Utilization of interval-valued intuitionistic fuzzy soft set in the health sector

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Abstract---The soft set theory plays a key role for dealing with uncertainty, fuzziness and vagueness. The concept of fuzzy soft set which can be seen as a new mathematical approach to vagueness is used in many applications including reliability evaluation, multi criteria decision making and medical diagnosis problems. Later, it is generalized to an interval-valued intuitionistic fuzzy soft set. In this attempt we present the definition and operations of an interval-valued intuitionistic fuzzy soft set. Furthermore, based on the analysis of several operations on interval-valued intuitionistic fuzzy soft set in the study, we provide some notions such as the restricted intersection and restricted union of two interval-valued intuitionistic fuzzy soft sets for selection of appropriate hospital for patient affected by specific diseases.

Keywords---soft set, intuitionistic fuzzy soft set, interval-valued intuitionistic fuzzy soft set, restricted union, restricted intersection.

Introduction

(Zadeh, L. A., 1965) introduced fuzzy set theory, for managing imprecise and vague information, where vagueness is reflected by the membership degree of the objects belonging to a concept. After that an intuitionistic fuzzy set (IFS) was introduced by (Atanassov, K.T., 1986), which allow to unified simultaneously the membership degree and the non-membership degree of each element. For dealing with uncertainty, Molodtsov initiated the concept of soft set theory, which can be used as an important mathematical tool. Soft set theory is different from many traditional tools for dealing with uncertainties, such as the theory of fuzzy sets (Zadeh, L. A., 1965), the theory of intuitionistic fuzzy sets (Atanassov, K.T., 1986) (Atanassov, K. et al., 1989) and the theory of rough sets (Pawlak, Z., 1991), is...
that it is free from the inadequacy of the parameterization tools of those theories (Yang, X. B., et al. 2009). The basic concept and properties of soft set theory are presented in (Molodtsov, D., 1999). Several operations on soft sets introduced in (Maji, P. K. et al. 2003). (Ali et al. 2009) present some new algebraic operations for soft sets and prove that certain De Morgan's laws hold in soft set theory with respect to these new definitions. It has been demonstrated that soft set theory brings about a rich potential for applications in many fields such as decision making problem in health sector, function smoothness, Reliability theory, Riemann integration, measurement theory, game theory, etc.(Xu, W. et al. 2010).

(Maji, P. K. et al., 2001 and 2003) have done a theoretical study on the soft set theory in more detail and contributed towards the fuzzification of the notion of it and described the application of soft set theory to a decision making problem using rough sets. Recently (Kong, Z. et al., 2008 and 2009) applied the soft set theoretic approach in decision making problems. Soft fuzzy set was studied by (Yao et al., 2008) followed by intuitionistic fuzzy soft set defined by (Xu Yong et al., 2010). Alkhazaleh et al. 2011, examined the concept of fuzzy parameterized interval-valued fuzzy soft set and gave its application in decision making. Alkhazaleh et al., 2011 studied soft multisets as a generalization of Molodtsov's soft set and proposed the concept of possibility fuzzy soft set. Maji et al. 2001 and 2004 presented the notion of the intuitionistic fuzzy soft set theory which is based on a combination of the intuitionistic fuzzy set (Atanassov, K.T. 1986) and soft set models. Yang et al. 2009, presented the concept of the interval-valued fuzzy soft sets by combining the interval-valued fuzzy set and soft set models.

The purpose of this paper is to combine the interval-valued intuitionistic fuzzy sets (Atanassov, K. et al. 1994) and soft sets, from which we can obtain a new soft set model: interval-valued intuitionistic fuzzy soft set theory which we have used for appropriate hospital selection problem. Intuitively, interval-valued intuitionistic fuzzy soft set theory presented in this paper is an interval-valued fuzzy extension of the intuitionistic fuzzy soft set theory or an intuitionistic fuzzy extension of the interval-valued fuzzy soft set theory (Alkhazaleh, et al., 2011).

