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Benders decomposition approach to solve a Bi-objective cell formation in dynamic conditions

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***Abstract*---**Cellular Manufacturing System (CMS) is an alternative to production systems based on process and product layout that combines the advantage of high throughput rate of flow shop system with the flexibility of job shop layout. CMS design has four steps: 1) Cell formation: grouping the parts with similar geometric design features and processing requirements in part families to use similarities to build, manufacture and assemble machines in machine lines and cells, 2) group layout: placing machines in each cell that includes intra-cell and inter-cell layout, 3) group scheduling: part family scheduling, and 4) source allocation: allocation of tools and primary resources and human force. Dynamic CMS (DCMS) can be divided into smaller periods, each of which is different from the previous one as these periods have various demands and compositions. The study presented a mixed-integer programming model for DCMS. The model considered some significant real-world conditions. In this model, the first objective function tries to minimize the costs related to the machines purchase cost, machines variable cost, the cost of movement workers, the cost of cell formation, the cost of inter-cell and intra-cell part movement, the cost of overtime, and the cost of reconfiguration. Moreover, the deviation of cell workload in each period is minimized in the second objective function. The model is then resolved and validated in GAMS 25.1.3 the exact method of Benders Decomposition solving on the problem was implemented given the NP-hard nature of DCMS problems.

***Keywords*---**DCMS, cell formation, group layout, mixed integer programming, benders decomposition algorithm.

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Introduction

Nowadays, cellular manufacturing is a production approach to reduce costs and increase system flexibility in the environment of converting small to medium-sized production groups. The advantage of cell production reported from the literature is related to reduce inventory under construction, preparation time, improved flexibility, better production control, and shorter delivery time (Askin & Estrada, 1999). With the introduction of the machine-part grouping problem by Burbidge (1984), which was the first step in the analysis of production flow, the problem of cell formation was considered by scholars with many advances in the theory, tools, and techniques of this problem since then. DCMS involves several operational and structural problems. Besides the problem of cell formation, another key step to consider is designing the machine and cellular layout.

The efficient design of this step greatly relies on considering related design dimensions. As proposed by Arvindh and Irani (1994), an integrated approach in designing CMS has to be considered, since the design characteristics of a system are interdependent in various ways. Accordingly, this section will present a new integrated mathematical model for the problem of cell formation in DCMS. Extensive studies have been carried out on the problem of cell manufacturing and many solutions have been proposed to this problem. The comprehensive classifications of the studies conducted on designing cellulite production systems have been done by many scholars.

Defersha and Chen (2006) incorporated the dynamic cell configuration, alternative routing, operation sequencing, workload balancing, lot splitting, machine adjacency, subcontracting, and tool consumption in an integrated problem. Ahi et al. (2009) applied the concept of multi-criteria decision-making and suggested a two-step method for cellular layout, intra-cell layout, and inter-cell layout, as the three basic characteristics in designing a CMS. Ahkioon (2009) modeled the problem of designing cell manufacturing along with multi-cycle product planning, cell reconfiguration, operation sequence, multiple machine versions, machine capacity, and introducing the processing path flexibility by forming possible processing paths in modeled alongside alternating main processing paths.

Ahkioon et al. (2009) formulated a hybrid approach to CMS design as a nonlinear mixed-integer model with production planning and system reconfiguration decisions with alternative process routing, operation sequence, machine capacity, and component divisibility. Deljou et al. (2010) solved the problem of dynamic cell formation using genetic algorithm (GA). They presented a mathematical formulation that eliminated some of the shortcomings of previous mathematical models (Arianjad et al., 2009). A new mathematics method was developed to simultaneously interact with the problem of dynamic cell formation and labor allocation by considering processing path flexibility, machine flexibility, and worker upgrading from one skill level to another. Objective functions include cost components like production, transportation between cellular materials, machine overhead, salaries, recruitment, dismissal, and training of human force.

Aramoon Bajestani (2009) suggested a dynamic multi-objective model of cell formation, where the sum of the costs of changes in cell formation and the sum of miscellaneous costs (machine cost, inter-cell movement cost, and machine relocation cost) are minimized simultaneously. They designed a multi-purpose model of a scattered search to find the optimal boundary. Ghotboddini and al. (2011) suggested a dynamic cell formation and human resource assignment, they focused on process routing, firing and hiring labors, inter-cell and intra-cell movement of parts, and cell formation during planning periods, and they presented an exact Benders' decomposition approach to generate an optimal solution for intended (DCMS) model.

Saxena and Jain (2011) suggested a DCMS model that integrates the problem of reliability by considering the effect of machine failure and production planning with consideration of inventory maintenance, domestic production, and outsourcing. Design features include production batch size, intra-cell motion batch size, production batch division, alternative processing path, operation sequence, multiple machine versions, machine capacity, cutting tool requirements, workload balance, machine proximity constraints, machine purchase, and cellular reconfiguration.

