How to Cite:
Hashemi, S. F., & Khorsandi, H. (2022). Two-stage network structure with undesirable outputs approach. *International Journal of Health Sciences*, *6*(S7), 1499–1518.
https://doi.org/10.53730/ijhs.v6nS7.11557

Two-stage network structure with undesirable outputs approach

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***Abstract*---**Articles in which nonparametric data envelopment analysis (DEA) is applied to estimate the efficiency based on a network structure usually consider the desirable intermediary parts that are the first stage's outputs to be used as the second stage's inputs. In many real-life situations, the intermediary parameters include desirable and undesirable outputs. This issue has drawn much attention from DEA researchers in recent years. The main motivation for this application is weak disposability for modeling the networks with undesirable intermediary parts. Undesirable outputs have been investigated in this study in two forms, i.e., the final outputs or intermediary parameters. In both cases, the non-cooperative game theory is offered for evaluating the relative efficiency of the executive units. In 2008, a real case was explicated for 39 airports in Spain in applied analysis of the proposed approaches.

Keywords*---DEA, efficiency, desirable and undesirable output, leader-follower game theory, concentrated model.*

Introduction

DEA is the science of mathematical planning based on the performance of the decision-making units. In most DEA articles, a two-stage network structure is considered with desirable scales for the first stage's outputs to be used as the second stage's inputs [1]. In many real-life situations, the middle sections have desirable and undesirable outputs. This issue has recently been taken into account by DEA researchers. DEA was started by Charnes et al. [2] and expanded by Bengar et al. [3]. Currently, DEA plays an essential role in the analysis of undesirable outputs. The modeling's undesirable factors have been considered

Manuscript submitted: 9 April 2022, Manuscript revised: 18 June 2022, Accepted for publication: 27 July 2022

not only for assessing efficiency and productivity but also for doing research about, saying, pollution reduction. This approach has been challenged in the writings by Hailu and Veeman [4], Grosskopf and Fare [5] and, also Hailu [6] and Kuosmanen [7]. Based on the traditional approach, weak disposability modeling (reduction of the undesirable outputs by lowering the production activities) has been proposed by Shephard [8], who uses the unit abatement factor for all activities observed in a sample. Kuosmanen [7] points out that applying an integrated abatement factor with a concentration on reductive factors in the companies to decrease emission costs is unreasonable. Podinovski and Kuosmanen [9] created two other technologies for modeling weak disposability based on allocated convexity theory. At present, inappropriate network structures with undesirable outputs are deemed as the causes of weak disposability and, to the best of the author's knowledge, few works have been done based on DEA in this regard with the consideration of the undesirable variables in the network structure-based production systems. In real cases, the common generation of the desirable and undesirable outputs causes the overall assessment and two-stage network structure application to become problematic. In the studies by Cook et al. [10], four topics have been offered for assessing the two-stage systems: the standard DEA approach, efficiency analysis, network data envelopment analysis and game theory approaches [11, 12]. As Hwang and Kao [13] pointed out, in a topological study of standard DEA, every process is discussed as an independent system. The implementation of these two processes is shown by the common envelopment constraints in a calculation of unit system efficiency and overall system efficiency; the product is the efficiencies of both of the two stages. These two aspects have been considered in many of the researches such as those done by Zhu about international companies [14], by Seiford and Zhu about banking firms [15], by Lewis and Sexton on baseball [16] as well as by Wang et al. [17] on IT articles. Liang et al. [18] performed more precise investigations on the two-stage network structure for which a concept of the non-cooperative approach is applied. This vista is specified through Stackelberg's game theory or leader-follower game theory.

Since a decade ago, increasing daily attention has been paid to productivity and efficiency management methods. In this regard, the polluting undesirable outputs should be taken into consideration. The primary objective of this research is the application of the "weak disposability approach" for modeling a two-stage DEA network through intermediaries' undesirability measurements. The modeling's undesirable factors should be considered concerning efficiency and productivity measurements and the abatement factor for air pollution. In real cases, production accompanied by desirable and undesirable outputs causes problems in assessing the efficiency and productivity in the two-stage network structures' modeling. Shephard has used a traditional modeling approach (reducing undesirable outputs by lowering the production activity rate); he points out that the integrated reduction of production does not bring about cost reductions. This research examines the DEA approach by applying weak disposability assumption to the two-stage network structure with fundamental undesirability scales.

Weak Disposability Technology:

Modeling the production activities' undesirable outputs like emission of harmful particles into the air and energy wastage in the power plants has drawn much attention from researchers. Hailu and Veeman [4] expanded the non-parametrical productivity analysis models that include undesirable outputs. They introduced an informal uniformity condition to the technology and claimed that it could better serve the weak disposability concept in DEA. Fare and Grosskopf showed that the use of uniformity conditions for weak disposability modeling conflicts with a law in physics.

According to Hailu and Veeman [4], the outputs are denoted by $x = \{x_1, \dots, x_N\}$, good or desirable outputs by $v = \{v_1, \dots, v_N\}$, or undesirable outputs by $w = \{w_1, \dots, w_N\}$. The technology includes all the possible entities (x, v, w) as shown below:

Y={(x, v, w)} if x can generate (v, w).