In the present paper we introduced some concepts about an interval-valued intuitionistic fuzzy soft set for solving appropriate hospital selection problem. There has been incredible interest in the subject due to its diverse applications, ranging from engineering and health sector to social behaviour studies. Here we define an interval-valued intuitionistic fuzzy soft set and its operations of union, intersection and complement operators. We also provide some notions such as the restricted union and restricted intersection of two interval-valued intuitionistic fuzzy soft sets for selection of appropriate hospital.

**Preliminaries**

**Definition 2.1.** Let $U$ be an initial set and $E$ be a set of parameters. Let $P(U)$ denotes the power set of $U$ and $A \subseteq E$ then a pair $(F, A)$ is called a soft set over $U$ if $F$ is a mapping given by $F : A \rightarrow P(U)$. 
Definition 2.2. Let $U$ be an initial set and $E$ be a set of parameters. Let $F(U)$ denotes the fuzzy power set of $U$ and $A \subseteq E$ then a pair $(F, A)$ is called a fuzzy soft set over $U$ if $F$ is a mapping given by $F : A \rightarrow F(U)$.

Definition 2.3. Consider $U$ and $E$ as a universe set and a set of parameters respectively. Let $IFS(U)$ denotes the intuitionistic fuzzy power set of $U$ and $A \subseteq E$ then a pair $(F, A)$ is an intuitionistic fuzzy soft set over $U$ if the mapping $F$ is given by $F : A \rightarrow IFS(U)$.

By combining the interval-valued intuitionistic fuzzy sets (Atanassov, K.T., 1986, 1989 and 1994) and soft sets, it is natural to define the interval-valued intuitionistic fuzzy soft set model. First, let us briefly introduce the concept of the interval-valued intuitionistic fuzzy sets. Interval-valued intuitionistic fuzzy set (IVIFS) was first introduced by Atanassov and Gargov, 1994. It is characterized by an interval-valued membership degree and an interval-valued non-membership degree.

Definition 2.4. An interval-valued intuitionistic fuzzy set is an object of the form $A = \left\{ (x, \mu_a(x), \nu_a(x)) \right\}_{x \in X}$, where $\mu_a(x) : X \rightarrow \text{Int}([0,1])$ and $\nu_a(x) : X \rightarrow \text{Int}([0,1])$, $(\text{Int}([0,1]))$ stands for the set of all closed subintervals of $[0,1]$, satisfy the following conditions:

$$\forall x \in X, \sup \mu_a(x) + \sup \nu_a(x) \leq 1.$$ 

The class of all IVIFS on $X$ will be denoted by IVIFS $(X)$.

Interval-valued intuitionistic fuzzy soft set

Definition 3.1. Let $U$ be an initial universe and $E$ be a set of parameters. IVIFS $(U)$ denotes the set of all interval-valued intuitionistic fuzzy sets of $U$. Let $A \subseteq E$. A pair $(F, A)$ is an interval-valued intuitionistic fuzzy soft set over $U$, where $F$ is a mapping given by $F : A \rightarrow \text{IVIFS} (U)$. In other words, an interval-valued intuitionistic fuzzy soft set is a parameterized family of interval-valued intuitionistic fuzzy subsets of $U$, its universe is the set of all interval-valued intuitionistic fuzzy sets of $U$, i.e., IVIFS $(U)$. An interval-valued intuitionistic fuzzy soft set is also a special case of a soft set because it is a still a mapping from parameters to IVIFS $(U)$. For any parameter $\varepsilon \in A$, $F(\varepsilon)$ is referred as the interval intuitionistic fuzzy value set of parameter $\varepsilon$, it is actually an interval-valued intuitionistic fuzzy set of $U$ where $x \in U$ and $\varepsilon \in A$, it can be written as:

$$F(\varepsilon) = \left\{ (x, \mu_{F(\varepsilon)}(x), \nu_{F(\varepsilon)}(x)) \right\},$$

where $\mu_{F(\varepsilon)}(x)$ is the interval-valued fuzzy membership degree that object $x$ holds on parameter $\varepsilon$, $\nu_{F(\varepsilon)}(x)$ is the interval-valued fuzzy membership degree that object $x$ does not hold on parameter $\varepsilon$. Now we define some operations on interval-valued intuitionistic fuzzy soft sets.
Definition 3.2. Complement of interval valued intuitionistic fuzzy soft set

Let $E = \{e_1, e_2, ..., e_n\}$ be a parameter set. The not set of $E$ denoted by $-E = \{-e_1, -e_2, ..., -e_n\}$ where $-e_i = \text{not } e_i$.