Previous studies by King and Nakornchai (1982) and Ballakur (1985) have revealed that cell formation problem belongs to NP-hard group problems. Thus, the model considered in the study, developed from previous studies on cell formation, is of this sort. Liu, Wang, and Leung JYT (2018) studied the advantages and disadvantages of lot splitting in the dynamic CMS (Liu et al.). The study used the problem presented by (Rafiei and Ghodsi, 2013) to solve the model of the cell formation problem. As the model is considered to be a dynamic bi-objective model for cell formation, in similar studies, a precise approach to solving the model is not presented and only meta-heuristic methods are used. This is the first time that an exact solution approach is designed to solve this model. Hence, the main research question is "Are there logical reasons and causes for using Benders Decomposition algorithm in the problem of bi-objective cell formation?

Methods

The proposed mathematical model

The study proposed a new mathematical model to design the problem of cell formation in dynamic conditions. Given the structure governing the problem - linearization of the objective function and existing constraints, as well as conversion to a single-objective mode using the standardization approach - one can use GAMS Software and run Benders solution algorithm in this software to solve the model in the following conditions. This example was solved in GAMS 25.1.3 on a dual-core Intel 3.2 GHz processor with 4 GB of RAM on a personal computer to validate this proposed model. Objective functions have different units, so combining them without standardization is useless. Thus, dividing each objective function by its optimal value is an effective way to overcome this problem. The first objective function is of the total cost type and the second is of the time type, so by standardizing them we will have a new objective function without any complex units. Thus, we have to solve each objective function once considering all available constraints.

Model features

In the first objective function, we have tried to minimize costs like purchase cost, cost of inter-cell labor moving, inter-cell, and intra-cell transfer costs of parts, overtime costs, and cost of cell manufacturing. Moreover, the second objective function is associated with the total load deviation of the cells to balance manual workload (by labor force) and machine (time done by machine), reduce inventory in work-in-progress (WIP), modification of the flow of parts between or inside the cell and prevention of cell-related abuse (overuse or less than the expectation) (Baykasoglu et al., 2001). The section presents a bi-objective programming model mixed-integer programming (MIP) from a DCMS.

Parameters

The number of planning periods	H
The number of part types	P
The number of operations related to the part	Op
The number of machine types	M
Total number of labor (human force)	L
The maximum number of working cells that can be formed	C
Demand for Part p in time h	D_{ph}
If the Part p is programmed to be produced in period h 1, otherwise it gets a value of zero	v_{ph}
The size of the Part p clusters in inter-cell movements	B_p^{inter}
The size of the Partp clusters in intra-cell movements	B_p^{intra}
The cost of moving component packages between work cells	γ_p^{inter}
The cost of moving parts packages inside work cells	γ_p^{intra}
Cost of buying an m-type machine	ϕ_m
The sale price of the m-type machine	Wm
Fixed cost of m-type machines in each planning period	α_m
Cost of inter-cell labor moving	ρ_h
The variable cost of m-type machines for each unit of time in normal working time	βm
The cost of moving an m-type machine	δ_m
Time capacity of m-type machines in period h at normal working time	Tmh
Time capacity of m-type machines in period h in overtime	T'_{mh}
The variable cost of processing operations on m-type machines per hour during overtime in period h	θ_{mh}
Maximum cell size	UB
Minimum cell size	LB
The processing time needed to perform operation j from Part p on machine m	t_{jpm}
Manual processing time (loading) needed to perform operation j Part from p on m-type machines	t'_{jpm}

If the operation j can be performed from the Part p on m-type machine 1, otherwise it gets a zero	a_{jpm}
Cost of cell manufacturing in period h	FC_h
Working time available to operators (human force) during the period h	WT_h

Decision variables

The number of m-type machines assigned to cell c over a period	N_{mch}
The number of m-type machines added to cell c in period h	K^{+}_{mch}
The number of m-type machines removed from cell c in period h	K^{-}_{mch}
The number of m-type machines purchased in period h	I^{+}_{mh}
The number of m-type machines sold in period h	I^{-}_{mh}
If operation j is performed from Part p on machine m in period 1, otherwise it gets a zero	X_{jpmch}
The number of human forces assigned to cell c in period h	L_{ch}
If the cell is formed 1 otherwise it gets a zero	Y_{ch}
Extra time needed for the m-type machine in cell c in period h	T'_{mch}

A linearized model of the objective function

Here, we rewrite the proposed formula as a linear programming model mixed with integers. This re-formulation needs several alternative uses and various operations that end in a precise deformation of the problem. The following bi-objective model is formulated as follows.