Hailu and Veeman's condition of uniformity can be applied as below:

If $\acute{x} \geq x, \acute{w} \geq w, \acute{v} \geq v, (x.v.w) \in Y$, then $\acute{x}, \acute{v}, \acute{w} \in Y$ (1)

To showcase the assumption, the outputs are themselves defined as below:

$$(v, \acute{w}) \in p(x)$$

According to Shephard, outputs are weakly accessible. In case the reduction in the ratio of the outputs is found possible:

$$if\ (v, w) \in p(x) \quad and \quad 0 \leq \theta \leq 1 \quad then \quad (\theta v, \theta w) \in p(x)$$

This presumption by Fare and Primont [17] for the weak disposability of the outputs somewhat means that one of the outputs is possibly undesirable.

Assume that there are k numbers of DMUs; and, for DMU$_k$, the data related to the input vectors and the desirable and undesirable outputs include:

$$x_k = (x_{1k}, \dots, x_{Nk}) \geq 0, v_k = (v_{1k}, \dots, v_{Nk}) \geq 0, w_k = (w_{1k}, \dots, w_{Nk}) \geq 0$$

Also, assume that $x_k \neq 0$, $v_k \neq 0, w_k \neq 0$. The production technology can be presented as shown in the following formula:

$$x \in R_{+¿^{N}¿}$$

P(x)={(v, w) if x can produce (v, w) with

Definition 1: the outputs (desirable and undesirable) are weakly available if and only if $0 \leq \theta \leq 1 \quad then \quad (\theta v, \theta w) \in p(x)$then $x \in R_+^N, (\theta v, \theta w) \in p(x)$

Fare and Grosskopf [5] suggested the following technology assuming the variable ratio of output to scale for weak disposability:

$T_{FG} = \{(v, w, x) | \sum_{k=1}^{k} \theta z^k v_m^k \geq v_m, m = 1, \dots, M$ (2) $\sum_{k=1}^{k} \theta z^k w_j^k = w_j \quad j = 1, \dots, J$

$$\sum_{k=1}^{k} z^k x_n^k \leq x_n \qquad n = 1, \dots, N$$

$$\sum_{k=1}^{k} z^k = 1 \qquad z^k \geq 0, \quad 0 \leq \theta \leq 1 \ , \ k = 1, \dots, K$$

Blending the parameter θ into formula (2) depends on Shephard's definition of weak disposability. This parameter allows the simultaneous reduction of both the good and bad outputs.

However, Kuosmanen claims that there is no special justification for applying identical constriction factors to all the units. He also expresses that the correct exertion of the principle of "weak disposability" entails using a distinct constriction factor. These stated axioms are:

(A1): strong disposability between the inputs and good outputs; if $\acute{x} \geq x, 0 \leq \acute{v} \leq v, (x, v, w) \in T$, then $\acute{x}, \acute{v}, w) \in T$

(A2): weak disposability between the desired and undesired outputs; if $0 \leq \theta \leq 1, (x, v, w) \in T$, then $(\acute{x}, v\theta, w) \in T$

(A3): T is convex.

As pointed out by Kuosmanen, this model uses an identical abatement coefficient for all companies. To allow the use of a non-identical abatement coefficient for private companies, he suggests the following construction technology:

$$T_K = \{(v, w, x) | \sum_{k=1}^{k} \theta^k z^k v_m^k \geq v_m, m = 1, \dots, M \qquad (3)$$

$$\sum_{k=1}^{k} \theta^k z^k w_j^k = w_j \qquad j = 1, \dots, J$$

$$\sum_{k=1}^{k} z^k x_n^k \leq x_n \qquad n = 1, \dots, N$$

$$\sum_{k=1}^{k} z^k = 1$$

$$0 \leq \theta \leq 1 \qquad k = 1, \dots, K z^k \geq 0, \quad k = 1, \dots, K$$

It must be noted that formula (3) is a special case of (3) or $\theta^k = \theta^1$. To calculate weak disposability, the above formula introduces the abatement factor θ^k that leads to the gradual reduction of both the good and bad outputs by the same section; hence it is in accordance with Shephard's definition. It is noteworthy that this is a certain section's abatement factor, but it can also be applied for bringing about non-uniform abatement in the whole company. Resultantly, formula (3) is consistent with Shephard's assumptions and definitions of weak disposability and supports them. Free disposability of the goods' inputs and outputs has been modelled through unequal limitations according to x and v. Nonlinear T_k technology can be recovered in a rectilinear form using a simpler and more effective method.

Weak Disposability in the Process of Two-Stage Decision-Making:

A two-stage decision-making process, including mean desirable and undesirable outputs' measurements, has been introduced in this part. Imagine a two-stage production process as exhibited in figure (1).

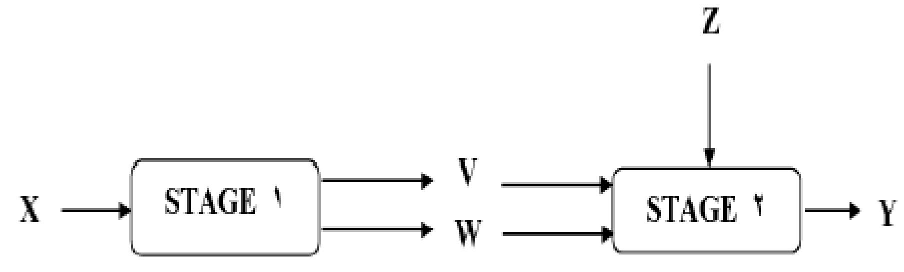

Figure (1): the overall structure of the two-stage network

Now, assume that there are k numbers of DMUs and, for the first stage, DMUs are the data observed in the input vectors regarding the desirable and undesirable outputs in the following order:

$$x_k = (x_{1k}, \dots, x_{Nk}) \geq 0, v_k = (v_{1k}, \dots, v_{Mk}) \geq 0, w_k = (w_{1k}, \dots, w_{Jk}) \geq 0$$

(v_k, w_k) are the outputs used as the inputs to the next stage. The second stage is fed with

(x_k, w_k) and an external input vector $z_k = (z_{1k}, \dots, z_{Tk})$.

