{0i},P_{1i},,P_{2i},P_{3i}). the static probability probility matrix is given by PQ=0
And therefore , we have

$B_{00}P_0+P_1A_{20}+A_{10}[P_2+P_3+\ldots]=0$ ---1

$B_{01}P_0+P_1A_1+P_2A_0=0$ ---2

$P_1A_2+P_2A_1+P_3A_0=0$

$P_{i-1}A_2+P_iA_1+P_{i+1}A_0=0$

$P_i=P_1R^{i-1}$ ---3

Where R is a rate matrix.
And so in general , we get

$$R_{n+1}=-A_1^{-1}(A_2+R_n^2A_0)$$

From equation 1 and 2 , we arrive at the condition

$P_0e+P_1(I-R)^{-1}e=1$ ---4

Where e is the unit matrix.
The static condition of such a QBD (Quasi-Birth Death), (See Neuts (1981)) can be obtained by the drift condition

$P\,A_2e < P\,A_0e$

Where P=(P0,P1,P2,P3) is got from the generator X and
A is given by A = A_0 +A_1+A_2 and so

$$X=\begin{pmatrix}Q&0&\alpha_0&0\\0&R&\alpha_1&0\\\beta_0&\beta_1&S&\lambda\\0&0&0&T\end{pmatrix}$$

Where Q= -(ξ+α_0) , R=-(ξ+α_1) , S= -(λ+β_0+β_1+ξ) and T= -(μ+μ_1+ξ)
A is irreducible and P can be shown to be unique such that PA=0 and Pe=1 ---- 5

By using the equation 5, we have

$$P_0 = \frac{\beta_0}{\xi + \alpha_0} P_2$$

$$P_1 = \frac{\beta_1}{\xi + \alpha_1} P_2$$

$$P_3 = \frac{\lambda}{\mu + \mu_1 + \xi} P_2$$

$$P_2 = \left[1 + \frac{\beta_0}{\xi + \alpha_0} + \frac{\beta_1}{\xi + \alpha_1} + \frac{\lambda}{\mu + \mu_1 + \xi} \right]^{-1}$$

The static condition takes the format

$$\lambda(P_0 + P_1 + 2P_2 + P_3) < (P_0 + P_1)\mu + P_3\mu_1 \qquad \text{---6}$$

The equation 6 is the static probability of A and the probability vectors are obtained by utilizing the equations 3&4 and the rate matrix.

III Numerical Study

Here, we have made the numerical analysis of the above described model. The parameters λ and μ varied. Therefore eight illuatrations are presented below , four in each.
The variation of arrival rate(0.6 to 0.9) is done from Illustration A to illustration D.

Illustration A
Let us assume $\lambda=0.6, \mu=1.5, \mu_1=1, \xi=0.05, \gamma=0.06, \alpha_0=0.1$, $\alpha_1=0.2$, $\beta_0=0.6$, $\beta_1=0.7$

The rate matrix is given by $\begin{pmatrix} 0.3617 & 0.0151 & 0.0108 & 0.0319 \\ 0.0264 & 0.3480 & 0.0207 & 0.0581 \\ 0.1471 & 0.1539 & 0.2435 & 0.3815 \\ 0 & 0 & 0 & 0.5409 \end{pmatrix}$

Table I Probability vectors

	P_{0i}	P_{1i}	P_{2i}	P_{3i}	Total
P_{00}	0.1666				0.1666
P_1	0.0734	0.0721	0.0553	0.1798	0.3806
P_2	0.0365	0.0347	0.0157	0.1249	0.2118
P_3	0.0164	0.0150	0.0049	0.0767	0.1130
P_4	0.0071	0.0062	0.0017	0.0448	0.0598
P_5	0.0030	0.0025	0.0006	0.0255	0.0316
P_6	0.0012	0.0010	0.0002	0.0142	0.0166
P_7	0.0005	0.0004	0.0001	0.0079	0.0089
P_8	0.0002	0.0001	0.0000	0.0043	0.0046
P_9	0.0001	0.0001	0.0000	0.0024	0.0026
P_{10}	0.0000	0.0000	0.0000	0.0013	0.0013