Min Z_1=

$\sum_{h}^{H}\sum_{m}^{M}\sum_{c}^{C} N_{mch}\alpha_m$	(1)
$+\sum_{h}^{H}\sum_{m}^{M} I^{+}_{mh}\,\Phi_m$	(2)
$-\sum_{h}^{H}\sum_{m}^{M} I^{-}_{mh}\,w_m$	(3)
$+\sum_{h}^{H}\sum_{c}^{C}\sum_{p}^{P}\sum_{j}^{Op} \beta_m D_{ph}\, t_{jpm}\, X_{jpmch}$	(4)
$+\frac{1}{2}\sum_{h}^{H}\sum_{p}^{P} \gamma_p^{inter}\left\lceil\frac{Dph}{B_p^{inter}}\right\rceil\sum_{j}^{Op-1}\sum_{c}^{C}(z^1_{jpch}+z^2_{jpch})$	(5)
$+\frac{1}{2}\sum_{h}^{H}\sum_{p}^{P} \gamma_p^{intra}\left\lceil\frac{Dph}{B_p^{intra}}\right\rceil\sum_{j}^{Op-1}\sum_{c}^{C}((y^1_{jpmch}+y^2_{jpmch}))-(z^1_{jpch}+z^2_{jpch}))$	(6)

$+\sum_{h}^{H}\sum_{m}^{M}\sum_{c}^{C} T'_{mch}\,\Theta_{mh}$	(7)
$+\frac{1}{2}\sum_{h}^{H}\sum_{c}^{C} \rho_h\,(w_{ch}^{1} + w_{ch}^{2})$	(8)
$+\frac{1}{2}\sum_{h}^{H}\sum_{c}^{C}\sum_{m}^{M} \delta\mathrm{m}\,(K_{mch}^{+} + K_{mch}^{-})$	(9)
$+\sum_{c}^{C}\sum_{h}^{H} Y_{ch}FC_h$	(10)

Min Z_2=

$\sum_{c}^{C}\sum_{h}^{H}(F_{ch}^{1} + F_{ch}^{2})$	(11)
However, the new variables are added to the previous variables $Z_{jpch}^{1}, Z_{jpch}^{2}, Y_{jpmch}^{1}, Y_{jpmch}^{2}, W_{ch}^{1}, W_{ch}^{2} \geq 0$ and integer	
$F_{ch}^{1}, F_{ch}^{2} \geq 0$	

In the first objective function, numbered from (1) to (10), simultaneous cost minimization is the question. The total fixed and variable costs of the machine, the costs of inter-cell and intra-cell movement, costs including overtime, labor, purchasing, and cell manufacturing are included in the planning period. Section (1) considers the total fixed costs of the machine-like energy and maintenance used in each planning period. Section (2) is the total cost of purchasing machines in all planning periods. Section (3) avoids imposing additional costs on the same machine (maximizing sales revenue).

$\sum_{c}^{C}\sum_{m}^{M} a_{jpm} X_{jpmc\,h} = v_{ph}$	$\forall\, j, p, m, c, h$	(12)
$X_{jpmc\,h} \leq a_{jpm}$	$\forall\, j, p, m, c, h$	(13)
$X_{jpmc\,h} \leq N_{mc\,h}$	$\forall\, j, p, m, c, h$	(14)
$\sum_{p}^{P}\sum_{j}^{Op} D_{ph} t_{jpm} X_{jpmc\,h} \leq T_{mh} N_{mc\,h} + T'_{mc\,h}$	$\forall\, m, c, h$	(15)
$\sum_{c}^{C} N_{mc\,h} - \sum_{c}^{C} N_{mc\,(h-1)} = I^{+}_{mh} - I^{-}_{mh}$	$\forall m, h$	(16)
$\sum_{c}^{C} T'_{mc\,h} \leq T'_{mh}$	$\forall\, m, h$	(17)
$\sum_{c}^{C} L_{ch} = L$	$\forall\, h$	(18)
$\sum_{c}^{C} N_{mc\,h} \leq UB \times Y_{ch}$	$\forall\, c, h$	(19)
$\sum_{c}^{C} N_{mc\,h} \geq LB \times Y_{ch}$	$\forall\, c, h$	(20)
$N_{mc\,(h-1)} + K^{+}_{mc\,h} - K^{-}_{mc\,h} = N_{mc\,h}$	$\forall\, m, c, h$	(21)
$\sum_{p}^{P}\sum_{j}^{Op} D_{ph} t'_{jpm} X_{jpmc\,h} \leq L_{ch}\, WT_{h}$	$\forall\, m, c, h$	(22)

Section (4) is all the variable costs of the machine in all cells and programming cycles. This cost, calculated from the time of operation on the machine, can be calculated for each machine in each period. There are direct relationships between machine fixed and variable costs. Sections (5) and (6), the costs of intra-cell and inter-cell movement of parts, can be expressed from the sequence of operations of the desired part on various or similar machines in the planned periods, comprehensively described by Safaei et al. (2008). Section (7) calculates the cost of overtime needed to generate a small fraction of demand. Section (8) calculates the set of cost of inter-cell labor moving during the planning period, including different parameters like training, incoming rate, skill, and the inter-cell labor movement. Section (9) shows the set of machine relocation costs for reconfiguration. Section (10) is the set of costs related to the manufacturing of a work cell in each planning period. The coefficients (1.2) given in Sections (5), (6), (8), and (9) are because each movement in the model is calculated twice. Section (11) in the second objective function is related to the total loading deviation of the cells.