The final DMU_k product is specified $y_k = (y_{1k}, \dots, y_{Tk})$. There are two different methods introduced for this decision-making process. A non-cooperative game theory was introduced in the first method, and in the second method, attention was directed at a concentrated model.

Leader-Follower Game (Play) Theory:

In this part, the leader-follower method has been developed for analyzing this expanded two-stage structure. In the non-cooperative leader-follower game theory, the leader is given a higher priority than the follower. The leader is a lot better than the follower in this case.

The algebraic model of the first stage's production technology has been given beneath:

$$T_1 = \{(v,w) | \sum_{k=1}^{k} (\rho^k + \mu^k) x_n^k \le x_n, n = 1, \dots, N \qquad (4)$$

$$\sum_{k=1}^{k} \rho^k v_m^k \ge v_m \qquad m = 1, \dots, M$$

$$\sum_{k=1}^{k} \rho^k w_j^k = w_j \qquad j = 1, \dots, J$$

$$\sum_{k=1}^{k} (\rho^k + \mu^k) = 1$$

$$\rho^k \ge 0 \qquad k = 1, \dots, K$$

$$\mu^k \ge 0, \qquad k = 1, \dots, K$$

Considering the uncertain variables μ and ρ, the above technology is linear. Attention should be paid to radial size when using this well-described model by which DMU_0 output is intended to be measured under the conditions of undesirable outputs' abatement potentials. This optimum value is obtained based on the following model:

$$e_0^{(1)} = Min\theta_0$$

$$s.t. \quad \sum_{k=1}^{k} (\rho^k + \mu^k) x_n^k \le x_n^0, n = 1, \dots, N \qquad (5)$$

$$\sum_{k=1}^{K} \rho^k v_m^k \ge v_m^0 \qquad m = 1, \dots, M$$

$$\sum_{k=1}^{k} \rho^k w_j^k = \theta_0 \ \ w_j^0 \qquad j = 1, \dots, J$$

$$\sum_{k=1}^{k} (\rho^k + \mu^k) = 1$$

$$\rho^k \ge 0 \qquad k = 1, \dots, K$$

$$\mu^k \ge 0, \qquad k = 1, \dots, K$$

The intended function minimizes the abatement factor to an equal ratio of all the undesirable outputs along with preserving the current level of the desired inputs and outputs. Model (5) is a linear programming problem and is always possible and constrained.

Considering this point, a proposed network is efficient in one stage according to the non-cooperative game theory if both stages are found separately efficient. In case of the inefficiency of DMU_0, we will have the following in the first stage:

$$\sum_{k=1}^{k}(\rho^k+\mu^k)x_n^k = x_n^0 - S_n^{(x)} \qquad (6)$$

$$\sum_{k=1}^{K}\rho^k v_m^k = v_m^0 + S_m^{(v)}$$

$$\sum_{k=1}^{k}\rho^k w_j^k = \theta_0 \quad w_j^0$$

Where, $S_n^{(x)}$ and $S_m^{(v)}$ are the first and second constraints' slack variables. Stage 1 can be improved by erasing undesirable inputs and outputs surplus and compensating for the output shortages.

It is easy to show that the improved leader is now effective. Having the first stage's output enables the evaluation of the second stage while keeping the first stage's output. Following the lead of Kuosmanen, weak disposability can be formulated in parts of numerical factor θ in all of the DMUs. Based on these presumptions, the production set $P_2(z,y)$ can be written in the following form:

$$T_2 = \{(v,w,x)| \sum_{k=1}^{k}\theta^k z^k v_m^k \geq v_m, m=1,\ldots,M \qquad (7)$$

$$\sum_{k=1}^{k}\theta^k z^k w_j^k = w_j \qquad j=1,\ldots,J$$

$$\sum_{k=1}^{k}\lambda^k y_r^k \geq y_r \qquad r=1,\ldots,S$$

$$\sum_{k=1}^{k}\lambda^k z_t^k \leq z_t \qquad t=1,\ldots,T$$

$$\sum_{k=1}^{k} z^k = 1$$

$$\lambda^k \geq 0 \qquad k=1,\ldots,K$$

$$\theta^k \geq 0, \qquad k=1,\ldots,K$$

Like the weak disposability behaviour, formula (7) uses the abatement factor θ^k , which reduces both the good and bad outputs in the same fraction based on Fare and Graskopf's definition.

Now, the very Kuosmanen's method is applied for transforming the nonlinear technology posited in (7) into a linear form.

$$\theta^k\lambda^k = \beta^k, \lambda^k(1-\theta^k) = \alpha^k$$

Then, $,\lambda^k = \beta^k + \alpha^k$ are considered. Rearranging the conditions in (7), an equivalent display of the construction technology in (7) is obtained as shown underneath:

$$\acute{T}_2 = \{(v,w)|\sum_{k=1}^{k} \beta^k v_m^k \geq v_m, m = 1, \dots, M \tag{8}$$

$$\sum_{k=1}^{k} \beta^k w_j^k = w_j \qquad j = 1, \dots, J$$

$$\sum_{k=1}^{k} (\beta^k + \alpha^k) y_r^k \geq y_r \qquad r = 1, \dots, S$$

$$\sum_{k=1}^{k} (\beta^k + \alpha^k) z_t^k \leq z_t \qquad t = 1, \dots, T$$

$$\sum_{k=1}^{k} \beta^k + \alpha^k = 1$$

$$\beta^k \geq 0 \qquad k = 1, \dots, K$$

$$\alpha^k \geq 0, \qquad k = 1, \dots, K$$

This technology is linear under the conditions that α and β variables are unclear. Based on Stackelberg's game theory for a two-stage process, the second stage only considers the favorable solutions that can keep the first stage's productive configurations. The second stage uses a tripartite (made of three components) (x, y and z) for a range within which the first stage's outputs are scored desirable. The following linear model is to be solved to evaluate the second stage.