P_{11}	0.0000	0.0000	0.0000	0.0007	0.0007
P_{12}	0.0000	0.0000	0.0000	0.0004	0.0004
P_{13}	0.0000	0.0000	0.0000	0.0002	0.0002
P_{14}	0.0000	0.0000	0.0000	0.0001	0.0001
P_{15}	0.0000	0.0000	0.0000	0.0000	0.0000
Total					0.9988

The probability vectors are given by P_i. The P_0 =0.1666 and P_1= (0.0734,0.0721,0.0553,0.1798) is found by normality condition (equation 4). The balance probability vectors are obtained by the recurrence relation (equation 3) . Each row consists of four elements. And the last row is the total of four. The last column gives the total probability and which is validated to be 0.9988 $\cong$ 1

Illustration B

Let us assume λ=0.7,μ=1.5,μ_1=1,§=0.05,γ=0.06,α_0=0.1, α_1=0.2, β_0=0.6, β_1=0.7

The rate matrix is given by $\begin{pmatrix} 0.4714 & 0.0172 & 0.0111 & 0.0362 \\ 0.0306 & 0.3997 & 0.0214 & 0.0657 \\ 0.1616 & 0.1675 & 0.2624 & 0.4198 \\ 0 & 0 & 0 & 0.6187 \end{pmatrix}$

Table II Probability vectors

	P_{0i}	P_{1i}	P_{2i}	P_{3i}	Total
P_{00}	0.1307				0.1307
P_1	0.0651	0.0637	0.0468	0.1605	0.3361
P_2	0.0367	0.0344	0.0144	0.1255	0.2110
P_3	0.0187	0.0168	0.0049	0.0872	0.1276
P_4	0.0091	0.0078	0.0018	0.0578	0.0765
P_5	0.0043	0.0036	0.0007	0.0374	0.0460
P_6	0.0020	0.0016	0.0003	0.0238	0.0277
P_7	0.0009	0.0007	0.0001	0.0151	0.0168
P_8	0.0004	0.0003	0.0000	0.0095	0.0102
P_9	0.0002	0.0001	0.0000	0.0059	0.0062
P_{10}	0.0001	0.0001	0.0000	0.0037	0.0039
P_{11}	0.0000	0.0000	0.0000	0.0023	0.0023
P_{12}	0.0000	0.0000	0.0000	0.0014	0.0014
P_{13}	0.0000	0.0000	0.0000	0.0009	0.0009
P_{14}	0.0000	0.0000	0.0000	0.0005	0.0005
P_{15}	0.0000	0.0000	0.0000	0.0003	0.0003
P_{16}	0.0000	0.0000	0.0000	0.0002	0.0002
P_{17}	0.0000	0.0000	0.0000	0.0001	0.0001
P_{18}	0.0000	0.0000	0.0000	0.0001	0.0001
Total					0.9985

The probability vectors are given by P_i. The P_0 =0.1307 and P_1= (0.0651,0.0637,0.0468,0.1605) is found by normality condition (equation 4). The balance probability vectors are obtained bythe recurrence relation (equation 3) .

Each row consist of four elements. And the last row is the total of four. The last column gives the total probability and which is validated to be 0.9985$\cong$1

Illustration C

Let us assume $\lambda=0.8, \mu=1.5, \mu_1=1, \xi=0.05, \gamma=0.06, \alpha_0=0.1$, $\alpha_1=0.2$, $\beta_0=0.6$, $\beta_1=0.7$

The rate matrix is given by $\begin{pmatrix} 0.4709 & 0.0194 & 0.0113 & 0.0399 \\ 0.0344 & 0.4487 & 0.0218 & 0.0717 \\ 0.1729 & 0.1779 & 0.2786 & 0.4508 \\ 0 & 0 & 0 & 0.6889 \end{pmatrix}$