Constraints

$L_{ch}, N_{mch}, K^{+}_{mch}, K^{-}_{mch}, I^{+}_{mh}, I^{-}_{mh} \geq 0$ and integer
Y_{ch} و $X_{jpmch} \in \{0,1\}$
$T'_{mch} \geq 0$

Constraint (12) ensures that each operation is assigned to only one machine and one cell. Moreover, if the left constraint is equal to one, the whole part production operation is carried out on the machine in the cell in the same period and no combination is possible. Constraint (13) will not allow the variable x to take a value if a parameter like a (previously defined) has a value of zero. This means that variable x can take a value if the desired operation of the Part p can be performed on machine m. Constraint (14) ensures that operations related to Part p component are assigned to a machine in the cell in question, either during normal working hours or during overtime. Constraint (15) ensures that the operation time to meet the demand does not exceed the capacity of the machine. In this model, two types of time capacity are considered for the machine, one of which is associated with the capacity in normal working time and the other with the capacity in overtime. If an operation cannot be done in the usual time, it will be done in overtime. Constraint (16) is the number of machines purchased or sold during each planning period, which is related to the cost of sections (3) and (2) of the objective function. Constraint (17) is the total time allocated to each cell on any type of machine that cannot exceed the capacity of that machine during overtime, in this model; overtime is defined for each machine.

Section (18) of this constraint shows the sum of the number of labor forces available in all planning periods (already known). Constraints (19) and (20) apply to the upper and lower borders of the cell (cell size) in a case formed. Constraint (21) states that the number of machine types in the current period in a particular cell is equal to the number of one machine type in the previous period, plus the number of machines added minus the number of machines deducted from the same cell. Constraint (22) states that the loading time for the production of parts in period h does not exceed the time available to the labor forces.

Multi-objective solution approach

The standardized method is proposed as a solution to convert multiple objective functions to single-objective functions. At the beginning of each of the solution approaches stated, each objective function has to be solved independently, by considering all the constraints and then using the real value of all objective functions to build a standardized objective function. Considering the following example function, we have:

Min $f_1(x)$,
Min $f_2(x)$,
$x \in s$

We can have the standardized model as follows where s is a space that is possible and f_i (x) is the ith of our objective function.

$Minf(x) = \frac{1}{f_1^*(x)} \times f_1(x) + \frac{1}{f_2^*(x)} \times f_2(x),$

Here, because of the existence of two objective functions, the maximum value for i is two, and $f_i^*(x)$ is the optimal value of the ith objective function.

Benders solution approach (Benders Decomposition)

In this approach, the correct answer is obtained from the master problem and sent to the problem of the sub-dual function, and then the sub-dual function produces a scalable or optimal solution for the corresponding correct solution that may be added to the master problem. These iterations continue so that the top and bottom are equal or the answers are as close as we want. Moreover, in each iteration, we do not need to solve the master problem to reach the optimal solution, yet in each iteration, we need only one solvable (Kot and Lawton, 1984). The general state of MIP problems can be presented as follows:

P_{main} :Min C_1X+C_2Y

$$AX \geq b1$$
$$BX + DY \geq b2$$

$X \in Z^+$, $Y \in R^+$

In the model above A ($m_1 \times n_1$), B ($m_2 \times n_1$), and D ($m_2 \times n_2$) are considered as matrices with real numbers, and C_1 and C_2 are cost vectors, and b_1 and b_2 are defined values on the right, defined as matrices with real numbers. The master problem is written as follows:

P_{master}MinC_1X+Z

$$AX \geq b1$$

$w_k^t(b2 - BX) \leq Z \quad k = 1, \dots, |k|$

$\Omega_f^t(b2 - BX) \leq 0 \quad f = 1, \dots, |f|$

$X \in Z^+$, $Z \in R$

F and K present a set of feasible and optimal cuts, and Z is a continuous variable acting as a continuous part of the main objective function (C2Y). By solving the master problem, we bring the integer variables to a constant value and obtain an LP problem as a sub-dual function. We use the symbol ($\overline{\ }$) above the variables that have been fixed in their value.