$$e_0^{(1)} = Min\varphi_0$$

$$s.t. \qquad \sum_{k=1}^{k} \beta^k v_m^k = \sum_{k=1}^{K} \rho^k v_m^k, m = 1, \dots, M \tag{9}$$

$$\sum_{k=1}^{k} \beta^k w_j^k = \theta_j^* w_j^0 \qquad j = 1, \dots, J$$

$$\sum_{k=1}^{k} (\beta^k + \alpha^k) y_r^k \geq y_r^0 \qquad r = 1, \dots, S$$

$$\sum_{k=1}^{k} (\beta^k + \alpha^k) z_t^k \leq \varphi_t z_t^0 \qquad t = 1, \dots, T$$

$$\sum_{k=1}^{k} \beta^k + \alpha^k = 1$$

$$\beta^k \geq 0 \qquad k = 1, \dots, K$$

$$\alpha^k \geq 0, \qquad k = 1, \dots, K$$

In this stage, the m-th desirable output, i.e. $\sum_{k=1}^{K}\rho^k v_m^k$, uses j-th undesirable output, i.e. $\theta_j^* w_j^0$, as a constant. These constants are the first stage's DMU_0 outputs. Therefore, the right side of the two first formulas in the model (9) can keep the first stage's output. It must be noted that the system is effective if and only if the two components' processes are effective.

Concentrated Model:

Under real-world circumstances, there are many cases in which sub-DMUs work together to achieve a comprehensive system's overall performance. For example, the marketing and construction departments cooperate to maximize the company's profits. This section considers a concentrated method for evaluating the relative performance of two-stage structures. In these methods, the two-stage process is envisioned as a one-stage process that continuously estimates an optimal plan for maximizing the overall output of the whole system. The mean size is an undesirable output that should be abated in both stages. Although the mean size is the first stage's desirable output to be inputted into the second stage, it might logically seem that it has to have been increased in the first stage and reduced in the second stage. Considering the good outputs, v, there are two logical behaviors:

i) One can remain unchanged because it has to be increased on the one side and reduced simultaneously on the other side;

ii) The mean size is a system's good product that its system can apply.

Thus, it appears that it can be used for the whole system to increase the desirable output v. Herein, the author has used the second method for evaluating the two-stage process. In case that it is intended to measure the performance of DMU_0 regarding bringing about reductions in the undesirable outputs' potentials and decreases in outputs' potentials, the following linear programming problem should be solved:

$$Min\ e_0 = \frac{1}{2}[\frac{1}{N+j}\left[\sum_{n=1}^{N}\beta_n + \sum_{j=1}^{J}\theta_j\right] + \frac{1}{J+T}\left[\sum_{j=1}^{J}\theta_j + \sum_{t=1}^{T}\varphi_t\right]]$$

S.T.

Stage One's Constraints:

$$\sum_{k=1}^{k}(\rho^k + \mu^k)x_n^k \le \beta_n x_n^0 \qquad n = 1, \dots, N$$

$$\sum_{k=1}^{K}\rho^k v_m^k \ge v_m^0 \qquad m = 1, \dots, M$$

$$\sum_{k=1}^{k}\rho^k w_j^k = \theta_j\ w_j^0 \qquad j = 1, \dots, J$$

S.T.
Stage Two's Constraints:

$$\sum_{k=1}^{K} \rho^k v_m^k \geq v_m^0 \qquad m = 1, \dots, M$$

$$\sum_{k=1}^{k} \rho^k w_j^k = \theta_j \ w_j^0 \qquad j = 1, \dots, J$$

$$\sum_{k=1}^{k} (\rho^k + \mu^k) y_r^k \geq y_r^0 \qquad r = 1, \dots, S \sum_{k=1}^{k} (\rho^k + \mu^k) z_t^k \leq \varphi_t z_t^0 \qquad t = 1, \dots, T$$

Generic Constraints:

$$\sum_{k=1}^{k} (\rho^k + \mu^k) = 1$$

$$\rho^k \geq 0 \qquad k = 1, \dots, K$$

$$\mu^k \geq 0, \quad k = 1, \dots, K$$

$$0 \leq \theta_j, \beta_n, \varphi_t \leq 1 \quad \forall j, n, t$$

In the above formulas $0 \leq \theta_j, \beta_n, \varphi_t \leq 1$ are constraints of the interventions required for remaining dominant. The intended function can be decomposed into two periods: the first period is $[\frac{1}{N+j}\left[\sum_{n=1}^{N} \beta_n + \sum_{j=1}^{J} \theta_j\right]$ which is Russell's input size, the first stage's bad output, and the second period is $\frac{1}{J+T}\left[\sum_{j=1}^{J} \theta_j + \sum_{t=1}^{T} \varphi_t\right]$, the second stage's size.

