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Optimizing the minimum spanning tree (MST) and its relationship with the minimum cut

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Abstract--The ST ST is a sub-tree of the original network so that the network graph can contain more than one sub-tree that reaches all the vertices of the network and the minimum spanning tree MST is the smallest measure among all the weights of the sub-spanning trees. To get the MST in a simplified way the Kruskal algorithm and the prime algorithm have been applied to several examples. In this work, we applied the new technique to the same examples, and the results were equal or have less value comparing with the above algorithms. The main idea of this technique is to link between the subject of the MST and the subject of the minimum cutoff, so that all the paths of the original graph were extracted and we calculated the weight of each path and chose the path that carried the largest weight, then we can grade by taking the paths according to need from largest to smallest and we cut the edge with the largest weight from that path and continue this process until the conditions of the ST are met. In the end of process we get the same tree that obtained by the Kruskal algorithm and the prime algorithm so that the total cost that obtained by our technique is the same or slightly less than the cost obtained by the other two algorithms.

Keywords--optimizing, minimum spanning tree (MST), relationship, minimum cut.

Introduction

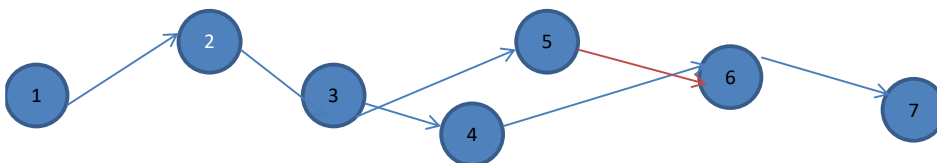
The ST of the graph is a sub tree that must contain all the nodes in the graph provided there is no edge rotation and the number of edges is $v - 1$ where v is the vertices and the graph can contain more than one ST [1, 2]. From general characteristics of a ST, the connected graph can contain more than one ST. A ST has no rotation at its edges such that all the STs we get from the original graph have the same number of nodes and edges, also removing one edge of the ST will separate the graph [3, 4]. The ST will be minimally connected adding a new edge to the tree will result in rotation while the ST is not max, and the MST is the lowest measure of weights or costs [5]. It is possible to find the MST by extracting all possible STs and calculating the weight of each tree so that the MST represents the least weight of them if the network is of a simple type [6, 7]. If it is somewhat complicated, we will discuss several algorithms for finding the MST, including Kruskal's algorithm and prime's algorithm.

MST

The MST is the lowest measure of the weights or costs of the ST. It is a special type of tree that reduces the lengths or weights of the edges of the tree, the graph must be connected in order to find the MST [8, 9]. If the graph is unconnected, we can calculate the MST for each of its connected components, known as the spanning minimum. The weights of the edges may be different or they may be the same (if they are different, the MST is not unique because the cost of the possible edges that reach the target point may be similar) [10, 11]. The ST must contain all the vertices of the original graph, and its edges must also be non- periodic.

The Basic Principles of the ST [12, 13]

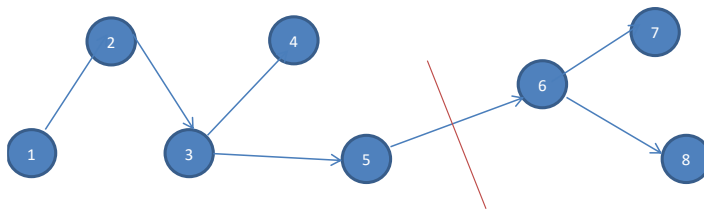
Adding an edge connecting two vertices in a tree creates another path (i.e. a single cycle), see the following example:



Path 1: 1_2_3_4_6_7

Path 2: 1_2_3_5_6_7

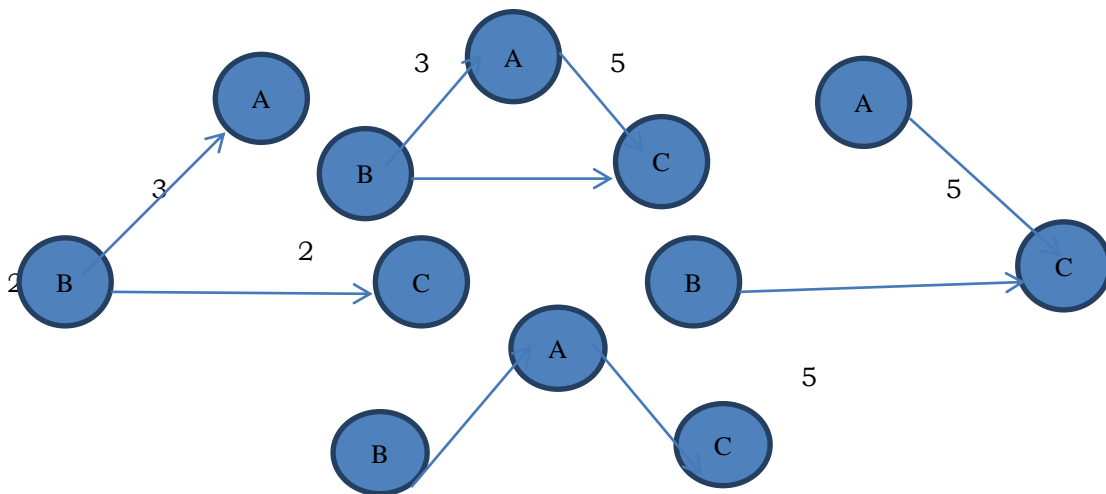
Cutting an edge split the vertices into two separate groups and the intersecting edge is the edge that connects a vertex of one group with a vertex of another, see the following example:



Finding the MST

There are two types of graphs either simple or complex so that larger graphs contain more nodes and many other possibilities of sub graph can reach millions or billions making them, which is very difficult to get a MST [14, 15]. In addition, the lengths are usually have different weights, that is one edge can be give 8 weight and a length of 5 meters while another edge of the same length can give 6 weights. To find the MST we have to find all the possible STs of the graph then we find the weight of each tree and take the lowest weight, which is considered the minimum of the ST [16, 17]. We denote the MST with the symbol (MST). This technique is useful for simple and uncomplicated graphs, the following example clarify this idea.

Example 1: Find the MST of the following graph [18].



If the graph is complex and contains a lot of nodes and edges, we need to use Kruskal's algorithm and prime's algorithm to get the MST from the indirect graph, which are simple and intuitive strategy used to solve some optimization problems [19, 20]. Algorithms do the optimum test at every step because they try to find the way to the overall point to solve the entire problem and succeed in solving some problems because in many cases they do not guarantee the optimal solution [21, 22].

Kruskal's Algorithm

This algorithm is used to find the MST, it depends on the edges where the ST is built by adding the edges with the least weight and gradually to the growing ST with the possibility of no closed cycle within the tree as well as calling all the vertices [23, 24].

Kruskal's Algorithm Steps [25]

- Delete all the edges, leaving only the vertices.
- Keep all edges in order from least to heaviest.
- List the edges in order to the graph from least weight to heaviest, ensuring that no cycle occurs inside any one of them until we meet the stopping condition, which represents: the number of vertices $- 1$, $(V - 1)$.

Prime Algorithm

The prime algorithm is a well-known algorithm for finding the MST, it depends on the nodes, where we choose any node randomly. In the ST, it is considered a partial tree of the original graph [26].

Prime Algorithm Steps

- Choose one of the nodes randomly.
- Distinguish the edge with the least weight that is connected to the first node chosen in the first step because this node can contain more than one edge.
- Search for all edges that connect the tree to the new nodes that were found, determine the minimum edge and add it to the tree and refer to repeat this step until the minimum weight of the ST is found and until the stop condition is met which is: the number of vertices $- 1$.

Suggested New Technique

The authors introduce many papers in various kinds of sciences [27- 43], but in this work we suggest a new technique to find the MST and its relationship to the minimum cut. In this technique, we get the same ST that was obtained by Kruskal algorithm, prime algorithm, or any similar algorithm, and the total cost of the tree will be the same or less than the cost obtained by the previous algorithms. In this technique, we link the issue of the ST with the subject of the minimum cut so that they are in the form of paths where it requires us to extract all the paths and know their weights in order to choose the path that carries the largest weight and cut off the edge with the it according to the idea of the minimum cut, then we continue to use these paths as needed from the largest to the smallest until we meet the conditions of the ST.

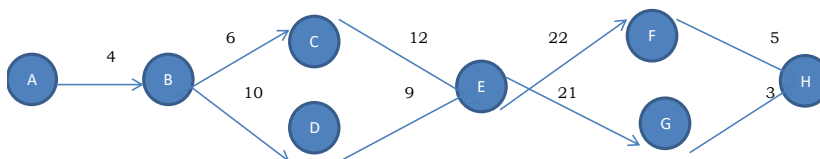
The Steps of the Suggested Method

- Extract all paths from the original graph.
- Calculate the weight of each path.