Table III Probability vectors

	P_{0i}	P_{1i}	P_{2i}	P_{3i}	Total
P_{00}	0.1023				0.1023
P_1	0.0570	0.0555	0.0390	0.1382	0.2897
P_2	0.0355	0.0329	0.0127	0.1190	0.2001
P_3	0.0200	0.0177	0.0046	0.0915	0.1338
P_4	0.0108	0.0092	0.0019	0.0672	0.0891
P_5	0.0057	0.0047	0.0008	0.0482	0.0594
P_6	0.0030	0.0023	0.0004	0.0342	0.0399
P_7	0.0016	0.0012	0.0002	0.0240	0.0270
P_8	0.0008	0.0006	0.0001	0.0168	0.0183
P_9	0.0004	0.0003	0.0000	0.0117	0.0124
P_{10}	0.0002	0.0001	0.0000	0.0081	0.0084
P_{11}	0.0001	0.0001	0.0000	0.0056	0.0058
P_{12}	0.0000	0.0000	0.0000	0.0039	0.0039
P_{13}	0.0000	0.0000	0.0000	0.0027	0.0027
P_{14}	0.0000	0.0000	0.0000	0.0018	0.0018
P_{15}	0.0000	0.0000	0.0000	0.0013	0.0013
P_{16}	0.0000	0.0000	0.0000	0.0009	0.0009
P_{17}	0.0000	0.0000	0.0000	0.0006	0.0006
P_{18}	0.0000	0.0000	0.0000	0.0004	0.0004
P_{19}	0.0000	0.0000	0.0000	0.0003	0.0003
P_{20}	0.0000	0.0000	0.0000	0.0002	0.0002
P_{21}	0.0000	0.0000	0.0000	0.0001	0.0001
P_{22}	0.0000	0.0000	0.0000	0.0001	0.0001
P_{23}	0.0000	0.0000	0.0000	0.0001	0.0001
P_{24}	0.0000	0.0000	0.0000	0.0000	0.0000
				Total	0.9986

The probability vectors are given by P_i. The P_0 =0.1023 and P_1= (0.0570, 0.0555, 0.0390,0.1382) is found by normality condition (equation 4). The balance probability vectors are obtained by the recurrence relation (equation 3) . Each row consist of four elements. And the last row is the total of four. The last column gives the total probability and which is validated to be 0.9986$\cong$1

Illustration D

Let us assume λ=0.9,μ=1.5,μ_1=1,§=0.05,γ=0.06,α_0=0.1, α_1=0.2, β_0=0.6, β_1=0.7

The rate matrix is given by $\begin{pmatrix} 0.5218 & 0.0212 & 0.0114 & 0.0425 \\ 0.0376 & 0.4950 & 0.0220 & 0.0760 \\ 0.1814 & 0.1854 & 0.2928 & 0.4747 \\ 0 & 0 & 0 & 0.7498 \end{pmatrix}$