Final Undesirable Outputs:

This section considers the concentrated method for evaluating the temporal two-stage process's performance with the undesirable outputs as the final output. As seen in figure (2), the first stage's desirable size has been used for the second stage. The mean undesirable size of the system is left unchanged in this stage. Therefore, in this method, improving the first stage's output through increasing the number of the outputs influences the second stage's output. In other words, the size quantities from the mean to the optimal are considered the final output and are not viewed as the second stage's inputs.

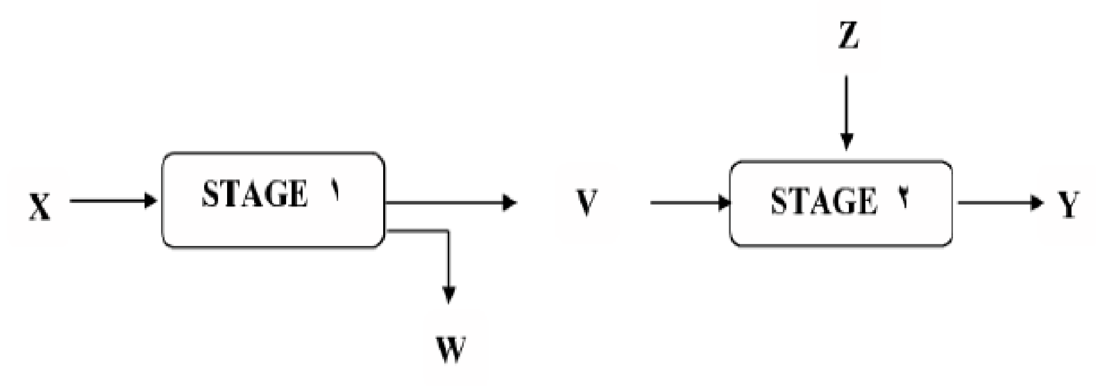

Figure (2): two-stage network structure with final undesirable outputs

The process can be seen in situations like evaluating the relationships between micro-and macro-level productions in the supply chain management methods or airport modelling operations. The use of DEA for modelling such situations decreases the inputs, increases the desirable outputs, and decreases the undesirable outputs. Therefore, the upcoming sections present a concentrated DEA network model with variable output ratio to scale and weak disposability attributes for optimal final outputs. The first stage of this model takes the following form:

$$Min\ e_0 = \frac{1}{2}[\frac{1}{N+j}\left[\sum_{n=1}^{N}\beta_n + \sum_{j=1}^{J}\theta_j\right] + \frac{1}{T}[\sum_{t=1}^{T}\varphi_t]] \quad (11)$$

S.T.

Stage One's Constraints:

$$\sum_{k=1}^{k}(\rho^k + \mu^k)x_n^k \leq \beta_n x_n^0 \qquad n = 1, \ldots, N$$

$$\sum_{k=1}^{K}\rho^k v_m^k \geq v_m^0 \qquad m = 1, \ldots, M$$

$$\sum_{k=1}^{k}\rho^k w_j^k = \theta_j\ \ w_j^0 \qquad j = 1, \ldots, J$$

S.T.

Stage Two's Constraints:

$$\sum_{k=1}^{K}\rho^k v_m^k \geq v_m^0 \qquad m = 1, \ldots, M$$

$$\sum_{k=1}^{k}\rho^k w_j^k = \theta_j\ \ w_j^0 \qquad j = 1, \ldots, J$$

$$\sum_{k=1}^{k}(\rho^k+\mu^k)z_t^k \leq \varphi_t z_t^0 \qquad t=1,\dots,T$$

$$\sum_{k=1}^{k}(\rho^k+\mu^k)y_r^k \geq y_r^0 \qquad r=1,\dots,S$$

Generic Constraints:

$$\sum_{k=1}^{k}(\rho^k+\mu^k)=1$$

$$\rho^k \geq 0 \qquad k=1,\dots,K$$

$$\mu^k \geq 0, \qquad k=1,\dots,K$$

$$0 \leq \theta_j,\beta_n,\varphi_t \leq 1 \quad \forall j,n,t$$

In the above formulas, the constraints $0 \leq \theta_j,\beta_n,\varphi_t \leq 1$ are the requirements for gaining domination. The objective function can be attached to the middle of Amin in the Thatcher Section: the first part is Russell's input size and the first stage's undesirable output as the input to the second stage; the second part is Russell's input size for the second stage.

Problem Solving:

To specify the practical concept of the proposed methods, we will apply methods to real cases like 39 Spanish airports in 2008, as cited in Lozano et al.

After formulating the method's framework, it has to be showcased through empirical methods. It has been found in an analysis of the productivity based on parametric and nonparametric techniques that the airport's performance is suffering now for a long time, with the initial research interests having been incited by the high socioeconomic significance of the issue.

The existing DEA studies about the airport's framework have considered an airport a single process. The slack-based DEA network method has been proposed by Yu et al., but it does not consider the undesirable outputs. As stated by Lozano et al., consideration of undesirable outputs increases the analysis ratio and ends in a fairer performance evaluation. This research has been motivated by testing the application of disposability in DEA network modelling based on mean undesirable sizes. To reach a second suggestion, the methods posited in a case study of 39 Spanish airports have been taken into account, as cited in Lozano et al.