The complement of an interval-valued intuitionistic fuzzy soft set soft $\langle F, A \rangle$ is denoted by $\langle F, A \rangle'$ and is defined by $\langle F, A \rangle' = \langle F', -A \rangle$ where $F' : -A \rightarrow \text{IVIFS} (U)$ is a mapping given by $F' (e) = \left\{ \left( x, \nu_{F_{\lvert e}} (x), \mu_{F_{\lvert e}} (x) \right) \right\}$ for all $x \in U$ and $e \in -A$.

Definition 3.3. Union of two interval-valued intuitionistic fuzzy soft sets

The union of two interval-valued intuitionistic fuzzy soft sets $\langle F, A \rangle$ and $\langle G, B \rangle$ over a common universe $U$ is an interval-valued intuitionistic fuzzy soft set $\langle H, C \rangle$, where $C = A \cup B$ and $\forall e \in C$,

\[
\mu_{\mu(e)} (x) = \begin{cases} 
\mu_{F_{\lvert e}} (x), & \text{if } e \in A - B, x \in U \\
\mu_{G_{\lvert e}} (x), & \text{if } e \in B - A, x \in U \\
\sup \left( \mu_{F_{\lvert e}} (x), \mu_{G_{\lvert e}} (x) \right), & \text{if } e \in A \cap B, x \in U,
\end{cases}
\]

\[
\nu_{\mu(e)} (x) = \begin{cases} 
\nu_{F_{\lvert e}} (x), & \text{if } e \in A - B, x \in U \\
\nu_{G_{\lvert e}} (x), & \text{if } e \in B - A, x \in U \\
\inf \left( \nu_{F_{\lvert e}} (x), \nu_{G_{\lvert e}} (x) \right), & \text{if } e \in A \cap B, x \in U.
\end{cases}
\]

we denoted it by $\langle F, A \rangle \cup \langle G, B \rangle = \langle H, C \rangle$.

Definition 3.4. Intersection of two interval-valued intuitionistic fuzzy soft sets

The intersection of two interval-valued intuitionistic fuzzy soft sets $\langle F, A \rangle$ and $\langle G, B \rangle$ over a common universe $U$ is an interval-valued intuitionistic fuzzy soft set $\langle H, C \rangle$, where $C = A \cup B$ and $\forall e \in C$,

\[
\mu_{\mu(e)} (x) = \begin{cases} 
\mu_{F_{\lvert e}} (x), & \text{if } e \in A - B, x \in U \\
\mu_{G_{\lvert e}} (x), & \text{if } e \in B - A, x \in U \\
\inf \left( \mu_{F_{\lvert e}} (x), \mu_{G_{\lvert e}} (x) \right), & \text{if } e \in A \cap B, x \in U,
\end{cases}
\]

\[
\nu_{\mu(e)} (x) = \begin{cases} 
\nu_{F_{\lvert e}} (x), & \text{if } e \in A - B, x \in U \\
\nu_{G_{\lvert e}} (x), & \text{if } e \in B - A, x \in U \\
\sup \left( \nu_{F_{\lvert e}} (x), \nu_{G_{\lvert e}} (x) \right), & \text{if } e \in A \cap B, x \in U.
\end{cases}
\]
we denoted it by \( \langle F, A \rangle \cap \langle G, B \rangle = \{ H, C \} \).

