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2-Metro Domination Number of Open Ladder Graph

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***Abstract*---**A subset D of the vertex set V of the graph $\boldsymbol{G(V,E)}$ is said to be a dominating set if every vertex in $\boldsymbol{V-D}$ is adjacent to at least one vertex in D. The minimum cardinality of the dominating set is called the domination number. The metro domination number is the order of a minimum dominating set which resolves as a metric as a metric set. It is denoted by $\boldsymbol{\gamma_\beta(G)}$. In this paper we determine the 2-metro domination number of open ladder graph.

***Keywords*---**dominating set, domination number, metric dimension, metro domination.

Introduction

Every graph "considered here are simple, finite, undirected and connected. A graph $G=(V,E)$ and $u,v\in V$, $d_G(u,v)$ is denoted as distance between u and v in G. We refer [5,6,7,8,9,10,11] for the works on metro domination".
A set $S\subseteq V$ is called resolving set if for every $u,v\in V$ there exist $w\in S$, such that $d(u,w)\neq d(v,w)$. The resolving set with minimum cardinality is called metric basis and its cardinality is called metric dimension and it is denoted by

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$\beta(G)$ A set D of vertices in a graph G is called a dominating set of G, If every vertex in $V-D$ is adjacent to some vertex in D, The minimum number cardinality of a dominating set in G is called the domination number of G and denoted by $\gamma(G)$

A dominating set D of $V(G)$ having the property that for each pair of vertices u,v there exists a vertex w in D such that $d(u,w)\neq d(v,w)$ is called the metro dominating set of G or simply $MD-$ set. The minimum cardinality of a metro dominating set of G is called metro domination number of G and is denoted by $\gamma_\beta(G)$

Corollary 1.1: For any integer n, $\beta[O(L_n)]=2$

Corollary1.2: For any integer n, $\gamma[O(L_n)]=\left\lfloor \frac{n+4}{2} \right\rfloor$

2 Main Results

Theorem 1: For any integer n, $\gamma_\beta[O(L_n)]=\left\lfloor \frac{n+4}{2} \right\rfloor$, $n\geq 4$.

Proof: Let $G=O(L_n)$ be an open ladder graph on $2n$ vertices with $V(G)=\{u_i,v_i : 1\leq i\leq n\}$ and
$E(G)=\{u_i,u_{i+1},v_i,v_{i+1} : 1\leq i\leq n-1\}\cup\{u_i,v_i : 2\leq i\leq n-1\}$.

By using the corollary 1.2 and corollary 1.1, $\gamma[O(L_n)]=\left\lfloor \frac{n+4}{2} \right\rfloor$ and $\beta[O(L_n)]=2$. Since a metro dominating set D is also a dominating set,

Thus $\gamma_\beta[O(L_n)]\geq\left\lfloor \frac{n+4}{2} \right\rfloor$ ----(1)

To prove the reverse inequality, we find a metro dominating set of cardinality $\left\lfloor \frac{n+4}{2} \right\rfloor$

We find a set D as follows

$$D_1=\{u_{4l-2} : l\geq 1\}\ n\equiv 2(\text{mod } 4)$$

$$D_2=\{v_{4l} : l\geq 1\}\ n\equiv 4(\text{mod } 4)$$

Choose D in the above cases, then D is a dominating set and $|D|=\left\lfloor \frac{n+4}{2} \right\rfloor$. By using corollary 1.1, the dominating set also serves a metric set.

Thus, $\gamma_\beta[O(L_n)] \le \left\lfloor \frac{n+4}{2} \right\rfloor$ ----(2)

From equation (1) and (2), $\gamma_\beta[O(L_n)] = \left\lfloor \frac{n+4}{2} \right\rfloor$

Example 2.2: The metro domination number of open ladder graph is 4

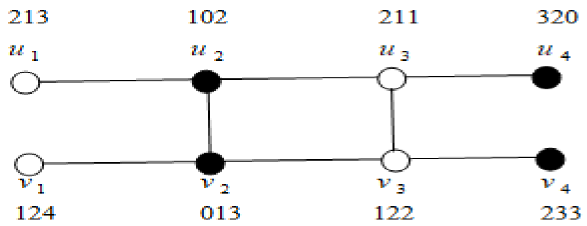

Figure:1 $\gamma_\beta[O(L_4)] = 4$

Theorem 2: For any integer $n, \gamma_{\beta_2}[O(L_n)] = \left\lceil \frac{n+1}{4} \right\rceil, n \ge 8$

Proof: "Let $G = O(L_n)$ be an open ladder graph on $2n$ vertices with $V(G) = \{u_i, v_i : 1 \le i \le n\}$ and $E(G) = \{u_i, u_{i+1}, v_i, v_{i+1} : 1 \le i \le n-1\} \cup \{u_i, v_i : 2 \le i \le n-1\}$ with for each i, $u_i v_i$ is the only edge between two" paths.
$W = V - D$, now each $v_i \in W$ is either adjacent to any of the vertex of D.
Any vertex $v_k \in D$, will dominates at least five vertices including itself.

Since metric dimension of an open ladder graph is 2, D itself serves as a metric set.

Thus $\gamma_{\beta_2}[O(L_n)] \ge \left\lceil \frac{n+1}{4} \right\rceil$ ----------(1)

To prove $\gamma_{\beta_2}[O(L_n)] \le \left\lceil \frac{n+1}{4} \right\rceil$

We define a set D as follows $D_1 = \{u_{8l-6} : l \ge 1\}\ n \equiv 2 \pmod 8$

$D_2 = \{v_{8l-2} : l \ge 1\}\ n \equiv 6 \pmod 8$

"We note that D is 2 dominating set for $O(L_n)$ and also D will serve a metric set" of $O(L_n)$ as in corollary 2.2. Thus, $\gamma_{\beta_2}[O(L_n)] \le \left\lceil \frac{n+1}{4} \right\rceil$ ---------(2)

From equation (1) and (2) $\gamma_{\beta_2}[O(L_n)] = \left\lceil \frac{n+1}{4} \right\rceil$

Example 2.1: The 2-metro domination number of open ladder graph is 3

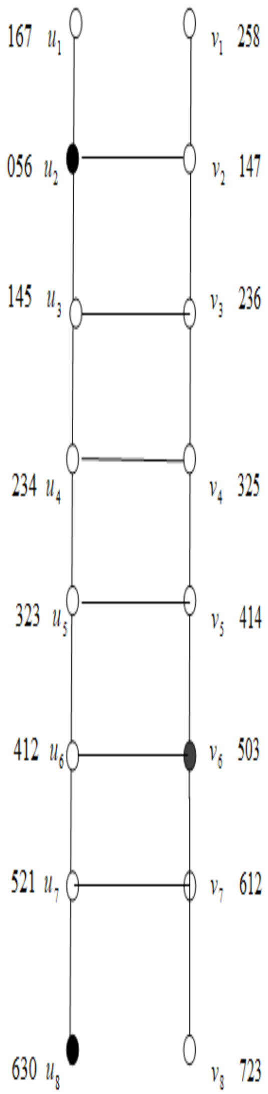

Figure: 2 $\gamma_{\beta_2}[O(L_8)] = 3$

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