Similar to prior research on airport performance evaluation, airport processes are divided into two stages:

Airplane movement process

Airplane loading process

The three inputs of the first stage are seminally specified: the whole of a region's airport (x_1), the precinct's capacity (x_2) and the number of exit gates (x_3). As for the second stage's output, annual passenger motions (y_1) and the number of landing times (y_2) have been considered. Amongst the inputs added for the second stage, the number of luggage belts (z_1) and the number of check-in-out counters (z_2) can be pointed out. The cross-mean of the airplanes' takeoff and landing numbers has been considered as the mean desirable size, the mean number of the delayed flights (w1), and the density of passengers until the announcement of the second flight (w_2) have been considered as the mean undesirable sizes. As discussed by Lozano et al., these mean w_1 and w_2 sizes are the first stage's final outputs. The important point of this research and the primary difference between the method proposed herein and the other methods is that this research's proposed method considers that the number of delayed flights and the density of delayed flights' passengers in the airport directly influence the second stage (airport loading process). These undesirable outputs have been used herein as the inputs of the second stage. To see how the weak disposability assumption influences the two-stage structures, both the cooperative and non-cooperative game methods have been considered. At first, the leader-follower game method proposed in the related articles was applied, and it is assumed that the leader should be the airplane movement process.

The following Table presents the first stage's desirable scores as drawn on the non-cooperative method posited herein in leader-follower game theory with desirable amounts for the objective points.

Table 1
The result of the stage one's efficiency and planning evaluation

Airport	$e1a$	v_1	w_1	w_2	x_1	x_2	x_3
Acoruna	0.3251	17.7190	395.9391	7731.3225	78375.8286	5.000	4.000
Albacete	0.2014	2.1130	11.6833	277.2762	112666.8227	2.000	1.9939
Alicante	0.5564	81.0970	4242.3424	79263.3603	135000.000	25.7824	16.000
Almeria	0.2688	18.2800	399.4928	5416.9756	80251.9378	7.000	3.6089
Asturias	0.2728	18.3710	357.4269	6519.2202	76282.5283	7.000	3.6089
Badajoz	1	4.0330	137.0000	2365.4000	171000.000	1.000	2.000
Barcelona	1	321.6930	33036.000	645924.6000	475020.0000	121.000	65.0000
Bilbao	0.3730	61.6820	1713.0203	30159.9749	133926.9648	13.8523	11.9293
Cordoba	1	9.6040	14.0000	254.4000	62100.000	23.000	1.000
El Hierro	1	4.07750	27.0000	641.6000	37500.000	3.000	2.000

Fuerteventura	0.2782	44.5520	1090.7215	20083.6616	116641.8814	18.8145	10.000
Girona-Costa Brav	0.3481	49.9270	1737.7146	34.916.3675	108000.000	12.6857	6.6387
Canaria Gran.	1	116.2520	7463.00000	136380.7000	139500.000	55.000	38.000
Granada-Jaen	0.4368	19.2790	415.3811	7804.7963	711154.5099	11.000	3.000
Ibiza	1	572330	6193.000	152840.1000	126000.000	25.000	12.000
Jerez	1	50.5510	1174.0000	19292.2000	103500.000	9.000	5.000
La Gomera	1	3.3930	17.0000	420.7000	45000.000	3.000	2.000
La Palma	1	20.1090	423.0000	8286.0000	99000.000	5.000	5.000
Lazarote	0.3736	53.3750	1906.9521	37991.6874	108000.000	11.9547	6.3978
Leon	0.1344	5.7050	59.4138	966.6842	66542.6079	5.000	1.9039
Madrid Barajas	1	469.7460	52526.0000	903860.0000	927000.000	263.000	230.000
Malaga	1	119.8210	15548.0000	277663.8000	144000.000	43.000	30.000
Melilla	1	109590	218.000	2979.6000	64260.000	5.000	2.000
Murcia	0.2925	19.3390	395.77226	7097.7288	73638.7112	5.000	3.7105
Palmade Mallorca	0.6552	193.3790	17060.9591	328590.2194	267737.2523	78.9153	47.0421
Pamplona	1	12.9710	666.0000	11691.8000	99315.000	7.000	2.000
Reus	1	26.6760	943.0000	18240.8000	110475.000	5.000	5.000
Salamanca	1	12.4500	427.0000	6626.1000	150000.000	6.000	2.000
San Sebastian	0.3076	12.2820	219.3020	3439.9342	57029.6777	6.000	2.2220
Santander	0.3691	19.1980	370.6255	6586.3551	73730.1553	8.00	3.2718
Santiago	0.1918	21.9250	382.9760	6583.5885	79197.0276	16.000	2.7566
Saragossa	0.2023	14.5840	221.5252	3954.9568	65860.7582	12	2.2826
Seville	0.9956	65.0670	2555.6539	50859.1052	120646.3647	17.6125	10
Tenerife North	1	67.8000	1783.0000	32637.0000	153000.000	16.000	16.000
Tenerife	0.44	60.779	2357.03	49715.24	140286.7	18.13	13.97

South	86	0	39	72	337	09	79
Valencia	1	96.795 0	4998.00 00	102719.2 00	144000.0 00	35.00 0	18.00 0
Valladolid	0.25 36	13.002 0	213.812 3	3743.770 2	54999.44 63	7.000	2.491 3
Vigo	0.21 89	17.934 0	236.052 8	5603.127 0	60209.18 49	8.000	2.652 5
Vitoria	0.17 51	12.225 0	117.117 9	2028.257 5	64601.99 37	18.00 0	1.615 5

Similar results have been reported for the second stage, as exhibited in Table (2).