P_{sub}Min C_2Y

$$DY \geq b2 - B\overline{X}$$

, $Y \in R^+$

The problem of sub-dual function is as follows:

$P_{sub\text{-}dual}$ Max $w^T(b2 - B\overline{X})$

$$w^T D \leq C2$$

$W \geq 0$

The master problem may be destroyed if the problem is a function of infinite duality. Thus, any small value of infinity Ω will be selected and added to F to be cut by Benders.

$$\Omega_f^t(\mathrm{b2} - \mathrm{BX}) \leq 0$$

In other words, if the problem is a double-bounded function, an optimal solution of the optimal bands section will be added to the set K.

$$w_k^t(\mathrm{b2} - \mathrm{BX}) \leq \mathrm{Z}$$

This algorithm may be iterated and end in computational time related to the software or will be carried out in the needed number of iterations. The method of deriving the master problem and the sub-dual function of the basic problem as stated in the previous section, we have to separate the problem into two master problems and a sub-dual function to run the Benders Decomposition algorithm. This is initially done based on the circumstances of the forthcoming MIP problem.

Sub-problem

To write the sub-problem of the problem of the terms and constraints, we delete the ones that contain only the integer variable and write the remaining terms.

Min Z=

$$(1/opt1) \times \sum_h^H \sum_m^M \sum_c^C T'_{mch}\,\Theta_{mh} + (1/opt2) \sum_c^C \sum_h^H (F_{ch}^1 + F_{ch}^2)$$

Constraints

$T'_{mch} \geq \sum_p^P \sum_j^{Op} \bar{X}_{jpmch} t_{jpm} D_{ph} - T_{mh}\,\bar{\mathrm{N}}_{mch}\ \ , Q_{mch}$	$\forall_{m,c,h}$	(23)
$-\sum_c^C T'_{mch} \geq -T'_{mh}\ \ , U_{mh}$	$\forall_{m,h}$	(24)
$F_{ch}^1 - F_{ch}^2 = \sum_j^{Op} \sum_p^P \sum_m^M D_{ph}\,\bar{X}_{jpmch} t_{jpm} - \frac{\sum_j^{Op} \sum_p^P \sum_m^M D_{ph}\,\bar{X}_{jpmch} t_{jpm}}{NC_h}\ \ , R_{ch}$	$\forall_{c,h}$	(25)

Constraint 25 breaks into the following two constraints to standardize for writing a sub-dual model.

$F^1_{ch} - F^2_{ch} \geq \sum_j^{Op} \sum_p^{P} \sum_m^{M} D_{ph}\, \bar{X}_{jpmch} t_{jpm} - \frac{\sum_j^{Op} \sum_p^{P} \sum_m^{M} D_{ph}\, \bar{X}_{jpmch} t_{jpm}}{NC_h}$, R^1_{ch}	$\forall c,h$	(26)
$F^1_{ch} - F^2_{ch} \geq \frac{\sum_j^{Op} \sum_p^{P} \sum_m^{M} D_{ph}\, \bar{X}_{jpmch} t_{jpm}}{NC_h} - \sum_j^{Op} \sum_p^{P} \sum_m^{M} D_{ph}\, \bar{X}_{jpmch} t_{jpm}$, R^2_{ch}	$\forall c,h$	(27)

The sub-dual variables are added as follows.

$$Q_{mch}\ ,\quad U_{mh}\ ,\quad R^1_{ch}\ ,\quad R^2_{ch} \geq 0$$

In the standard model of the problem (in the minimal state) and considering the definition of the sub-dual variables (Q_{mch}، U_{mh}،$R1_{ch}$، $R2_{ch}$) defined, one can write the sub-dual of the mentioned problem. Sub-dual function problem: Based on the sub-dual variables defined above, we use the usual method in sub-dual writing based on operations research. The problem is maximized and we will have constraints as many as the number of problem variables and problem constraints, obtained as follows

Max Z=

$\sum_m^{M} \sum_c^{C} \sum_h^{H} Q_{mch} [\sum_j^{Op} \sum_p^{P} \bar{X}_{jpmch} t_{jpm} D_{ph} - T_{mh}\, \bar{\text{N}}_{mch}]$
$-\sum_m^{M} \sum_h^{H} T'_{mh}\, U_{mh}$
$+\sum_c^{C} \sum_h^{H} R^1_{ch} [\sum_j^{Op} \sum_p^{P} \sum_m^{M} D_{ph}\, \bar{X}_{jpmch} t_{jpm} - \frac{\sum_j^{Op} \sum_p^{P} \sum_m^{M} D_{ph}\, \bar{X}_{jpmch} t_{jpm}}{NC_h}]$

$$+\sum_{c}^{C}\sum_{h}^{H} R_{ch}^{2}\left[\frac{\sum_{j}^{Op}\sum_{p}^{P}\sum_{m}^{M} D_{ph}\,\bar{X}_{jpmch}t_{jpm}}{NC_h} - \sum_{j}^{Op}\sum_{p}^{P}\sum_{m}^{M} D_{ph}\,\bar{X}_{jpmch}t_{jpm}\right]$$