- Take the path that carries the largest weight (take the paths as needed) in descending order from largest to smallest.
- Cut off the edge with more weight than that path.
- Continue with this process until the conditions of the ST are met so that its edges do not form any rotation. We must reach all the vertices in the original graph with the number of edges in the ST is less than the number of vertices by one degree.

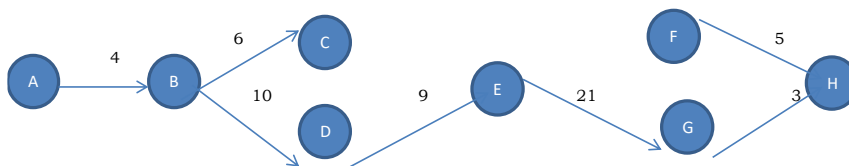
In the following examples, we'll apply the suggested technique.

Example 2: Find the MST.



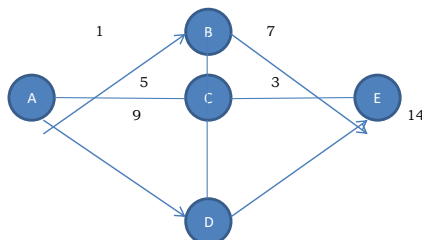
Solution:

Path 1: A-B-C-E-F-H	the weight = 49
Path 2: A-B-C-E-G-H	the weight = 46
Path 3: A-B-D-E-F-H	the weight = 50
Path 4: A-B-D-E-G-H	the weight = 47



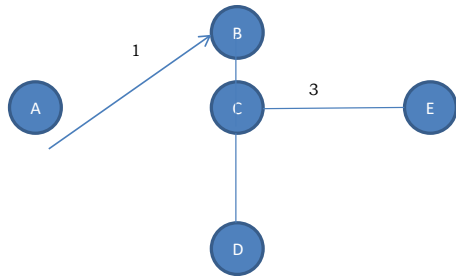
The cost = 58

Example 3: Find the MST.



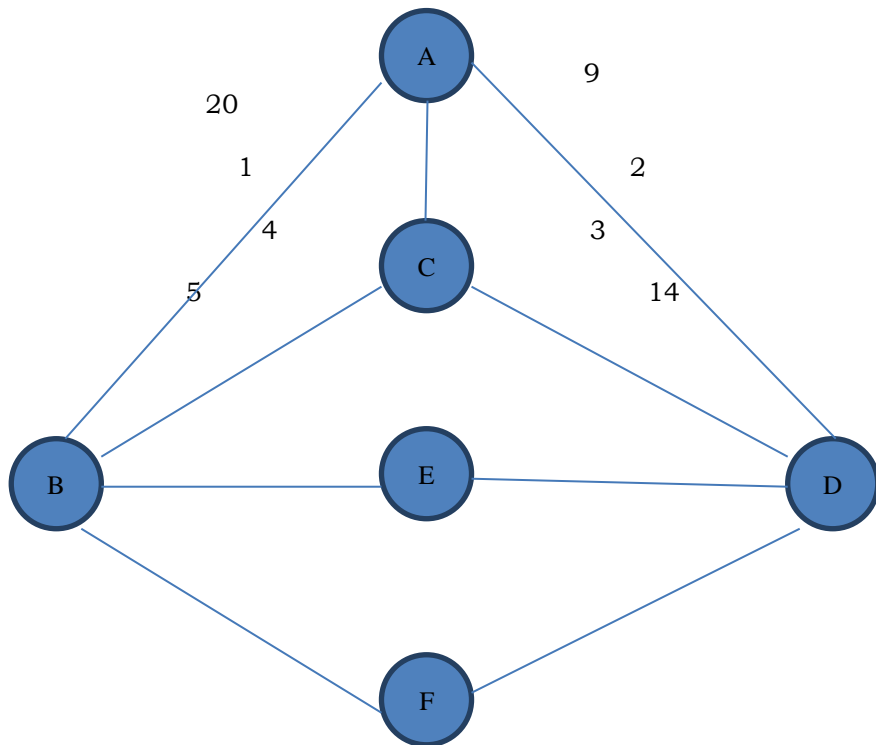
Solution:

Path 1: A-B-E	the weight = 8
Path 2: A-B-C-E	the weight = 6
Path 3: A-C-E	the weight = 8
Path 4: A-D-C-E	the weight = 13
Path 5: A-D-E	the weight = 23



The cost = 7

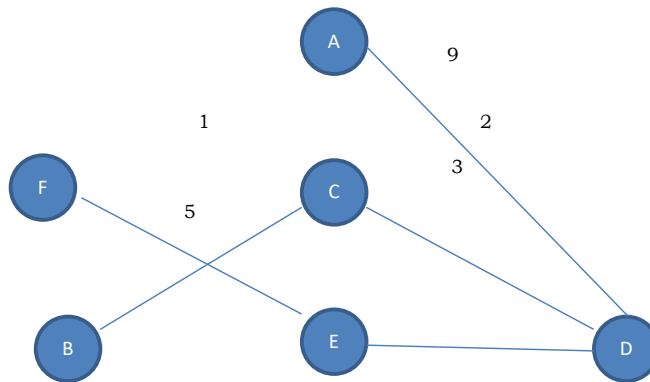
Example 4: Find the MST.



Solution:

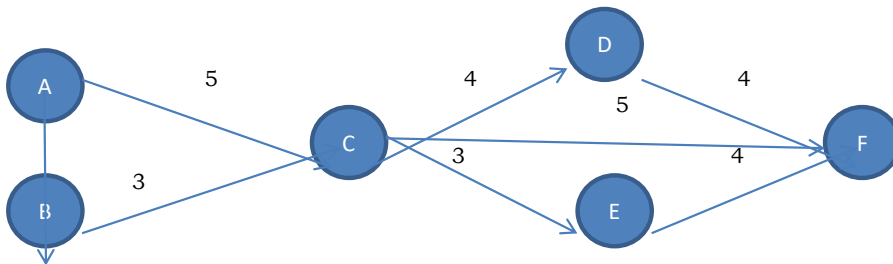
Path 1: B-A-D	the weight = 29
Path 2: B-A-C-D	the weight = 16

Path 3: B-C-D	the weight = 3
Path 4: B-E-D	the weight = 7
Path 5: B-F-D	the weight = 19



The cost = 20

Example 5: Find the MST.

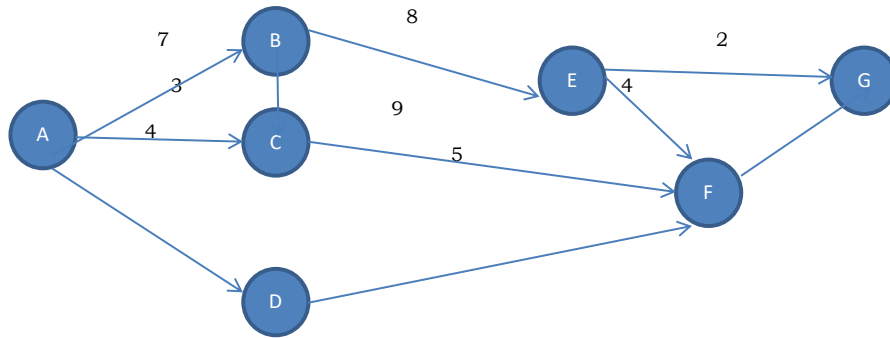


Solution:

Path 1: A-C-D-F	the weight = 13
Path 2: A-C-F	the weight = 10
Path 3: A-C-E-F	the weight = 12
Path 4: A-B-C-D-F	the weight = 16
Path 5: A-B-C-F	the weight = 13
Path 6: A-B-C-E-F	the weight = 15

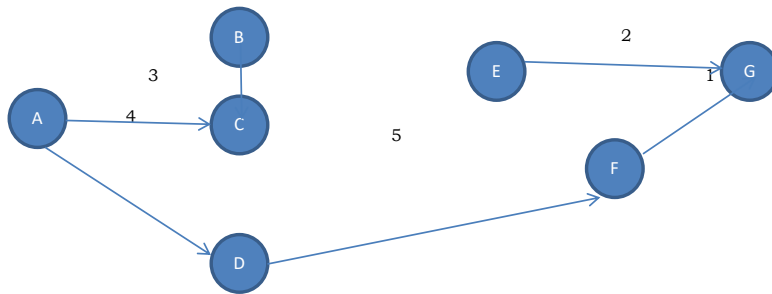
The cost = 19

Example 6: Find the MST.