Table 2
Results of the second stage's efficiency and planning evaluation

Airport	$e2\alpha$	z_1	z_2	y_1	y_2
Acoruna	0.4607	4.6072	0.6493	1174.97	283.571
Albacete	0.2504	1.0015	0.0005	22.2416	8.924
Alicante	0.9174	38.5329	5.6282	9578.304	17236.65
Almeria	0.2617	4.4384	0.6436	1024.303	35.593
Asturias	0.5167	5.6841	0.8312	1530.245	139.465
Badajoz	0.2955	1.1821	0.0321	81.01	1.9719
Barcelona	1	143	19	30272.08	103996.5
Bilbao	0.5672	20.41197	3.6713	4172.903	4486.831
Cordoba	1	1	0	22.23	0
El Hierro	0.3123	1.5614	0.1077	195.425	171.717
Fuerteventura	0.4631	15.745	2.7926	4492.003	2722.661
Girona-Costa Brava	1	18	3	5510.97	184.127
Gran Canaria	0.5619	48.324	7.1008	10212.12	33695.25
Granada-Jaen	0.4449	5.3385	0.7667	1422.014	66.889
Ibiza	0.4165	19.9924	3.3321	4647.36	4992.297
Jerez	1	12	2	1203.817	90.428
La Gomera	0.2124	1.062	0.0113	41.89	7.863
La Palma	0.406	5.2782	0.812	1151.357	1277.264
Lanzarote	0.4055	19.8717	3.2444	5438.178	5429.589
Leon	0.4382	1.3146	0.0562	123.183	15.979

As shown in Table (2), 17 airports are found effective in the first stage when the airplane movement process is considered the leader. Preserving these effective airports for the second stage, seven other airports are found effective in the second stage. Having a glance at the second column in Tables (1) and (2) makes it clear that only four airports are efficient in the general sense of the term (Barcelona, Cordoba, Jerez and Madrid Airports). It is seen in the observation of columns 4 and 5 in Table (1) that the first and second undesirable outputs have undergone reductions, respectively, for 973.05 and 18441.61.

A concentrated model [10] has also been used for the airport data according to the results reported in tables (3) and (4). The first three columns in Table (3) report the outputs' scores in the course of the stages. The seven last columns of this Table show the influence factors. As seen therein, out of 39 airports, 24 are completely efficient. It is discerned in a test of columns 6 and 7 in Table (1), columns 4 and 5 in Table (2) and columns 6 and 7 in Table (4) that the intermediaries' reductions in the first and second undesirable outputs' abatement stages are considerably larger in non-cooperative method than the intermediaries' reductions through the use of the cooperative method meaning that, in this example, the non-cooperative method, significantly reduces the undesirable outputs.

We use the concentrated model for the airport data based on the results reported in tables (3) and (4). The first three columns in these tables give the outputs' scores during the stages. The seven last columns of these tables display the influence factors.

Table 3
Total output along with the total results

Airport	eα(total)	eα(1)	eα(2)	β1	β2		θ1β3	θ2	φ2	φ1
Acoruna	0.7654	0.8383	0.6925	1	1	1	0.6	0.59	0.82	0.76
Albacete	0.7698	0.7858	0.7537	0.91	1	0.99	0.65	0.38	1	0.99
Alicante	1	1	1	1	1	1	1	1	1	1
Almeria	0.4154	0.4573	0.3735	0.53	0.47	0.62	0.36	0.3	0.43	0.2
Asturias	0.5718	0.5764	0.5671	0.82	1	0.48	0.3	0.49	1	0.68
Badajoz	1	1	1	1	1	1	1	1	1	
Barcelona	1	1	1	1	1	1	1	1	1	1
Bilbao	0.6734	0.7017	0.6451	0.65	0.8	1	0.52	0.52	0.81	0.71
Cordoba	0.875	1	0.75	1	1	1	1	1	1	0
El Hierro	1	1	1	1	1	1	1	1	1	1
Fuerteventura	0.5945	0.6513	0.5376	0.84	0.56	1	0.22	0.44	0.72	0.58
Girona-Costa Brava	1	1	1	1	1	1	1	1	1	1
Gran Canaria	1	1	1	1	1	1	1	1	1	1
Granada-Jaen	1	1	1	1	1	1	1	1	1	1
Ibiza	0.5885	0.68	0.4971	1	0.69	0.92	0.45	0.33	0.57	0.63
Jarez	1	1	1	1	1	1	1	1	1	1
La Gomera	1	1	1	1	1	1	1	1	1	1
La Palma	1	1	1	1	1	1	1	1	1	1
Lanzarote	0.7394	0.790	0.688	1	0.8	0.6	0.7	0.7	0.5	0.7

		3	4			8	6	1	7	1
Leon	1	1	1	1	1	1	1	1	1	1
Madrid Barajas	1	1	1	1	1	1	1	1	1	1
Malaga	1	1	1	1	1	1	1	1	1	1
Melilla	1	1	1	1	1	1	1	1	1	1
Murcia	1	1	1	1	1	1	1	1	1	1
Palma De Mallorca	1	1	1	1	1	1	1	1	1	1
Pamplona	1	1	1	1	1	1	1	1	1	1
Reus	1	1	1	1	1	1	1	1	1	1
Salamanca	1	1	1	1	1	1	1	1	1	1
San Sebastian	0.5552	0.6206	0.4897	0.84	0.88	0.72	0.35	0.32	0.74	0.55
Santander	0.5968	0.5866	0.607	0.73	0.83	0.6	0.42	0.36	0.86	0.79
Santiago	0.4022	0.4279	0.3764	0.66	0.64	0.36	0.25	0.23	0.57	0.45
Saragossa	1	1	1	1	1	1	1	1	1	1
Seville	1	1	1	1	1	1	1	1	1	1
Tenerife North	1	1	1	1	1	1	1	1	1	1
Tenerife South	0.7181	0.7946	0.6415	1	0.75	1	0.65	0.57	0.6	0.74
Valencia	1	1	1	1	1	1	1	1	1	1
Valladolid	0.439	0.2384	0.4396	0.37	0.78	0.46	0.32	0.26	0.6	0.58
Vigo	0.5056	0.5652	0.446	0.75	1	0.58	0.26	0.24	0.69	0.6
Vitoria	1	1	1	1	1	1	1	1	1	1

Table (4) presents the results of the concentrated points.

