How to Cite:
Ajith, P. S., Basavaraju, G. C., & Vishu, K. M. (2022). 2-Metro domination number of slanting ladder graph. *International Journal of Health Sciences*, *6*(S9), 3756–3760. Retrieved from https://sciencescholar.us/journal/index.php/ijhs/article/view/13455

2-Metro domination number of slanting ladder graph

Ajith P S
Dept of Mathematics, Brindavan College of Engineering, Bangalore.
Email: ajithps24x7@gmail.com

G C Basavaraju
Dept of Mathematics, Brindavan College of Engineering, Bangalore.
Email: basava.raju759@gmail.com

Vishu Kumar M
Dept of Mathematics, REVA University, Bangalore.
Email: vishukumarm@reva.edu.in

***Abstract*---**A subset D of the vertex set V of the graph $G(V,E)$ is said to be a dominating set if every vertex in $V-D$ is adjacent to at least one vertex in D. The minimum cardinality of the dominating set is called the domination number. The metro domination number is the order of a minimum dominating set which resolves as a metric as a metric set. It is denoted by $\gamma_\beta(G)$. in this paper we determine the 2-metro domination number of slanting ladder graphs.

***Keywords*---**dominating set, domination number, metric dimension, metro domination.

1 Introduction

Every graph considered here are simple, finite, undirected and connected. A graph $G=(V,E)$ and $u,v\in V$, $d_G(u,v)$ is denoted as distance between u and v in G. We refer [5,6,7,8,9,10,11] for the works on metro domination. A set $S\subseteq V$ is called resolving set if for every $u,v\in V$ there exist $w\in S$, such that $d(u,w)\neq d(v,w)$. The resolving set with minimum cardinality is called metric basis and its cardinality is called metric dimension and it is denoted by $\beta(G)$.

Manuscript submitted: 9 May 2022, Manuscript revised: 18 July 2022, Accepted for publication: 27 August 2022

A set D of vertices in a graph G is called a dominating set of G , If every vertex in $V-D$ is adjacent to some vertex in D, The minimum number cardinality of a dominating set in G is called the domination number of G and denoted by $\gamma(G)$. A dominating set D of $V(G)$ having the property that for each pair of vertices u, v there exists a vertex w in D such that $d(u,w) \neq d(v,w)$ is called the metro dominating set of G or simply $MD-$ set. The minimum cardinality of a metro dominating set of G is called metro domination number of G and is denoted by $\gamma_\beta(G)$.

Corollary1.1: The metric dimension of slanting ladder graph is 2.

Corollary1.2: For all n, $\gamma(S(L_n)) = \left\lceil \frac{n+1}{2} \right\rceil$

2 Main Results

Theorem 1: For any integer n, $\gamma_\beta[S(L_n)] = \left\lceil \frac{n+1}{2} \right\rceil, n \geq 5,\ where n \neq 4l, l \geq 1.$

Proof: Let $G = S(L_n)$ be an slanting ladder graph on $2n$ vertices with $V(G) = \{u_i, v_i : 1 \leq i \leq n\}$ and
$E(G) = \{(v_i, v_{i+1}), (v_i, v_{i+1})(u_i, u_{i+1}) : 1 \leq i \leq n-1\}$,
By using corollary 1.1 and 1.2, since metro dominating set D also a dominating set.

Thus $\gamma_\beta[S(L_n)] \geq \left\lceil \frac{n+1}{2} \right\rceil$ (1)

To prove the reverse inequality, we find a metro dominating set of cardinality $\left\lceil \frac{n+1}{2} \right\rceil$

We find a set D as follows

$$D_1 = \{u_{4l-1} : l \geq 1\}, n \equiv 3 (\mathrm{mod}\ 4)$$
$$D_2 = \{v_{4l-2} : l \geq 1\}, n \equiv 2 (\mathrm{mod}\ 4)$$

We note that D is also dominating set for $S(L_n)$ and also D will serves as metric set of $S(L_n)$ as in 1.1.

Thus $\gamma_\beta[S(L_n)] \leq \left\lceil \frac{n+1}{2} \right\rceil$ (2)

From (1) and (2)

$$\gamma_\beta[S(L_n)] = \left\lceil \frac{n+1}{2} \right\rceil, n \geq 5.$$

Example 1.1: The metro domination number of slanting ladder graph is 3.

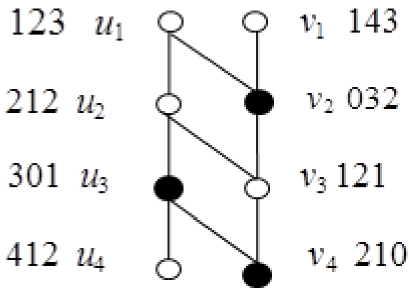

Figure:1 $\gamma_\beta[S(L_4)]=3$

Theorem 2: For any integer n, $\gamma_{\beta_2}[S(L_n)]=\left\lceil\frac{n+1}{4}\right\rceil, n\geq 9$, $where n \neq 8l, l\geq 1.$

Proof: Let $G=S(L_n)$ be an slanting ladder graph on $2n$ vertices with $V(G)=\{u_i, v_i : 1\leq i\leq n\}$ and $E(G)=\{(v_i,v_{i+1}),(v_i,v_{i+1})(u_i,u_{i+1}):1\leq i\leq n-1\}$, with for each i, u_i , v_i the only edges between two paths $W=V-D$, Now each $v_i\in W$ is either adjacent to any of the vertex D or at least at a distance 2 from at least one of the vertex D. Any vertex $v_K\in D$, will dominates at least 5 vertex including itself. Since the metric dimension of slanting ladder graph is 2, D itself serves as a metric set.

Thus $\gamma_{\beta_2}[S(L_n)]\geq\left\lceil\frac{n+1}{4}\right\rceil$ (1)

We find a set D as follows

$$D_1=\{u_{8l-2} : l\geq 1\} n\equiv 6(\text{mod } 8)$$
$$D_2=\{v_{8l-5} :\geq 1\} n\equiv 3(\text{mod } 8)$$

We note that D is also dominating set for $S(L_n)$ and also D will serves as metric set of $S(L_n)$ as in 1.1.

Thus $\gamma_{\beta_2}[S(L_n)]\leq\left\lceil\frac{n+1}{4}\right\rceil$ (2)

From (1) and (2)

$$\gamma_{\beta_2}[S(L_n)]=\left\lceil\frac{n+1}{4}\right\rceil, n\geq 9.$$

Example 1.2: The 2-metro domination number of slanting ladder graph is 3

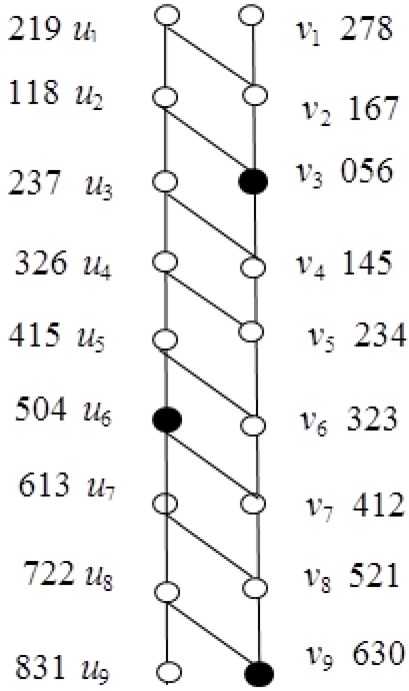

Figure:2 $\gamma_{\beta_2}[S(L_9)] = 3$

Acknowledgment

A part of the paper was written when both authors were visiting department of mathematics, Brindavan College of Engineering, Bengaluru, India. The authors thankful to the referees for helpful comments.

References

1. C. Berge, Graphs and hyper graphs, North-Holland, Amsterdam,1973
2. B. Zmazek and J. Zerovnik, "On domination number of graph bundles", Institute of mathematics, physics and mechanics, 43(2005),pp.1-10.
3. A. Sugumaran and E. Jayachandran, "Domination number of some graphs", International Journal of Scentific Development and Reserch, 3(2018),no.11,pp.386-391.
4. Raghunath P and Sooryanarayana B, "Metro domination number of a graph", Twentieth annual conference of Ramanujan Mathematics Society, July 25-30(2005) university of Calicut, Calicut.
5. G C Basavaraju, M Vihukumar and Raghunath, "Metro domination of square of path", Annals of Pure and Applied mathematics, 14(2017), no.3,pp.539-545.
6. G CBasavaraju, M Vishukumar and Raghunath, "Metro domination of powers of path", International Journal of Emerging Technology and Advance Engineering, 8(2018), no.1.

7. G CBasavaraju, M Vishukumar and Raghunath, “On the metro domination number of Cartesian product of P2X Pn”,International Journal of mathematical archieve,9(2018),no.3,pp.1-2
8. G CBasavaraju, M Vishukumar and Raghunath, “On the metro domination number of Cartesian product of P2X Pnand CmXCn”, Journal of Engineering and applied sciences, 14(2019), no.1, pp.114-119
9. G CBasavaraju, M Vishukumar and Raghunath, “On the K-metrodomination number of Cartesian product of P2X Pn”, Journal of Advanced Reserch in Dynamical and Control Systems, 11(2019), no.1, pp.425-431.
10. LalithaLamani and Yogalaksmi S, “Neighborhood, Locating Dominating and Almost Locating Dominating Set of a Ladder Graph”, International Journal of Mathematics Trends and Technology, 67(2021), no.1, pp.121-128.
11. G CBasavaraju, M Vishukumar and Raghunath, “Metro Domination of Square Cycle”, International Journal of Mathematics and its applications, 5(2017), no.1-E, pp.641-645