Constraints

$Q_{mch} - U_{mh} \leq (1/opt1) \times \Theta_{mh}$	$\forall m,c,h$
$R_{ch}^{1} - R_{ch}^{2} \leq (1/opt2)$	$\forall\ c,h$
$R_{ch}^{2} - R_{ch}^{1} \leq (1/opt2)$	$\forall\ c,h$

The inherent constraints of the problem are expressed as follows.

$$Q_{mch}\ ,\quad U_{mh}\ ,\quad R_{ch}^{1}\ ,\quad R_{ch}^{2} \geq 0$$

The master problem

In the basic model, we write all the constraints of integer type with the remainder as follows: Here is the master problem with cut or optimizing that should be added in each iteration:

$Min\ Z$

$\sum_{c}^{C}\sum_{m}^{M} a_{jpm}X_{jpmch} = v_{ph}$	$\forall j,p,h$
$X_{jpmch} \leq a_{jpm}$	$\forall j,p,m,c,h$
$X_{jpmch} \leq N_{mch}$	$\forall j,p,m,c,h$
$\sum_{c}^{C} N_{mch} - \sum_{c}^{C} N_{mc(h-1)} = I_{mh}^{+} - I_{mh}^{-}$	$\forall m,h$

$\sum_{c}^{C} L_{ch} = L$	$\forall h$
$\sum_{c}^{C} N_{mch} \leq UB \times Y_{ch}$	$\forall c,h$
$\sum_{c}^{C} N_{mch} \geq LB \times Y_{ch}$	$\forall c,h$
$N_{mc(h-1)} + K_{mch}^{+} - K_{mch}^{-} = N_{mch}$	$\forall m,c,h$
$z_{jpch}^{1} - z_{jpch}^{2} = \sum_{m}^{M} X_{(j+1)pmch} - \sum_{m}^{M} X_{jpmch}$	$\forall j,p,c,h$
$y_{jpmch}^{1} - y_{jpmch}^{1} = X_{(j+1)pmch} - X_{jpmch}$	$\forall j,p,m,c,h$
$w_{ch}^{1} - w_{ch}^{2} = L_{c(h+1)} - L_{ch}$	$\forall c, h \neq H$

Feasibility cut restriction which might be added in any iteration:

$$Z \geq \sum_{m}^{M} \sum_{c}^{C} \sum_{h}^{H} Q_{mch} [\sum_{j}^{Op} \sum_{p}^{P} \bar{X}_{jpmch} t_{jpm} D_{ph} - T_{mh} \bar{N}_{mch}] - \sum_{m}^{M} \sum_{h}^{H} T'_{mh} U_{mh} + \sum_{c}^{C} \sum_{h}^{H} R_{ch}^{1} [\sum_{j}^{Op} \sum_{p}^{P} \sum_{m}^{M} D_{ph} \bar{X}_{jpmch} t_{jpm} - \frac{\sum_{j}^{Op} \sum_{p}^{P} \sum_{m}^{M} D_{ph} \bar{X}_{jpmch} t_{jpm}}{NC_h}] + \sum_{c}^{C} \sum_{h}^{H} R_{ch}^{2} [\frac{\sum_{j}^{Op} \sum_{p}^{P} \sum_{m}^{M} D_{ph} \bar{X}_{jpmch} t_{jpm}}{NC_h} - \sum_{j}^{Op} \sum_{p}^{P} \sum_{m}^{M} D_{ph} \bar{X}_{jpmch} t_{jpm}]$$

Results

A numerical example for the proposed model

Table (1) shows the flow path of parts and the value of demand for the Part during each period. The last two rows show the size of the batch of parts for the inter-cell and intra-cell movement. Tables 2 is the value of model parameters and information about the machines.

Table 1
Sample problem input information (material flow path and operation time, demand for Parts, batch size in intercellular and intracellular movements)

The value of the model input parameters									
Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value	Parameter	Value
H	3	Op	3	D_{ph}	$U(100,1000)$	ρ_h	100$	B_p^{inter}	$U(10,50)$Unit
L	5	α_m	$\phi_m / 10$	ϕ_m	$U(10000,20000)$	β_m	$U(0,10)$$	B_p^{intra}	$B_p^{inter} / 10$
C	3	UB	4	δ_m	$2/\alpha_m$	t_{jpm}	$U(0,1)h$	γ_p^{inter}	50$
M	6	LB	2	θ_{mh}	1.78*	t'_{jpm}	$t_{jpm}/10$	γ_p^{intra}	5$
WT_h	50	FC_h	$U(7000,10000)$	* The cost of the machines during normal work hours					

Information and parameters related to machines

Table 2
The value of model input parameters and machine information and parameters

Machines	$T_{mh} \forall h$	$T'_{mh} \forall h$	ϕ_m($)	W_m ($)	($)$\beta_m$	θ_{mh}($)
Machine 1	500	200	12000	8400	6	15
Machine 2	500	200	14000	9800	3	10.35
Machine 3	500	200	15000	10500	7	17.85
Machine 4	500	200	14000	9800	4	12.15
Machine 5	500	200	12000	8400	2	7.85
Machine 6	500	200	16000	11200	8	20

Group	Part 1			Part 2			Part 3			Part 4			Part 5			Part 6			Part 7			Part 8		
Operation	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Machine 1												0.76					0.57							0.54
Machine 2	0.68	0.61		0.67	0.23	0.24		0.19		0.55	0.58		0.71			0.72		0.47	0.44		0.28		0.17	
Machine 3		0.61	0.88		0.48	0.57				0.82										0.97				0.15
Machine 4			0.63				0.13		0.58					0.49						0.47		0.84	0.86	
Machine 5	0.55			0.79				0.89	0.96			0.26	0.17		0.65			0.12	0.76		0.86	0.2		
Machine 6							0.36				0.78			0.45	0.59	0.81	0.48							
Demand	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Period 1	200			0			0			650			350			600			550			600		
Period 2	500			450			0			500			0			500			200			450		
Period 3	600			0			600			0			750			350			300			350		
Patch	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3
Inter-cell	25			25			20			35			20			40			40			30		
Intra-cell	5			5			4			7			4			8			8			6		

Examining the solution of the proposed model

First, considering all the existing constraints and the initial objective function related to cost alone in GAMS software with the conditions of MIP with integers has been solved by Cplex Solver to solve the mentioned dynamic cell manufacturing model. The result of solving this model is as shown in Table (1). Moreover, this work has been performed again on the second objective function, and the result of solving this model is written as mentioned in Table (2). Moreover, the model is solved again and is shown in Table (2) after standardizing the objective function described earlier. The results obtained in the tables are associated with the best values for the objective functions z_1 and z_2 plus costs available in the objective function is as follows: The fixed cost of machines, the cost of machine purchase, the variable cost of machines, cost of inter-cell movement of parts, cost of intra-cell movement of parts, cost of overtime machines, cost of inter-cell labor moving, cost of reconfiguring cells, cost of cell manufacturing as well as the revenue from the sale of machines, which is considered a negative sign in the objective function.

Table 3
Optimal solutions of the objective function

Cell formation cost	Reconfiguring cells	Labor moving	Overtime cost	Intra-cell movement	Inter-cell movement	The variable cost of machines	Machine sales cost	Machine purchase cost	Machine fixed cost	*Z2*	Best Z1 value
50000	3125	0	6731.775	12265	2750	39065	9800	111000	31900	5323.5	247036.78

First objective function

Cell formation cost	Reconfiguring cells	Labor moving	Overtime cost	Intra-cell movement	Inter-cell movement	The variable cost of machines	Machine sales cost	Machine purchase cost	Machine fixed cost	Best Z2 value	Z1
57000	6475	300	6778.3	8060	10350	40726.5	0	143000	36900	5100.5	309590

Standard objective function

Cell formation cost	Reconfiguring cells	Labor moving	Overtime cost	Intra-cell movement	Inter-cell movement	The variable cost of machines	Machine sales cost	Machine purchase cost	Machine fixed cost	*Z2*	*Z1*	Best standardized Z value
50000	3125	0	7256.1	12440	1500	40945.5	9800	111000	31900	5215.5	248380	2.028

Moreover, GAMS Software output is given in two forms in the figure associated with the standard model. Table 4 is the best configuration and flow of materials (parts) obtained from the output of GAMS Software. In the figure, the numbers inside the matrix show the sequence of operations and the numbers on the right the needed overtime hours, and the numbers on the left, the number of machines and human force inside the cell, as well as the cell number, formed. Only two cells will be formed for each planned period as already stated

Table 4
The best cell configuration in the periods plus overtime hours considering the model output in standard mode

First period	Number of manual labor force	The number of machines	Machines	Cell1	Cell1	Cell1	Cell2	Cell2	Cell2	Machine overtime
				Part5	Part6	Part8	Part1	Part4	Part7	
Cell 1		1	Machine 2		1	2		2 and 1		34 hours
Cell 1	2	1	Machine 3			3		3		
Cell 1		1	Machine 5	3 and 1	3	1		Cell 2		
Cell 1		1	Machine 6	2	2					
Cell 2		2	Machine 2				2	2 and 1	2	10.5 hours
Cell 2	3	1	Machine 4				3	3	3	
Cell 2		1	Machine 5				1	Cell 2	1	197 hours

second period	Number of manual labor force	The number of machines	Machines	Cell1	Cell1	Cell1	Cell2	Piece 3	Cell2	Machine overtime
				Part5	Part6	Part8	Part1	Part2	Part7	
Cell 1		1	Machine 2	3 and 2	1	2		3 and 1		148 hours
Cell 1	2	1	Machine 3			3		Cell2		
Cell 1		1	Machine 5	1	3	1				5.5 hours
Cell 1		1	Machine 6		2			2 and 1		
Cell 2		2	Machine 2				2	3	3 and 1	14 hours
Cell 2	3	1	Machine 4				3		2	
Cell 2		1	Machine 5				1			