**Selection of appropriate hospital for any patient using some operations in an interval-valued intuitionistic fuzzy soft set theory**

**Definition 4.1. Restricted union**

Let \( \langle F, A \rangle \) and \( \langle G, B \rangle \) be two interval-valued intuitionistic fuzzy soft sets over a common universe \( U \) such that \( A \cap B \neq \phi \). The restricted union of \( \langle F, A \rangle \) and \( \langle G, B \rangle \) is denoted by \( \langle F, A \rangle \cup_{r} \langle G, B \rangle \) and is defined as \( \langle F, A \rangle \cup_{r} \langle G, B \rangle = \{ H, C \} \) where \( C = A \cap B \) and \( \varepsilon \in C, H_{\varepsilon} = F(\varepsilon) \cup G(\varepsilon) \),

\[
\mu_{H_{\varepsilon}}(x) = \sup \left( \mu_{F(\varepsilon)}(x), \mu_{G(\varepsilon)}(x) \right), \\
\nu_{H_{\varepsilon}}(x) = \inf \left( \nu_{F(\varepsilon)}(x), \nu_{G(\varepsilon)}(x) \right)
\]

**Example 1**

Given two interval-valued intuitionistic fuzzy soft sets \( \langle F, A \rangle \) and \( \langle G, B \rangle \) Here \( U = \{ h_{1}, h_{2}, h_{3}, h_{4}, h_{5} \} \) is the set of hospitals, \( A \) and \( B \) are parameter sets, where \( A = \{ e_{1}, e_{2}, e_{3} \} = \{ \) expensive, availability of medicine, availability of doctors\}, \( B = \{ e_{2}, e_{3}, e_{4}, e_{5} \} = \{ \) availability of medicine, availability of doctors, in the green surroundings, emergency facility\}. Here two attributes availability of medicine and availability of doctors play key role for good treatment of any patient. So here we evaluate restricted union and restricted intersection for selection of good hospitals among all given hospitals.

\[
\begin{align*}
F(e_{1}) & = \langle h_{1}, [0.35, 0.55], [0.25, 0.45] \rangle, \langle h_{2}, [0.55, 0.6], [0.15, 0.4] \rangle, \langle h_{3}, [0.20, 0.30], [0.40, 0.70] \rangle, \\
& \quad \langle h_{4}, [0.30, 0.40], [0.50, 0.60] \rangle, \langle h_{5}, [0.20, 0.40], [0.50, 0.60] \rangle \\
F(e_{2}) & = \langle h_{1}, [0.13, 0.37], [0.41, 0.63] \rangle, \langle h_{2}, [0.15, 0.39], [0.39, 0.61] \rangle, \langle h_{3}, [0.17, 0.41], [0.37, 0.59] \rangle, \\
& \quad \langle h_{4}, [0.19, 0.43], [0.35, 0.57] \rangle, \langle h_{5}, [0.21, 0.45], [0.33, 0.55] \rangle \\
F(e_{3}) & = \langle h_{1}, [0.23, 0.47], [0.31, 0.53] \rangle, \langle h_{2}, [0.25, 0.49], [0.29, 0.51] \rangle, \langle h_{3}, [0.27, 0.51], [0.27, 0.49] \rangle, \\
& \quad \langle h_{4}, [0.29, 0.53], [0.25, 0.47] \rangle, \langle h_{5}, [0.31, 0.55], [0.23, 0.45] \rangle \\
G(e_{2}) & = \langle h_{1}, [0.23, 0.47], [0.31, 0.53] \rangle, \langle h_{2}, [0.25, 0.49], [0.29, 0.51] \rangle, \langle h_{3}, [0.27, 0.50], [0.27, 0.50] \rangle, \\
& \quad \langle h_{4}, [0.29, 0.53], [0.25, 0.47] \rangle, \langle h_{5}, [0.31, 0.45], [0.23, 0.55] \rangle \\
G(e_{3}) & = \langle h_{1}, [0.20, 0.44], [0.34, 0.56] \rangle, \langle h_{2}, [0.22, 0.46], [0.43, 0.54] \rangle, \langle h_{3}, [0.24, 0.47], [0.30, 0.53] \rangle, \\
& \quad \langle h_{4}, [0.36, 0.60], [0.18, 0.40] \rangle, \langle h_{5}, [0.28, 0.42], [0.26, 0.58] \rangle
\end{align*}
\]
G(e_4) = \{\langle h_1, [0.30, 0.54], [0.34, 0.46] \rangle, \langle h_2, [0.32, 0.56], [0.33, 0.44] \rangle, \langle h_3, [0.34, 0.57], [0.20, 0.43] \rangle,
           \langle h_4, [0.26, 0.50], [0.28, 0.50] \rangle, \langle h_5, [0.38, 0.52], [0.16, 0.48] \rangle\}

