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K-Metro domination number of open ladder graph

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***Abstract*---**A dominating set D of a graph $G = G(V, E)$ is called Metro dominating set of G. If for every pair of vertices u, v there exists a vertex w in D such that $d(u, w) \neq d(v, w)$. The K-Metro domination number of open ladder graph $(\gamma_{\beta_k}(O(L_n))$, is the order of smallest K-dominating set of $O(L_n)$ which serves as a matric set. In this paper we calculate K-Metro domination number of open ladder graph $(\gamma_{\beta_k}(O(L_n))$.

***Keywords*---**Dominating set, K-Dominating set, Domination number, Locating dominating set, Metric dimension, Metro dominating set.

1 Introduction

Every graph considered here are simple, finite, undirected and connected. A graph $G = (V, E)$ and $u, v \in V$, $d_G(u, v)$ is denoted as distance between u and v in G.

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A set $S \subseteq V$ is called resolving set if for every $u, v \in V$ there exist $w \in S$, such that $d(u,w) \neq d(v,w)$. The resolving set with minimum cardinality is called metric basis and its cardinality is called metric dimension and it is denoted by $\beta(G)$.

A set D of vertices in a graph G is called a dominating set of G , If every vertex in $V - D$ is adjacent to some vertex in D, The minimum number cardinality of a dominating set in G is called the domination number of G and denoted by $\gamma(G)$.

A dominating set D of $V(G)$ having the property that for each pair of vertices u, v there exists a vertex w in D such that $d(u,w) \neq d(v,w)$ is called the metro dominating set of G or simply $MD-$ set. The minimum cardinality of a metro dominating set of G is called metro domination number of G and is denoted by $\gamma_\beta(G)$.

Metro domination number introduced by B.Sooryanarayan and Raghunath.P[12]. A subset D of $V(G)$ is k-dominating in G if every vertex of $V - D$ has at least k neighbours in D. The cardinality of minimum k-dominating set is called k-domination number of G and is denoted by $\gamma_k(G)$. A dominating set D of a graph $G = (V, E)$ is called metro dominating set of G if for each pair of vertices u, v there exists a vertex w in D such that $d(u,w) \neq d(v,w)$.

Corollary 1.1: For any integer n, $\beta[O(L_n)] = 2$

Corollary1.2: For any integer n, $\gamma[O(L_n)] = \left\lfloor \frac{n+4}{2} \right\rfloor$

2 Main Results

Theorem 2.1: For any integer n,

$$\gamma_{\beta_3}[O(L_n)] = \left\lceil \frac{n+1}{6} \right\rceil, n \geq 12$$

Proof: Let $G = O(L_n)$ be an open ladder graph on $2n$ vertices with $V(G) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(G) = \{u_i, u_{i+1}, v_i, v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i, v_i : 2 \leq i \leq n-1\}$ with for each i, $u_i v_i$ is the only edge between two paths. $W = V - D$, now each $v_i \in W$

is either adjacent to any of the vertex of D. Any vertex $v_k \in D$, will dominates at least five vertices including itself. Since metric dimension of an open ladder graph is 2, D itself serves as a metric set.

Thus $\gamma_{\beta_3}[O(L_n)] \geq \left\lceil \frac{n+1}{6} \right\rceil$ (1)

To prove $\gamma_{\beta_3}[O(L_n)] \leq \left\lceil \frac{n+1}{6} \right\rceil$

We define a set D as follows

$$D_1 = \{u_{12l-9} : l \geq 1\}\ n \equiv 3(\bmod 12)$$
$$D_2 = \{v_{12l-3} : l \geq 1\}\ n \equiv 9(\bmod 12)$$

We note that D is 3- dominating set for O(L$_n$) and also D will serve a metric set of $O(L_n)$ as in 1.1.

Thus $\gamma_{\beta_3}[O(L_n)] \leq \left\lceil \frac{n+1}{6} \right\rceil$ (2)

From (1) and (2)

$$\gamma_{\beta_3}[O(L_n)] = \left\lceil \frac{n+1}{6} \right\rceil$$

Example: The 3-metro domination number of slanting ladder graph 3.

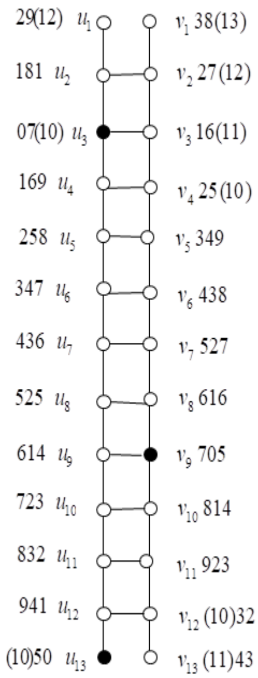

Figure 1: $\gamma_{\beta_3}[S(L_{13})] = 3$

Theorem 2.1: For any integer n,

$$\gamma_{\beta_4}[O(L_n)] = \left\lceil \frac{n+1}{8} \right\rceil,\ n \geq 16$$

Proof: "Let $G = O(L_n)$ be an open ladder graph on $2n$ vertices with $V(G) = \{u_i, v_i : 1 \le i \le n\}$ and $E(G) = \{u_i, u_{i+1}, v_i, v_{i+1} : 1 \le i \le n-1\} \cup \{u_i, v_i : 2 \le i \le n-1\}$ with for each i, $u_i v_i$ is the only edge between two paths. $W = V - D$, now each $v_i \in W$

is either adjacent to any of the vertex of D. Any vertex $v_k \in D$, will dominates at least five vertices including itself. Since metric dimension of an open ladder graph is 2, D itself serves as a metric set.

Thus $\gamma_{\beta_4}[O(L_n)] \ge \left\lceil \frac{n+1}{8} \right\rceil$ (1)

To prove $\gamma_{\beta_4}[O(L_n)] \le \left\lceil \frac{n+1}{8} \right\rceil$

We define a set D as follows

$$D_1 = \{u_{16l-12} : l \ge 1\}\ n \equiv 4 (\mathrm{mod}\ 16)$$
$$D_2 = \{v_{16l-4} : l \ge 1\}\ n \equiv 12 (\mathrm{mod}\ 16)$$

We note that D is 4- dominating set for $O(L_n)$ and also D will serve a metric set" of $O(L_n)$ as in 1.1.

Thus $\gamma_{\beta_4}[O(L_n)] \le \left\lceil \frac{n+1}{8} \right\rceil$ (2)

From (1) and (2)

$$\gamma_{\beta_4}[O(L_n)] = \left\lceil \frac{n+1}{8} \right\rceil$$

Example: The 4-metro domination number of slanting ladder graph is 3.

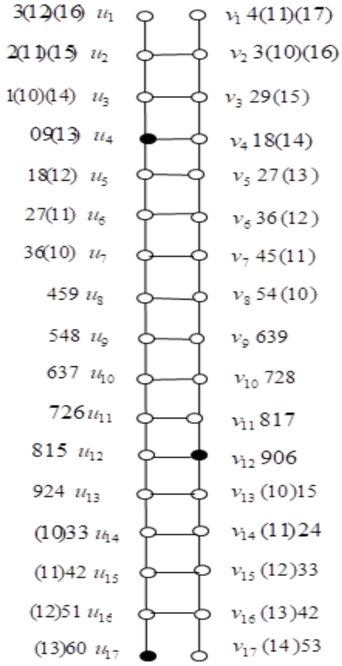

Figure 2: $\gamma_{\beta_4}[S(L_{17})] = 3$

Theorem 2.3: "For any integer n ,

Proof: "Let $G = O(L_n)$ be an open ladder graph on $2n$ vertices with $V(G) = \{u_i, v_i : 1 \le i \le n\}$ and $E(G) = \{u_i, u_{i+1}, v_i, v_{i+1} : 1 \le i \le n-1\} \cup \{u_i, v_i : 2 \le i \le n-1\}$ with for each i, $u_i v_i$ is the only edge between two paths. $W = V - D$, now each $v_i \in W$

is either adjacent to any of the vertex of D or a least at a distance 2 from at least one of the vertex D. Any vertex $v_K \in D$, will dominates at least $4k$ vertex including itself. The lower bound of $O(L_n)$, of order $n = 4kl$ for some $l \ge 1$.

We defines a set D as follows

$$D_1 = \{u_{4kl-3k} : l \ge 1\}\ n \equiv k (\mathrm{mod}\ 4k)$$

$$D_2 = \{v_{4kl-k} : l \ge 1\}\ n \equiv 3k (\mathrm{mod}\ 24k)$$

We note that D is also k-dominating set for $O(L_n)$ and also D will serves as metric set of $O(L_n)$ as in 1.3.

Thus $\gamma_{\beta_K}[O(L_n)] \le \left\lceil \frac{n+1}{2k} \right\rceil, \quad n \ge 4k.$

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