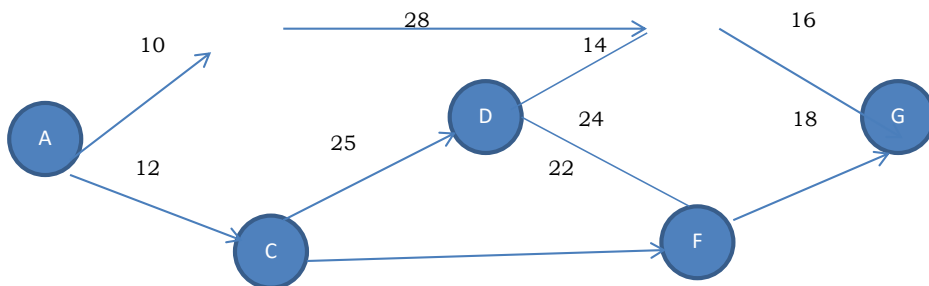
Solution:

Path 1: A-B-E-G	the weight = 17
Path 2: A-B-C-F-G	the weight = 19
Path 3: A-B-C-F-E-G	the weight = 24
Path 4: A-B-E-F-G	the weight = 20
Path 5: A-C-B-E-F-G	the weight = 18
Path 6: A-C-B-E-G	the weight = 15
Path 7: A-C-F-E-G	the weight = 18
Path 8: A-C-F-G	the weight = 13
Path 9: A-D-F-G	the weight = 10
Path 10: A-D-F-E-G	the weight = 15



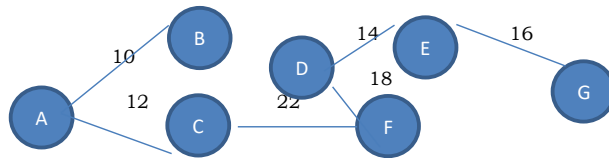
The cost = 17

Example 7: Find the MST.



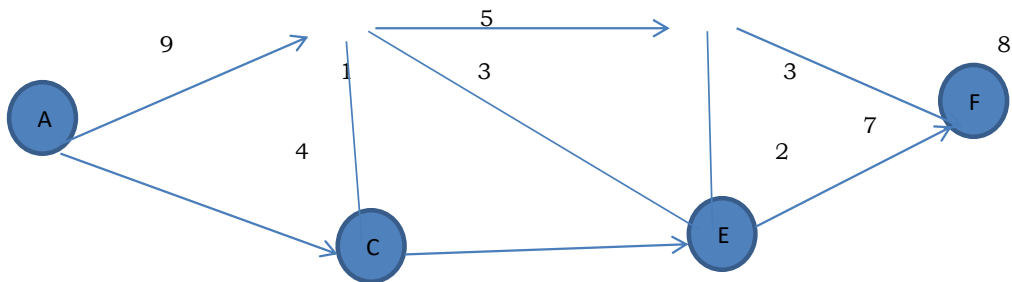
Solution:

Path 1: A-B-E-G	the weight = 54
Path 2: A-B-E-D-F-G	the weight = 95
Path 3: A-C-D-F-G	the weight = 79
Path 4: A-C-F-G	the weight = 59



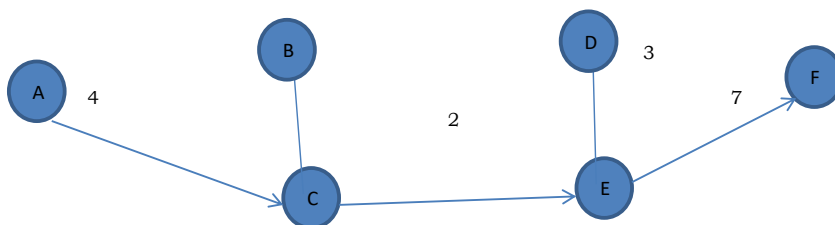
The cost = 92

Example 8: Find the MST.



