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On the k-metro domination number of cartesian product of $C_3 \times C_n$

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***Abstract*---**A dominating set D of a graph $G=(V,E)$ is called metro dominating set G if for every pair of vertices u,v there exists a vertex w in D such that " $d(u,w) \neq d(v,w)$ .The K"-metro domination number of Cartesian product of $C_3 \times C_n (\gamma_{\beta_k}(C_3 \times C_n))$, is the order of smallest K-dominating set of $C_3 \times C_n$ which resolves as a metric set. In this paper we determine K-metro domination number of Cartesian product of $C_3 \times C_n$.

***Keywords*---**Distance matrix, metric dimension, Land mark, Dominating set, Metro dominating set, K-Dominating set.

1. Introduction

Let $G=(V,E)$ be a graph. A subset of vertices $D \subseteq V$ is called a dominating set $(\gamma - set)$ if every vertex n adjacent to at least one vertex in D. The minimum cardinality of dominating set is called domination number of graph G and it is denoted by $\gamma(G)$.

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“A subset $S \subseteq V$ is called resolving set if every pair of $u, v \in V$, there exist a vertex $w \in V$ such that the distance between vertices $u, v \in V(G)$ is represented as $d(u,w) \neq d(v,w)$. A set of vertices $S \subseteq V(G)$ resolves G, then S is a resolving set S of G and its minimum cardinality is a metric basis of G, and its cardinality is called metric dimension of G and it is denoted by $\beta(G)$.”

“A subset D of $V(G)$ is K-dominating set in G if every vertex of $V - D$ has at least K neighbours in D.The cardinality of minimum K- dominating set is called K-domination number of G and it is denoted by $\gamma_K(G)$. A dominating set D of a graph $G = (V, E)$ is called metro dominating set G if for every pair of vertices u, v there exists a vertex w in D such that $d(u,w) \neq d(v,w)$.The K-metro domination number of Cartesian product of graph G, is the order of smallest K-dominating set of graph G which resolves as a metric set.” Vizing V.G[5] was initiated by the domination number of cross products of graphs and also were intensively investigated in the past(See [9],[13],[23]).

”The Cartesian product of two graphs G, H is a graph with vertex set $V(G) \times V(H)$ and $((g_1,h_1),(g_2,h_2) \in E(G \times H)$ if and only if either $g_1 = g_2$ and $(h_1,h_2) \in E(H)$ or $(g_1,g_2) \in E(G)$ and $(h = h_1 \cup h_2)$.”

2 Main Results

Theorem2.1: For all $m, n,$ $\gamma_{\beta_2}(C_3 \times C_n) = \left\lceil \frac{n}{3} \right\rceil, n \geq 10.$

Proof: Let D be a dominating set of $C_3 \times C_n$.

Let $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n \,\&\, w_1, w_2, \ldots w_n$ are the vertices of $C_1, C_2 \,\&\, C_3$ respectively, such that for each $i, i = 1,2,\ldots,n-1$. $W = V - D$, now each $v_i \in W$ is either adjacent to any of the vertex D or at least at distance two from at least one of the vertex of D. Any vertex $v_k \in D$, will dominates at least 5 vertices including itself. Since the metric dimension of $C_3 \times C_n$ is 3 if m *or* n is odd, and 4 otherwise [2]. D itself serves as a metric set.

Thus $$\gamma_{\beta_2}(C_3 \times C_n) \geq \left\lceil \frac{n}{3} \right\rceil.$$

(1)

To Prove $\gamma_{\beta_2}(C_3 \times C_n) \leq \left\lceil \frac{n}{3} \right\rceil$

We define a set D as follows.

$$D_1 = \{u_{6l-5} : l \geq 1\}, n \equiv 1(\mathrm{mod}\, 6)$$
$$D_2 = \{w_{6l-2} : l \geq 1\}, n \equiv 4(\mathrm{mod}\, 6)$$

We note that D is a 2-dominating set of $C_3 \times C_n$, and also D will serves as a metric set of $C_3 \times C_n$ as in [2].

Thus $$\gamma_{\beta_2}(C_3 \times C_n) \leq \left\lceil \frac{n}{3} \right\rceil$$

(2)
From (1) and (2),

$$\gamma_{\beta_2}(C_3 \times C_n) = \left\lceil \frac{n}{3} \right\rceil$$

Theorem2.2: For all $m, n,$ $\gamma_{\beta_3}(C_3 \times C_n) = \left\lceil \frac{n}{4} \right\rceil, n \geq 12.$

Proof: Let D be a dominating set of $C_3 \times C_n$. Let $u_1, u_2, \ldots. u_n, v_1, v_2, \ldots. v_n \,\&\, w_1, w_2, \ldots. w_n$ are the vertices of $C_1, C_2 \,\&\, C_3$ respectively, such that for each $i, i = 1,2,\ldots., n-1$. $W = V - D$, now each $v_i \in W$ is either adjacent to any of the vertex D or at least at distance two from at least one of the vertex of D. Any vertex $v_k \in D$, will dominates at least 7 vertices including itself. Since the metric dimension of $C_3 \times C_n$ is 3 if m *or* n is odd, and 4 otherwise [2]. D itself serves as a metric set.

