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New technique to find the optimal solution to TPs based on graph theory

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***Abstract*---**A novel approach called the difference absolute method (DAM) is introduced in this paper to locate and test the optimal solution for diverse transportation challenges. The most appealing characteristic of this approach is that it just requires simple arithmetic and logical computations, making it accessible to even the most inexperienced user. This strategy will be incredibly beneficial for those who are capable decision-makers. Take care of logistics and supply chain difficulties. You may easily apply this approach to the present procedure because it is straightforward.

***Keywords*---**TPs, steepest stone method, modified method.

Introduction

The issue of TPs T.P. is one of the most essential issues in linear programming. The primary goal of the problem is to find an optimal cost for shipping goods while fulfilling orders in each goal. As a result, methods for solving the T.P. that lead to the ideal solution faster, more precisely, and better than the optimal solution achieved by previous techniques are needed. In 1941, Alfred Hitchcock [1] developed the basic T.P. as well as a beneficial arranging approach. Dantzig [2] proposed the TP as a direct programming problem in 1951, and he also provided a technique for solving it. Charnes and Cooper [3] developed the "Stepping Stone Method" in 1953 as a method for obtaining an optimal solution from an initial basic feasible solution (IBFS). In 2020, Hussein and Shiker published many studies to address TPs [4– 8]. The authors introduced many papers in a variety of science fields to discover the best solutions, including operation research [9- 12],

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reliability [13- 22] and optimization [23- 43], to find the optimal solution to nonlinear systems and optimization problems, but in this paper, we proposed an innovative method for testing and finding the optimal solution to TPs. This technique reduces effort and time factor by reducing the number of indexes required and cells that need to be treated.

Mathematical formulation of TPS

If we have $\boldsymbol{m}$ origins and $\boldsymbol{n}$ destinations, the plants $\boldsymbol{P_i}$ $(i = 1, 2, \dots, m)$ transport the products $\boldsymbol{a_i}$ to the storages $\boldsymbol{W_j}$ $(j = 1, 2, \dots, n)$ which requires $\boldsymbol{b_j}$ units. Let $\boldsymbol{C_{ij}}$ be the transporting cost of one unit of product from $\boldsymbol{i^{th}}$ origin to $\boldsymbol{j^{th}}$ destination and let $\boldsymbol{X_{ij}}$ be the amount transported from $\boldsymbol{i^{th}}$ origin to $\boldsymbol{j^{th}}$ destination. The following equation (1) represents the transportation cost which its objective is to determine the number of units to be transported from $\boldsymbol{i^{th}}$ origin to $\boldsymbol{j^{th}}$ destination:

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij} \qquad (1)$$

Subject to:

$$\sum_{j=1}^{n} x_{ij} = a_i \quad , \quad i = 1,2, \dots \dots m \text{ (supply constraints)}$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad , \quad j = 1,2, \dots \dots n \text{ (demand constraints)},$$

$$\text{where } x_{ij} \geq 0 \ , \ \forall \ i \text{ and } j$$

Algorithm of YM-method

- **Step 1:** Find (IBFS) by any method.
- **Step 2:** Represent the solution matrix with a weight graph by putting zero on the edges representing the allocation cell or unoccupied cells whose allocation cell in its row or column is greater than its cost, and the weight of other cells is the difference absolute between the cost of the empty cell with the largest allocation cell in its row or column.
- **Step 3:** Find any minimum spanning tree for this graph and take the complement of minimum spanning tree.
- **Step 4:** Select the cell with the largest weight edge. If the weights are equal, choose the one in its row or column has the largest allocation cell and then go to the next step,
- **Step 5:** Make a closed path for the cell with the largest weight in the minimum complement of the spanning tree, and give (+1) for the check cell, (-1) for the next cell in the path, and so on for all cells in the path. If the optimization index of the closed path is positive or zero, take the next cell with the largest weight in the minimum complement of the spanning tree and make a path for it as well:
  - If the optimization index for a closed path is positive or zero for all paths in the previous step, IBFS is the optimal solution.

- If there exist a negative improvement index for the closed path, then determine the least value of products in the path cells which has (-1) and subtract this value from these cells and add it to the cells with (+1).
- **Step 6:** Repeat steps 2 to 5 until we reach the optimal solution.

Example 1: Let's take the following TP matrix with three rows and four columns.

Table 1
TP with Three Rows and Four Columns

	A	B	C	D	Supply
1	11	13	17	14	250
2	16	18	14	9	300
3	21	24	13	10	400
Demand	200	225	275	250	

Firstly, to find IBFS by Vogel's method get:

Table 2
IBFS by Vogel's Method

	A	B	C	D	Supply	Row Penalty
1	11(200)	13(50)	17	14	250	2 | 2 | 2 | 13 | 13 | 13 |
2	16	18(50)	14	9(250)	300	5 | 2 | 2 | 18 | 18 | -- |
3	21	24(125)	13(275)	10	400	3 | 8 | 3 | 24 | -- | -- |
Demand	200	225	275	250		
Column Penalty	5 5 5 -- -- --	5 5 5 5 5 13	1 1 -- -- -- --	1 -- -- -- -- --		

IBFS= $11 \times 200 + 13 \times 50 + 18 \times 50 + 9 \times 250 + 24 \times 125 + 13 \times 275 =$ 12575.Represent the solution matrix and locate the minimal spanning tree with yellow edges by the following graph,

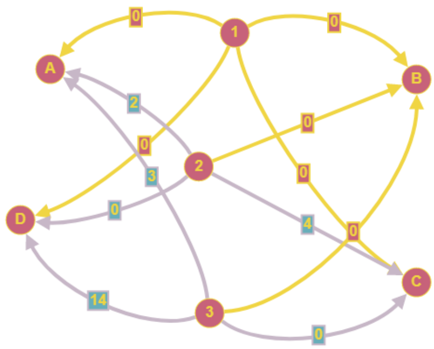

Figure 1. Solution Graph

So, the complement of the minimal spanning tree represents by the following figure.