G(e_5) = \{\langle h_1, [0.27, 0.51], [0.37, 0.49] \rangle, \langle h_2, [0.29, 0.53], [0.36, 0.47] \rangle, \langle h_3, [0.31, 0.54], [0.23, 0.46] \rangle,
           \langle h_4, [0.33, 0.57], [0.21, 0.43] \rangle, \langle h_5, [0.35, 0.49], [0.19, 0.51] \rangle\}

Thus, we can view an interval-valued intuitionistic fuzzy soft set as a collection of fuzzy approximations. So restricted union

\langle H, C \rangle_R = \langle F, A \rangle \cup_R \langle G, B \rangle

\langle H, C \rangle_R = \{\text{availability of medicine} = \langle h_1, [0.23, 0.47], [0.31, 0.53] \rangle, \langle h_2, [0.25, 0.49], [0.39, 0.51] \rangle,
           \langle h_3, [0.27, 0.50], [0.27, 0.50] \rangle, \langle h_4, [0.29, 0.53], [0.25, 0.47] \rangle, \langle h_5, [0.31, 0.45], [0.23, 0.55] \rangle\}

\langle h_2, [0.25, 0.49], [0.29, 0.51] \rangle, \langle h_3, [0.27, 0.51], [0.27, 0.49] \rangle, \langle h_4, [0.29, 0.53], [0.25, 0.47] \rangle,
           \langle h_5, [0.31, 0.55], [0.23, 0.45] \rangle\}

It is clear that hospital $h_5$ has superior value of membership function. So hospital $h_5$ is good for treatment among all given hospitals.

**Definition 4.2. Restricted intersection**

Let $\langle F, A \rangle$ and $\langle G, B \rangle$ be two interval-valued intuitionistic fuzzy soft sets over a common universe $U$ such that $A \cap B \neq \emptyset$. The restricted intersection of $\langle F, A \rangle$ and $\langle G, B \rangle$ is denoted by $\langle F, A \rangle \cap_R \langle G, B \rangle$ and is defined as $\langle F, A \rangle \cap_R \langle G, B \rangle = \langle H, C \rangle_R$ where $C = A \cap B$ and $\varepsilon \in C, H_\varepsilon = F(\varepsilon) \cap G(\varepsilon)$.

$\mu_{H_\varepsilon}(x) = \inf \left[ \mu_{F_\varepsilon}(x), \mu_{G_\varepsilon}(x) \right]$,
$v_{H_\varepsilon}(x) = \sup \left[ v_{F_\varepsilon}(x), v_{G_\varepsilon}(x) \right]$

**Example 2**

Consider $\langle F, A \rangle$ and $\langle G, B \rangle$ be two interval-valued intuitionistic fuzzy soft set over a common universe $U$ as defined in example 1.

$\langle H, C \rangle_R = \langle F, A \rangle \cap_R \langle G, B \rangle$

availability of medicine = $\langle h_1, [0.13, 0.37], [0.41, 0.63] \rangle, \langle h_2, [0.15, 0.39], [0.39, 0.61] \rangle$,
$\langle h_3, [0.17, 0.41], [0.37, 0.59] \rangle, \langle h_4, [0.19, 0.43], [0.35, 0.57] \rangle, \langle h_5, [0.21, 0.45], [0.33, 0.55] \rangle$,
availability of doctors = $\langle h_1, [0.20, 0.44], [0.34, 0.56] \rangle, \langle h_2, [0.22, 0.46], [0.43, 0.54] \rangle$.

It is clear from our result that hospital $h_5$ has superior value of membership function. So hospital $h_5$ is good for treatment among all given hospitals.
Conclusion

In this paper we introduced new concepts of an interval-valued intuitionistic fuzzy soft set. There has been incredible interest in the subject due to its diverse applications, ranging from health sector to social behaviour studies. In this study we defined an interval-valued intuitionistic fuzzy soft set and its operations of union, intersection and complement. We have given some notions such as the restricted intersection and the restricted union of two interval-valued intuitionistic fuzzy soft sets for selection of appropriate hospital for any patient affected by specific disease which play key role in the health sector.

References: