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K-Metro domination number of slanting ladder graph

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***Abstract*---**A dominating set D of a graph $G = G(V, E)$ is called Metro dominating set of G. If for every pair of vertices u, v there exists a vertex w in D such that $d(u,w) \neq d(v,w)$. The K-Metro domination number of slanting ladder graph $(\gamma_{\beta_k}(S(L_n)))$, is the order of smallest K-dominating set of $S(L_n)$ which serves as a matric set. In this paper we calculate K-Metro domination number of slanting ladder graph $(\gamma_{\beta_k}(S(L_n)))$.

***Keywords*---**Dominating set, K-Dominating set, Domination number, Locating dominating set, Metric dimension, Metro dominating set.

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1 Introduction

Every graph considered here are simple, finite, undirected and connected. A graph $G=(V,E)$ and $u,v\in V$, $d_G(u,v)$ is denoted as distance between u and v in G. We refer [5,6,7,8,9,10,11,13] for the works on metro domination. A set $S\subseteq V$ is called resolving set if for every $u,v\in V$ there exist $w\in S$, such that $d(u,w)\neq d(v,w)$. The resolving set with minimum cardinality is called metric basis and its cardinality is called metric dimension and it is denoted by $\beta(G)$.

A set D of vertices in a graph G is called a dominating set of G, If every vertex in $V-D$ is adjacent to some vertex in D, The minimum number cardinality of a dominating set in G is called the domination number of G and denoted by $\gamma(G)$. A dominating set D of $V(G)$ having the property that for each pair of vertices u,v there exists a vertex w in D such that $d(u,w)\neq d(v,w)$ is called the metro dominating set of G or simply $MD-$ set. The minimum cardinality of a metro dominating set of G is called metro domination number of G and is denoted by $\gamma_\beta(G)$.

Metro domination number introduced by B.Sooryanarayan and Raghunath.P[12]. A subset D of $V(G)$ is k-dominating in G if every vertex of $V-D$ has at least k neighbours in D. The cardinality of minimum k-dominating set is called k-domination number of G and is denoted by $\gamma_k(G)$. A dominating set D of a graph $G=(V,E)$ is called metro dominating set of G if for each pair of vertices u,v there exists a vertex w in D such that $d(u,w)\neq d(v,w)$.

Corollary 1.1: For any integer n, $\beta[S(L_n)]=2$

Corollary1.2: For any integer n, $\gamma[S(L_n)]=\left\lfloor \frac{n+4}{2} \right\rfloor$

2 Main Results

Theorem 2.1: For any integer n, $\gamma_{\beta_3}[S(L_n)]=\left\lceil \frac{n+1}{6} \right\rceil, n\geq 13\ , where\, n\neq 12l, l\geq 1.$

Proof: Let $G=S(L_n)$ be an slanting ladder graph on $2n$ vertices with $V(G)=\{u_i,v_i:1\leq i\leq n\}$ and $E(G)=\{(v_i,v_{i+1}),(v_i,v_{i+1})(u_i,u_{i+1}):1\leq i\leq n-1\}$, with for each i, u_i v_i the only edges between two paths $W=V-D$, Now each $v_i\in W$ is either adjacent to any of the vertex D or at least at a distance 3 from at least one of the vertex D. Any vertex $v_K\in D$, will dominates at least 7 vertexes including itself. Since the metric dimension of slanting ladder graph is 2, D itself serves as a metric set.

Thus $\gamma_{\beta_3}[S(L_n)] \geq \left\lceil \frac{n+1}{6} \right\rceil$ (1)

We find a set D as follows

$$D_1 = \{u_{12l-3} : l \geq 13\}\, n \equiv 9(\mathrm{mod}\,12)$$
$$D_2 = \{v_{12l-8} : l \geq 13\}\, n \equiv 4(\mathrm{mod}\,12)$$

We note that is also dominating set for $S(L_n)$ and also D will serves as metric set of $S(L_n)$ as in 1.1.

Thus $\gamma_{\beta_3}[S(L_n)] \leq \left\lceil \frac{n+1}{6} \right\rceil$ (2)

From equation (1) and (2), $\gamma_{\beta_3}[S(L_n)] = \left\lceil \frac{n+1}{6} \right\rceil, n \geq 13.$

Example: The 3-metro domination number of slanting ladder graph 3

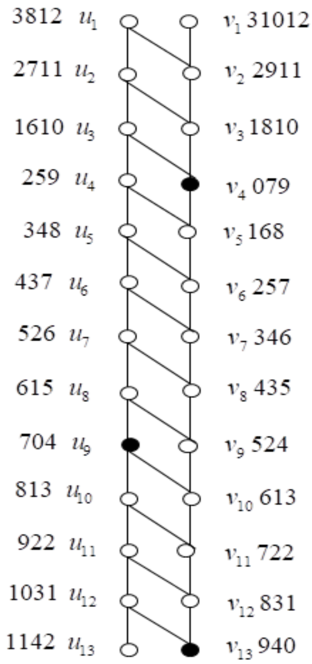

Figure 1: $\gamma_{\beta_3}[S(L_{13})] = 3$

Theorem 2.2: For any integer n, $\gamma_{\beta_4}[S(L_n)] = \left\lceil \frac{n+1}{8} \right\rceil$, $n \geq 17$ $, where\, n \neq 16l, l \geq 1.$

Proof: Let $G = S(L_n)$ be an slanting ladder graph on $2n$ vertices with $V(G) = \{u_i, v_i : 1 \leq i \leq n\}$ and

$E(G)=\{(v_i,v_{i+1}),(v_i,v_{i+1})(u_i,u_{i+1}):1\le i\le n-1\}$, with for each i, u_i v_i the only edges between two paths $W=V-D$, Now each $v_i\in W$ is either adjacent to any of the vertex D or at least at a distance 4 from at least one of the vertex D. Any vertex $v_K\in D$, will dominates at least 9 vertex including itself. Since the metric dimension of slanting ladder graph is 2, D itself serves as a metric set.

Thus $\gamma_{\beta 4}[S(L_n)]\ge\left\lceil\frac{n+1}{8}\right\rceil$ (1)

We find a set D as follows

$$D_1=\{u_{16l-4}:l\ge 17\}\, n\equiv 12(\mathrm{mod}\,16)$$
$$D_2=\{v_{16l-4}:l\ge 17\}\, n\equiv 5(\mathrm{mod}\,16)$$

We note that D is also dominating set for $S(L_n)$ and also D will serves as metric set of $S(L_n)$ as in 1.1.

Thus $\gamma_{\beta_4}[S(L_n)]\le\left\lceil\frac{n+1}{8}\right\rceil$ (2)

From equation (1) and (2), $\gamma_{\beta 4}[S(L_n)]=\left\lceil\frac{n+1}{8}\right\rceil, n\ge 17.$

Example: The 4-metro domination number of slanting ladder graph is 3

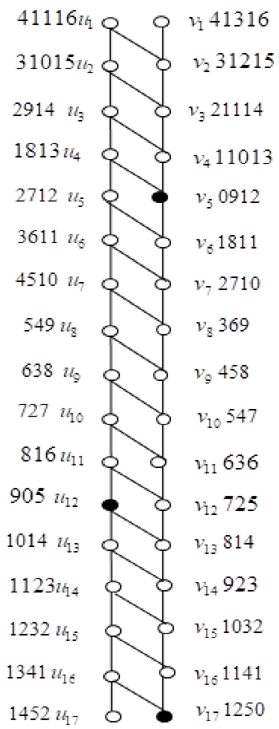

Figure 2: $\gamma_{\beta_4}[S(L_{17})]=3$

Theorem 2.3: For any integer n, $\gamma_{\beta_K}\left[S(L_n)\right] \le \left\lceil \frac{n+1}{2K} \right\rceil, n \ge 4K+1$ $, where n \ne 4Kl, l \ge 1.$

Proof: Let $G = S(L_n)$ be an slanting ladder graph on $2n$ vertices with $V(G) = \{u_i, v_i : 1 \le i \le n\}$ and $E(G) = \{(v_i, v_{i+1}), (v_i, v_{i+1})(u_i, u_{i+1}) : 1 \le i \le n-1\}$, with for each i, u_i v_i the only edges between two paths $W = V - D$, Now each $v_i \in W$ is either adjacent to any of the vertex D or at least at a distance 2 from at least one of the vertex D. Any vertex $v_K \in D$, will dominates at least $4K+1$ vertex including itself. The lower bound of $S(L_n)$ of order $n = (4K+1)l$ for some $l \ge 1$.

We defines a set D as follows

$$D_1 = \{u_{4Kl-K} : l \ge 1\}\; n \equiv 3K(\mathrm{mod}\, 2(K+1))$$
$$D_2 = \{v_{4Kl-3K+1} : l \ge 1\}\; n \equiv K+1(\mathrm{mod}\, 2(K+1))$$

We note that D is also k-dominating set for $S(L_n)$ and also D will serves as metric set of $S(L_n)$ as in 1.1.

Thus, $\gamma_{\beta_K}\left[S(L_n)\right] \le \left\lceil \frac{n+1}{2K} \right\rceil, n \ge 4K+1.$

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