Table IV Probability vectors

	P_{0i}	P_{1i}	P_{2i}	P_{3i}	Total
P_{00}	0.0801				0.0801
P_1	0.0495	0.0479	0.0322	0.1152	0.2448
P_2	0.0335	0.0307	0.0110	0.1074	0.1826
P_3	0.0206	0.0180	0.0043	0.0895	0.1324
P_4	0.0122	0.0101	0.0019	0.0714	0.0956
P_5	0.0071	0.0056	0.0009	0.0557	0.0693
P_6	0.0041	0.0031	0.0005	0.0429	0.0506
P_7	0.0023	0.0017	0.0002	0.0328	0.0370
P_8	0.0013	0.0009	0.0001	0.0250	0.0273
P_9	0.0007	0.0005	0.0001	0.0189	0.0202
P_{10}	0.0004	0.0003	0.0000	0.0143	0.0150
P_{11}	0.0002	0.0001	0.0000	0.0108	0.0111
P_{12}	0.0001	0.0001	0.0000	0.0081	0.0083
P_{13}	0.0001	0.0000	0.0000	0.0061	0.0062
P_{14}	0.0000	0.0000	0.0000	0.0046	0.0046
P_{15}	0.0000	0.0000	0.0000	0.0034	0.0034
P_{16}	0.0000	0.0000	0.0000	0.0026	0.0026
P_{17}	0.0000	0.0000	0.0000	0.0019	0.0019
P_{18}	0.0000	0.0000	0.0000	0.0014	0.0014
P_{19}	0.0000	0.0000	0.0000	0.0011	0.0011
P_{20}	0.0000	0.0000	0.0000	0.0008	0.0008
P_{21}	0.0000	0.0000	0.0000	0.0006	0.0006
P_{22}	0.0000	0.0000	0.0000	0.0004	0.0004
P_{23}	0.0000	0.0000	0.0000	0.0003	0.0003
P_{24}	0.0000	0.0000	0.0000	0.0002	0.0002
P_{25}	0.0000	0.0000	0.0000	0.0002	0.0002
P_{26}	0.0000	0.0000	0.0000	0.0001	0.0001
P_{27}	0.0000	0.0000	0.0000	0.0001	0.0001
P_{28}	0.0000	0.0000	0.0000	0.0001	0.0001
				Total	0.9983

The probability vectors are given by P_i. The P_0 =0.0801 and P_1= (0.0495,0.0479,0.0322,0.1152) is found by normality condition (equation 4). The balance probability vectors are obtained by the recurrence relation (equation 3) . Each row consist of four elements. And the last row is the total of four. The last column gives the total probability and which is validated to be 0.9983 $\cong$ 1

By varying the values of service (1.6 to 1.9), we get four illustrations from illustration E to illustration H.

Illustration E

Let us assume $\mu=1.6,\lambda=0.6,\mu_1=1,\xi=0.05,\gamma=0.06,\alpha_0=0.1$, $\alpha_1=0.2$, $\beta_0=0.6$, $\beta_1=0.7$

The rate matrix is given by $\begin{pmatrix} 0.3318 & 0.0131 & 0.0103 & 0.0282 \\ 0.0228 & 0.3304 & 0.0198 & 0.0529 \\ 0.1373 & 0.1483 & 0.2432 & 0.3770 \\ 0 & 0 & 0 & 0.5409 \end{pmatrix}$

Table V Probability vectors

	P_{0i}	P_{1i}	P_{2i}	P_{3i}	Total
P_{00}	0.1724				0.1724
P_1	0.0713	0.0706	0.0567	0.1866	0.3852
P_2	0.0330	0.0327	0.0159	0.1280	0.2096
P_3	0.0139	0.0136	0.0048	0.0779	0.1102
P_4	0.0056	0.0054	0.0016	0.0451	0.0577
P_5	0.0022	0.0021	0.0005	0.0254	0.0302
P_6	0.0008	0.0008	0.0002	0.0141	0.0159
P_7	0.0003	0.0003	0.0001	0.0078	0.0085
P_8	0.0001	0.0001	0.0000	0.0043	0.0045
P_9	0.0000	0.0000	0.0000	0.0023	0.0023
P_{10}	0.0000	0.0000	0.0000	0.0013	0.0013
P_{11}	0.0000	0.0000	0.0000	0.0007	0.0007
P_{12}	0.0000	0.0000	0.0000	0.0004	0.0004
P_{13}	0.0000	0.0000	0.0000	0.0002	0.0002
P_{14}	0.0000	0.0000	0.0000	0.0001	0.0001
				Total	0.9992

The probability vectors are given by P_i. The P_0 =0.1724 and P_1= (0.0713,0.0706,0.0567,0.1866) is found by normality condition (equation 4). The balance probability vectors are obtained by the recurrence relation (equation 3). Each row consist of four elements. And the last row is the total of four. The last column gives the total probability and which is validated to be $0.9992 \cong 1$

Illustration F

Let us assume $\mu=1.7$, $\lambda=0.6$, $\mu_1=1$, $\xi=0.05$, $\gamma=0.06$, $\alpha0=0.1$, $\alpha_1=0.2$, $\beta_0=0.6$, $\beta_1=0.7$

The rate matrix is given by $\begin{pmatrix} 0.3247 & 0.0122 & 0.0099 & 0.0263 \\ 0.0216 & 0.3145 & 0.0190 & 0.0486 \\ 0.1357 & 0.1437 & 0.2428 & 0.3738 \\ 0 & 0 & 0 & 0.5409 \end{pmatrix}$

Table VI Probability vectors

	P_{0i}	P_{1i}	P_{2i}	P_{3i}	Total
P_{00}	0.1768				0.1768
P_1	0.0695	0.0688	0.0576	0.1903	0.3862
P_2	0.0319	0.0307	0.0160	0.1296	0.2082
P_3	0.0132	0.0123	0.0048	0.0784	0.1087
P_4	0.0052	0.0047	0.0015	0.0451	0.0565
P_5	0.0020	0.0018	0.0005	0.0253	0.0296
P_6	0.0007	0.0006	0.0002	0.0140	0.0155
P_7	0.0003	0.0000	0.0000	0.0077	0.0080
P_8	0.0001	0.0001	0.0000	0.0042	0.0044
P_9	0.0000	0.0000	0.0000	0.0023	0.0023
P_{10}	0.0000	0.0000	0.0000	0.0012	0.0012
P_{11}	0.0000	0.0000	0.0000	0.0007	0.0007
P_{12}	0.0000	0.0000	0.0000	0.0004	0.0004
P_{13}	0.0000	0.0000	0.0000	0.0002	0.0002
P_{14}	0.0000	0.0000	0.0000	0.0001	0.0001
				Total	0.9988

The probability vectors are given by P_i. The P_0 =0.1768 and P_1= (0.0695,0.0688,0.0576,0.1903) is found by normality condition (equation 4). The balance probability vectors are obtained by the recurrence relation (equation 3) . Each row consist of four elements. And the last row is the total of four. The last column gives the total probability and which is validated to be 0.9988 $\cong$ 1

Illustration G

Let us assume μ=1.8,λ=0.6,μ_1=1,ξ=0.05,γ=0.06,α_0=0.1, α_1=0.2, β_0=0.6, β_1=0.7

The rate matrix is given by $$\begin{pmatrix} 0.3087 & 0.0112 & 0.0095 & 0.0241 \\ 0.0196 & 0.2999 & 0.0183 & 0.0450 \\ 0.1308 & 0.1393 & 0.2425 & 0.3709 \\ 0 & 0 & 0 & 0.5409 \end{pmatrix}$$

Table VII Probability vectors

	P_{0i}	P_{1i}	P_{2i}	P_{3i}	Total
P_{00}	0.1811				0.1811
P_1	0.0677	0.0671	0.0586	0.1945	0.3879
P_2	0.0299	0.0290	0.0161	0.1316	0.2066
P_3	0.0119	0.0113	0.0047	0.0792	0.1071
P_4	0.0045	0.0042	0.0014	0.0454	0.0555
P_5	0.0017	0.0015	0.0005	0.0254	0.0291
P_6	0.0006	0.0005	0.0001	0.0140	0.0152
P_7	0.0002	0.0002	0.0000	0.0077	0.0081
P_8	0.0001	0.0001	0.0000	0.0042	0.0044
P_9	0.0000	0.0000	0.0000	0.0023	0.0023

P_{10}	0.0000	0.0000	0.0000	0.0012	0.0012
P_{11}	0.0000	0.0000	0.0000	0.0007	0.0007
P_{12}	0.0000	0.0000	0.0000	0.0003	0.0003
P_{13}	0.0000	0.0000	0.0000	0.0002	0.0002
P_{14}	0.0000	0.0000	0.0000	0.0001	0.0001
				Total	0.9998

The probability vectors are given by P_i. The P_0 =0.1811 and P_1= (0.0677,0.0671,0.0586,0.1945) is found by normality condition (equation 4). The balance probability vectors are obtained by the recurrence relation (equation 3) . Each row consist of four elements. And the last row is the total of four. The last column gives the total probability and which is validated to be 0.9998 $\cong$ 1

Illustration H

Let us assume μ=1.9,λ=0.6,μ_1=1,ξ=0.05,γ=0.06,α_0=0.1, α_1=0.2, β_0=0.6, β_1=0.7

The rate matrix is given by $\begin{pmatrix} 0.2942 & 0.0103 & 0.009 & 0.0223 \\ 0.0179 & 0.2865 & 0.0176 & 0.0420 \\ 0.1265 & 0.1354 & 0.2423 & 0.3680 \\ 0 & 0 & 0 & 0.5409 \end{pmatrix}$

Table VIII Probability vectors

	P_{0i}	P_{1i}	P_{2i}	P_{3i}	Total
P_{00}	0.1851				0.1851
P_1	0.0659	0.0655	0.0594	0.1982	0.3890
P_2	0.0281	0.0275	0.0161	0.1331	0.2048
P_3	0.0108	0.0103	0.0046	0.0797	0.1054
P_4	0.0039	0.0037	0.0014	0.0455	0.0545
P_5	0.0014	0.0013	0.0004	0.0253	0.0284
P_6	0.0005	0.0004	0.0001	0.0140	0.0150
P_7	0.0002	0.0001	0.0000	0.0076	0.0079
P_8	0.0000	0.0000	0.0000	0.0041	0.0041
P_9	0.0000	0.0000	0.0000	0.0022	0.0022
P_{10}	0.0000	0.0000	0.0000	0.0012	0.0012
P_{11}	0.0000	0.0000	0.0000	0.0006	0.0006
P_{12}	0.0000	0.0000	0.0000	0.0003	0.0003
P_{13}	0.0000	0.0000	0.0000	0.0002	0.0002
P_{14}	0.0000	0.0000	0.0000	0.0001	0.0001
				Total	0.9988

The probability vectors are given by P_i. The P_0 =0.1851 and P_1= (0.0659, 0.0655 ,0.0594,0.1982) is found by normality condition (equation 4). The balance probability vectors are obtained by the recurrence relation (equation 3) . Each row consist of four elements. And the last row is the total of four. The last column gives the total probability and which is validated to be 0.9988 $\cong$ 1

IV Performance Measures

Probability that the system is empty, P(E) = P_0

Probability of mean number of customers in the system when the system reaches its capacity one time, P(OTR)= $\sum_{j=1}^{\infty} jP_{0j}$

Probability of mean number of customers in the system when the system reaches its capacity second time, P(TTR) = $\sum_{j=1}^{\infty} jP_{1j}$

Probability of mean number of customers when server undergoes breakdown, P(BD)= $\sum_{j=1}^{\infty} jP_{2j}$

Probability of mean number customers when the backup server services, P(BUS) = $\sum_{j=1}^{\infty} jP_{3j}$

Probability of total number of customers in the system,

P(N) = P(E)+P(OTR)+P(TTR)+P(BD)+P(BUS)

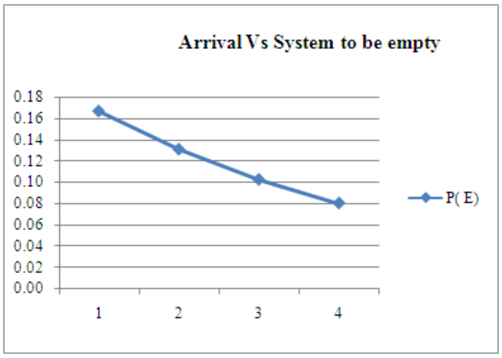

Fig.2. Arrival Vs P(E)

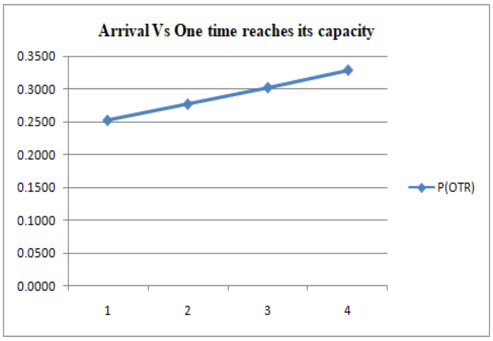

Fig.3. Arrival Vs P(OTR)

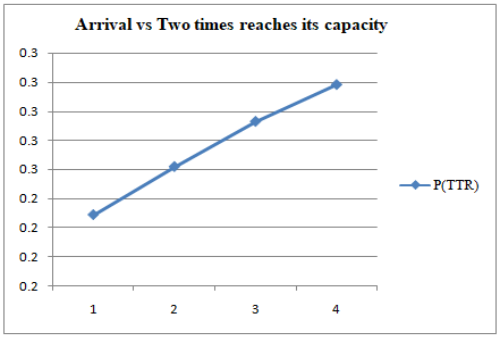

Fig.4.Arrival Vs P(TTR)

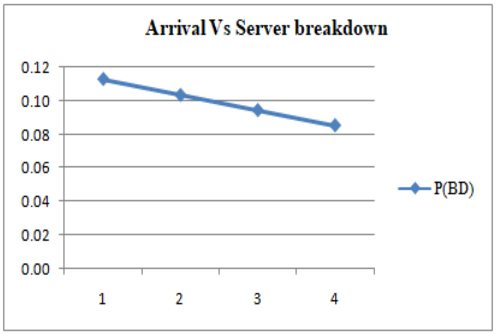

Fig.5.Arrival Vs P(BD)

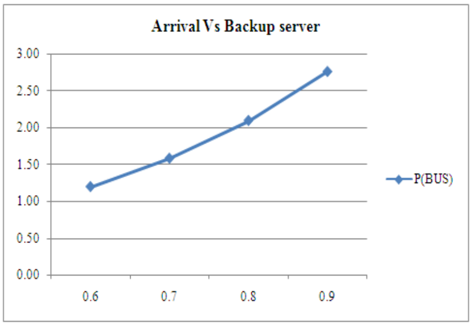

Fig.6. Arrival Vs P(BUS)

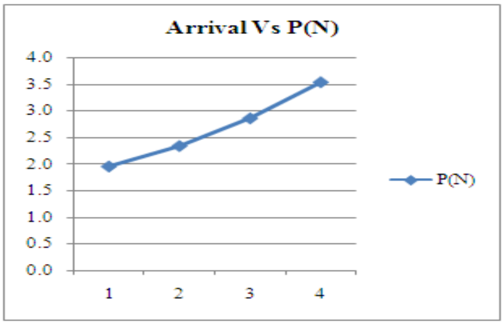

Fig.7.Arrival Vs P(n)

The variation of arrival rate from λ=0.6 to λ=0.9 is been shown in the above grafical figures. As arrival rate increases P(E) and P(BD) decreases gradually(Fig.2 and Fig.5 respectively), P(OTR), P(TTR), P(BUS) and P(N) increases steadily (Fig.3,Fig.4,Fig.6 and Fig.7 respectively).