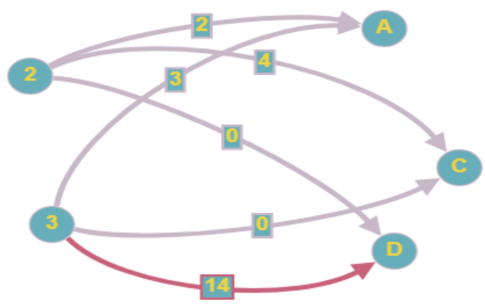

Figure 2. Minimal Spanning Tree

Cell with the largest weight in the complement of minimal spanning is determine red edge to test the solution, whether it is optimal solution or not, make a closed path to $3D$ cell only in the following table.

Table 3
Closed Paths of the Cells

Unoccupied Cell	Closed Path	Index of Net Cost Change
$3D$	$3D \rightarrow 2D \rightarrow 2B \rightarrow 3B$	10 - 9 + 18 - 24 = -5

Since index of net cost change < 0, so IBFS is not optimal solution. So the solution is:

Table 4
Matrix the Solution After Improvement

	A	B	C	D	Supply
1	11 **(200)**	13 **(50)**	17	14	250
2	16	18 **(175)**	14	9 **(125)**	300
3	21	24	13 **(275)**	10 **(125)**	400

Demand	200	225	275	250	

The solution= $11 \times 200 + 13 \times 50 + 18 \times 175 + 9 \times 125 + 13 \times 275 + 10 \times 125 = 11950$.

Now repeat steps 2 to 4 on matrix the solution after improvement, so the following figure represent the solution matrix and locate the minimal spanning tree with yellow edges by the following graph,

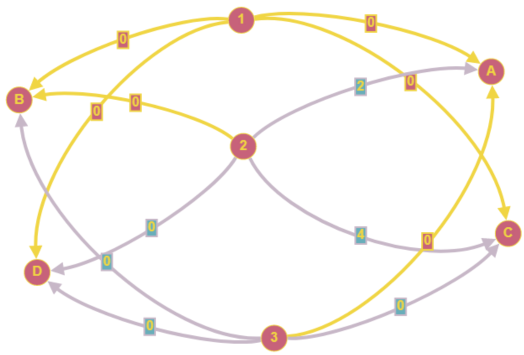

Figure 3. Solution Graph

So, the complement of the minimal spanning tree represents by the following

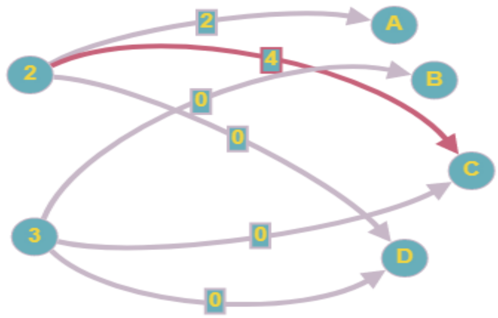

Figure 4. Solution Graph

Cell with the largest weight in the complement of minimal spanning is determine red edge to test the solution, whether it is optimal solution or not, make a closed path to $2C$ cell only in the following table.

Table 5
Closed Paths of The Cells

Unoccupied Cell	Closed Path	Index of Net Cost Change
$2C$	2C→2D→$3D$→$3C$	14 - 9 + 10 - 13 = 2

Since index of net cost change ≥ 0 , so IBFS is optimal solution. The following table contains the comparison among DAM- method, (SS) method, and (MODI) method in terms of the optimal solutions and the number of indexes.

Table 6
Comparison Among DAM, (SS) Method and (MODI)

(SS) Method		(MODI) Method		DAM-Method	
Optimal Solution	Number of Index	Optimal Solution	Number of Index	Optimal Solution	Number of Index
11950	6	11950	6	11950	1

Conclusion

In this study, we present the (Difference Absolute Approach (DAM)) as a novel method for finding the best solution to TPs. When compared to the old (SS) and (MODI) approaches for solving transportation issues, the new method is distinguished by simplicity, ease, and the absence of difficult computations, saving decision makers time and effort. We discovered that the optimal solution produced by DAM is equivalent to the optimal solution acquired by the (SS) and (MODI) techniques, plus the index number, when compared to the optimal solution obtained by the (SS) and (MODI) methods. The MST technique requires fewer indexes to obtain the best solution than the other two methods, indicating the efficiency and quality of our novel method. As a result, it is preferable to employ it as a novel, more efficient, and high-quality solution.

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