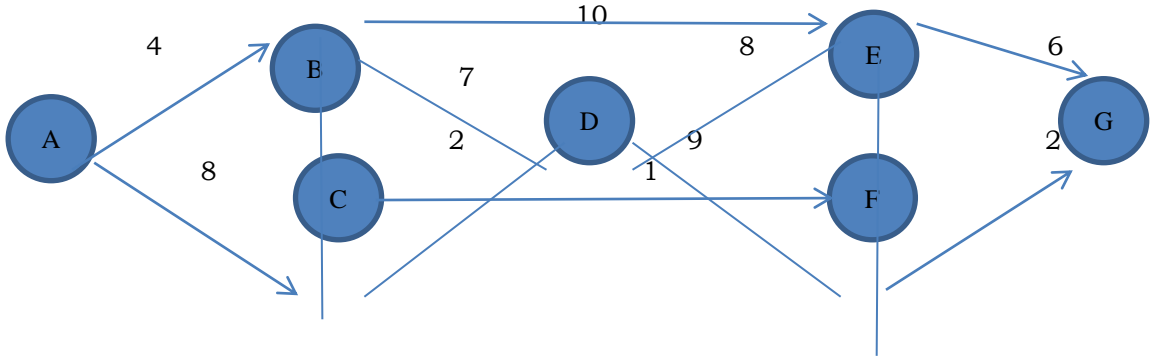
Solution:

Path 1: A-B-D-F	the weight = 22
Path 2: A-B-E-D-F	the weight = 23
Path 3: A-B-E-F	the weight = 19
Path 4: A-B-D-E-F	the weight = 24
Path 5: A-B-C-E-F	the weight = 19
Path 6: A-B-C-E-D-F	the weight = 23
Path 7: A-C-E-F	the weight = 13
Path 8: A-C-B-D-F	the weight = 18
Path 9: A-C-B-E-D-F	the weight = 19
Path 10: A-C-B-E-F	the weight = 15



The cost = 17

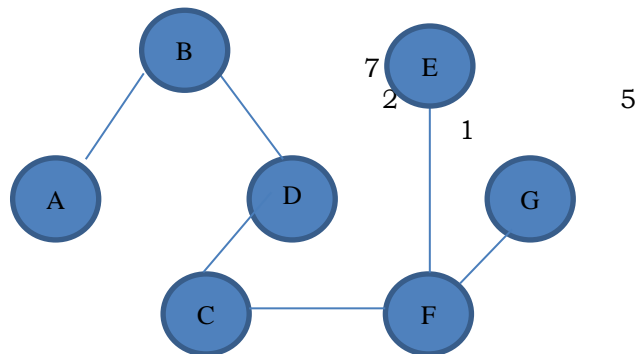
Example 9: Find the MST.



Solution:

Path 1: A-B-E-G	the weight = 20
Path 2: A-B-D-E-G	the weight = 25
Path 3: A-B-D-F-G	the weight = 22
Path 4: A-B-D-F-E-G	the weight = 31
Path 5: A-B-D-E-F-G	the weight = 26
Path 6: A-B-C-F-G	the weight = 16
Path 7: A-B-C-D-F-G	the weight = 26
Path 8: A-B-C-D-F-E-G	the weight = 35
Path 9: A-B-C-D-E-F-G	the weight = 30
Path 10: A-B-C-D-E-G	the weight = 29
Path 11: A-C-F-G	the weight = 7
Path 12: A-C-D-E-F-G	the weight = 21
Path 13: A-C-D-E-G	the weight = 24
Path 14: A-C-D-F-E-G	the weight = 30
Path 15: A-C-D-F-G	the weight = 21
Path 16: A-C-B-D-E-F-G	the weight = 39
Path 17: A-C-B-D-E-G	the weight = 38
Path 18: A-C-B-D-F-E-G	the weight = 44
Path 19: A-C-B-D-F-G	the weight = 35
Path 20: A-C-B-E-F-G	the weight = 34
Path 21: A-C-B-E-G	the weight = 33

2



The cost = 21

Comparison the results

In this section we compare the results that obtained by Kruskal's algorithm, Prime's algorithm, and our suggested technique. The results of all previous examples are recorded in the following table.

Table 1
Comparing the Results

Examples	Results of Kruskal's Algorithm	Results of Prime's Algorithm	Results of Suggested Technique
Example 2	17	19	17
Example 3	58	60	58
Example 4	7	18	7
Example 5	20	21	20
Example 6	21	22	21
Example 7	19	19	19
Example 8	17	21	17
Example 9	92	92	92

Conclusion

It has been shown by using the proposed technique to find the MST that the results are equal to the results obtained by the two methods, Kruskal's algorithm and prime algorithm, and sometimes less than them, and even the shape of the ST is similar to the shape that obtained by the two methods. The results indicated the efficiency and goodness of the new suggested technique.

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