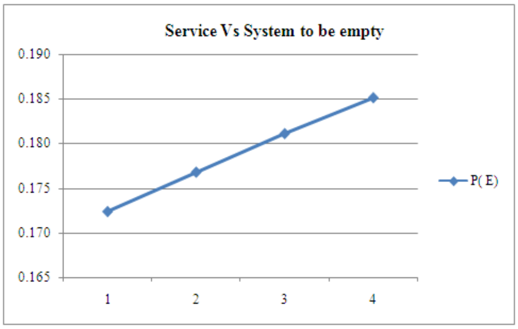

Fig.8. Service Vs P(E)

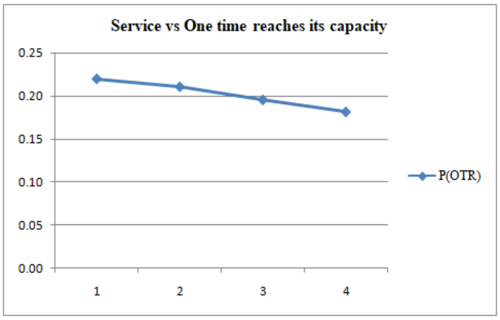

Fig.9. Service Vs P(OTR)

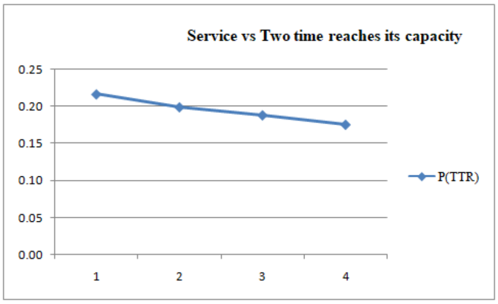

Fig.10.Service Vs P(TTR)

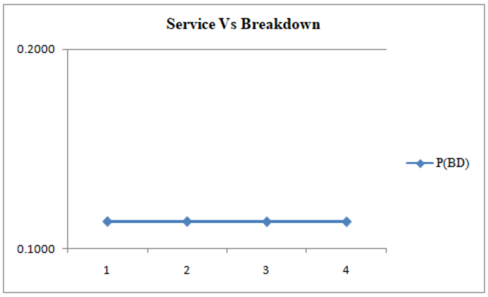

Fig.11. Service Vs P(BD)

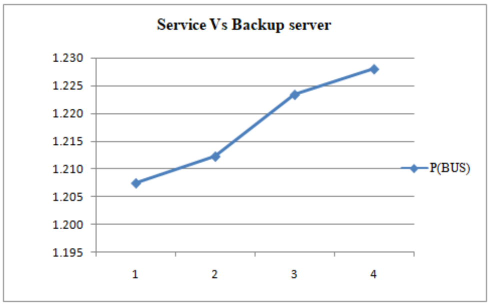

Fig.12.Service Vs P(BUS)

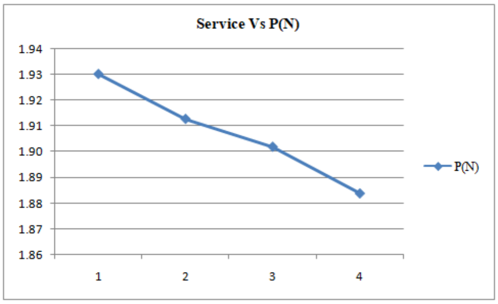

Fig.13. Service Vs P(N)

The service rate is varied from 1.6 to 1.9 and it is reprented by graph above. The rise in the value of service rate shows the gradual increase in P(E) (Fig.8) while there is a sudden increase in P(BUS) which is shown in the Fig.12. As thhe rate os service goes high , a gradual slow reduction is shown in P(OTR) (Fig.9) and P(TTR) (Fig.10) , while there is a sudden fall in P(N) ,given in the Fig.13. There is no change in P(BD) (Fig.11) , when the rate of service is getting increased.

Conclusion

A finite size Markovian queue is taken into consideration with single server. The concepts catastrophe , restoration , breakdown ,repair and backup server are included and solved by using matrix geometric approach. The numerical study is done by varying the rate of arrival and service. The corresponding graphical resentation are given.

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