Thus $$\gamma_{\beta_3}(C_3 \times C_n) \geq \left\lceil \frac{n}{4} \right\rceil.$$

(1)

To Prove $\gamma_{\beta_3}(C_3 \times C_n) \leq \left\lceil \frac{n}{4} \right\rceil$

We define a set D as follows.

$$D_1 = \{u_{8l-7} : l \geq 1\}, n \equiv 1(\mathrm{mod}\, 8)$$
$$D_2 = \{w_{6l-2} : l \geq 1\}, n \equiv 6(\mathrm{mod}\, 8)$$

We note that D is a 3-dominating set of $C_3 \times C_n$, and also D will serves as a metric set of $C_3 \times C_n$ as in [2].

Thus $$\gamma_{\beta_3}(C_3 \times C_n) \leq \left\lceil \frac{n}{4} \right\rceil$$

(2)
From (1) and (2),

$$\gamma_{\beta_3}(C_3 \times C_n) = \left\lceil \frac{n}{4} \right\rceil$$

Theorem2.3: For all $m,n,\ \gamma_{\beta_4}(C_3 \times C_n) = \left\lceil \frac{n}{5} \right\rceil, n \geq 14.$

Proof: Let D be a dominating set of $C_3 \times C_n$. Let $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n \,\&\, w_1, w_2, \ldots w_n$ are the vertices of $C_1, C_2 \,\&\, C_3$ respectively, such that for each $i, i = 1,2,\ldots,n-1$. $W = V - D$, now each $v_i \in W$ is either adjacent to any of the vertex D or at least at distance two from at least one of the vertex of D. Any vertex $v_k \in D$, will dominates at least 9 vertices including itself. Since the metric dimension of $C_3 \times C_n$ is 3 if m *or* n is odd, and 4 otherwise [2]. D itself serves as a metric set.

Thus $$\gamma_{\beta_4}(C_3 \times C_n) \geq \left\lceil \frac{n}{5} \right\rceil.$$

(1)

To Prove $\gamma_{\beta_4}(C_3 \times C_n) \leq \left\lceil \frac{n}{5} \right\rceil$

We define a set D as follows.

$$D_1 = \{u_{10l-9} : l \geq 1\}, n \equiv 1 (\text{mod } 10)$$
$$D_2 = \{w_{10l-2} : l \geq 1\}, n \equiv 8 (\text{mod } 10)$$

We note that D is a 3-dominating set of $C_3 \times C_n$, and also D will serves as a metric set of $C_3 \times C_n$ as in [2].

Thus $$\gamma_{\beta_4}(C_3 \times C_n) \leq \left\lceil \frac{n}{5} \right\rceil$$

(2)

From (1) and (2),

$$\gamma_{\beta_4}(C_3 \times C_n) = \left\lceil \frac{n}{5} \right\rceil$$

3 Generalization of K-MD set of $C_3 \times C_n$

Theorem3.1: For all $m,n\ \gamma_{\beta_K}(C_3 \times C_n) \leq \left\lceil \frac{n}{K+1} \right\rceil,\ n \geq 2K+6$

Proof: Consider $C_3 \times C_n$ as three canonical copies of C_n with vertices labelled $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n \,\&\, w_1, w_2, \ldots w_n$ are the vertices of $C_1, C_2 \,\&\, C_3$ respectively, such that for each $i, i = 1,2,\ldots,n-1$. $W = V - D$, now each $v_i \in W$ is either adjacent to any of the vertex D or at least at distance two from at least one of the vertex of D. Any vertex $v_k \in D$, will dominates at least $2K+6$ vertices

including itself. The lower bound of $C_3 \times C_n$ of order $n=(2K+6)l$ for some $l \geq 1$
.
We define a set D as follows.

$$D_1 = \{u_{2(K+1)l-(2K+1)} : l \geq 1\},\ n \equiv 1(\mathrm{mod}\ 2(K+1))$$

$$D_2 = \{w_{2(K+1)l-2} : l \geq 1\},\ n \equiv 2K(\mathrm{mod}\ 2(K+1))$$

We note that D is K-dominating set of $C_3 \times C_n$, and also D will serves as a metric set of $C_3 \times C_n$ as in [2] Thus $\gamma_{\beta_K}(C_3 \times C_n) \leq \left\lceil \frac{n}{K+1} \right\rceil$

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