Table 4
Results of the concentrated points

Airport	x_1^*	x_2^*	x_3^*	v_1^*	w_1^*	w_2^*	z_1^*	z_2^*	y_1^*	y_2^*
Acoruna	87.300	5	4	17.72	727.78	14132.09	8.23	2.27	1174.97	283.57
Albacete	147138.4	2	1.99	2.11	37.51	523.12	4	0.99	123.38	70.59
Alicante	135.000	31	16	81.1	7624	142445.8	42	9	9578.3	5982.31
Almeria	76773.82	7	3.12	18.28	399.26	6143.86	7.25	1.62	1024.3	304.26
Asturias	80824.01	7	4.3	18.37	390.88	6914.48	11	2.04	1530.24	472.33
Badajoz	171.000	1	2	4.03	137	2365.4	4	1	81.01	0

Barcelona	475.020	121	65	321.69	32.036	645924.6	143	19	30272.08	103996.5
Bilbao	3.213354	16.77	12	61.68	3410.8	43649.86	29.04	4.96	4173.9	10836.53
Cordoba	62.100	23	1	9.6	14	254.4	1	0	22.23	0
El Hierro	37.500	3	2	4.78	27	641.6	5	1	195.43	171.72
Fuerteventura	129137.3	19.07	10	42.55	1706.24	30046.04	22.53	4.62	4492	6765.66
Girona-Costa Brava	108.000	17	7	49.93	4992	100305.6	18	3	5510.97	184.13
Gran Canaria	139.500	55	38	116.25	7463	136380.7	86	19	10212.12	33695.25
Granada-Jaen	134.550	11	3	19.28	951	17868.8	12	3	1422.01	66.89
Ibiza	126.000	17.3	11.09	57.23	2788.87	80977.44	27.58	8.02	4647.36	7518.65
Jarez	103.500	9	5	50.55	1174	19292.2	13	3	1303.82	90.43
La Gomera	45.000	3	2	3.39	17	420.7	5	1	41.89	7.86
La Palma	99.000	5	5	20.11	423	8286	13	2	1151.36	1277.26
Lanzarote	108.000	19.12	10.94	53.38	3885.92	72199.4	28.05	5.68	5438.18	5429.59
Leon	94.500	5	2	5.7	442	7191.5	3	1	123.18	15.98
Madrid Barajas	927.000	263	230	269.75	52.526	908360	484	53	50826.49	329186.6
Malaga	144.000	43	30	119.82	15.548	2777663.8	85	16	12813.47	4800.27
Melilla	64.260	5	3	10.96	218	2979.6	4	1	314.62	386.34
Murcia	138.000	5	5	19.34	1344	24103.1	18	4	1876.26	2.73
Palma De Mallorca	295.650	86	68	193.38	26.038	501486	204	16	22832.86	21395.79
Pamplona	99.315	7	2	12.97	666	11691.8	4	1	434.48	52.94
Reus	110.475	5	5	26.68	943	18240.8	8	3	1278.07	119.85
Salamanca	150.000	6	2	12.45	427	6626.1	4	2	60.1	0
San Sebastian	66125.17	5.27	2.16	12.28	250.42	3540.28	4.47	1.09	403.19	373.57

Santander	75780.8	6.64	2.98	19.2	419.81	6468.58	6.89	1.57	856.61	307.3
Santiago	94974.94	10.18	4.36	21.94	497.53	7975.6	10.84	2.27	1917.47	2418.8
Saragossa	302.310	12	3	14.58	1095	19547.6	6	2	594.95	21438.89
Seville	151.200	23	10	65.07	2567	51084.9	42	6	4392.62	6102.26
Tenerife North	153.000	16	16	67.8	1783	22.637	37	5	4236.62	20781.67
Tenerife South	144.000	23.21	22	60.78	3419.17	62889.89	52.29	10.42	8251.99	17311.92
Valencia	144.000	35	18	96.8	4998	102719.2	42	8	5779.34	13225.8
Valladolid	67422.79	5.48	2.28	13	268.3	3853.36	4.81	1.16	479.69	365.15
Vigo	81426.74	8	3.46	17.93	392.26	6140.41	8.29	1.79	1278.86	1481.94
Vitoria	157500	18	3	12.23	669	11585.8	7	2	67.82	34989.73

Conclusion

The analysis of the two-stage network structure efficiency has drawn many DEA researchers' attention in recent years. The extant studies on the network DEA demonstrate the existence of undesirable and desirable products in two-stage processes. The previous approaches in network DEA could not evaluate the efficiency properly. The present study used a two-stage DEA approach for analyzing the implementation of these processes along with the undesirable intermediary parameters. Two different two-stage structures were considered and used in every structure for different cases (one with harmful outputs as the final outputs and the other with harmful outputs as the intermediary parameters). The present article's quotient of the research body in this field is its application of weak disposability assumption in two-stage processes in case of desirable and undesirable outputs. The approach above has been explicated through a set of real data procured about 39 Spanish airports for 2008.

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