Third period	Number of manual labor force	The number of machines	Machines	Cell1	Cell1	Cell1	Cell1	Cell 2	Cell2	Machine overtime
				Part1	Part8	Part5	Part6	Part 3	Part7	
Cell 1		1	Machine 2	2	2			2	3	9.5 hours
Cell 1		1	Machine 3	3	3			3		80.5 hours
Cell 1	2	1	Machine 5	1	1	1	3	2		69.5 hours
Cell 1		1	Machine 6			2	2	3 and 1		5.5 hours
Cell 2		1	Machine 2				1		1	
Cell 2		1	Machine 4						2	67 hours
Cell 2		1	Machine 5			3				

Implementation of Benders Decomposition solution

It is necessary to code according to the master problem and the sub-dual problem function previously distributed to solve the model of cell formation in dynamic conditions for implementing the Benders Decomposition algorithm in the GAMS Software environment. This coding is provided in the second appendix of the study. Primary is derived. These iterations continue until the master problem and the sub-dual function problem are as close as possible when the value of the variables extracted from the software is shown as the desired output. Table 5 is the output of GAMS Software with sample numbers as a matrix after running the Benders algorithm.

Table 5
The best cell configuration in periods plus overtime hours, after the implementation of the Benders Decomposition algorithm

First period	Number of manual labor force	The number of machines	Machines	Cell1	Cell1	Cell1	Cell1	Cell1	Cell2	Machine overtime
				Part5	Part1	Part4	Part6	Part8	Part7	
Cell 1	3	1	Machine2			1		2		183.25 hours
Cell 1		1	Machine3		2,3			3		43 hours
Cell 1		1	Machine5	1		3	3	1		5 hours
Cell 1		1	Machine6	3			2			138.5 hours
Cell 2	2	3	Machine2		1	2	1		1,3	57 hours
Cell2		2	Machine4	2					2	59.25 hours

Second period	Number of manual labor force	The number of machines	Machines	Cell1	Cell1	Cell1	Cell2	Cell2	Cell2	Machine overtime
				Part1	Part7	Part4	Part8	Part6	Part2	
Cell 1	3	1	Machine2		3	1	2			45 hours
Cell 1		1	Machine3	2			3			40 hours
Cell 1		1	Machine4	3	2					66.5 hours
Cell 1		1	Machine5	1		3	1	3		192.5 hours
Cell 2	2	3	Machine2		1	2		1	3, 2 and 1	7.5 hours
Cell2		1	Machine6					2		

Third period	Number of manual labor force	The number of machines	Machines	Cell1	Cell1	Cell1	Cell1	Cell2	Cell2	Machine overtime
				Part7	Part1	Part8	Part3	Part5	Part6	
Cell 1	3	1	Machine2	1		2	2		1	183.5 hours
Cell 1		1	Machine3		2	3				119.5 hours
Cell 1		2	Machine4	2	3		3 and 1			158 hours
Cell 2	2	3	Machine5	3	1	1		3 and 1	3	122.5 hours
Cell 2		1	Machine6					2	2	174.25 hours

Efficiency analysis of Benders Decomposition method

The numerical example stated in Chapter 3 was retested to evaluate the performance of the Benders Decomposition algorithm to solve the dynamic cell model. In this method, the solution to the problem of minimizing a possible distance for the optimal objective function value (Z*) is established that is constrained by LB and UB where $LB \leq Z^{*} \leq UB$. As already stated, LB is the lower bound that the software will be able to reach, and conversely, UB is the upper bound. In the execution of the Benders Decomposition algorithm in Gomez software as is seen in the second appendix, the value of the sub-dual function problem and the master problem, as the lower bound and the upper bound for the basic problem, respectively, are obtained. The solution time for the standardized MIP problem is about 570 seconds reduced to one-third in Benders solution. The costs of the objective function of the basic problem are shown in Table 5. This problem (Benders Decomposition) has 5000 constraints and 4824 decision variables. It has to be stated that it has succeeded in obtaining one of the upper and lower bounds for the optimal value of Z*, which will be further analyzed on the next page while implementing the Benders algorithm in GAMS software on the report.

Table 6
Upper and lower bounds for the optimal value of Z * and the costs of the objective function after solving Benders algorithm

Z1, lower bound for Z *	Z2, upper bound for Z *	Fixed machine cost	The cost of buying a machine	Machine selling price	Machine variable cost
248380	5215.5	31900	111000	9800	40954.5
Inter-cell movement	Intra-cell movement	Overtime cost	Labor relocation	Reconfiguration	Cell formation cost
1500	12440	7256.1	0	3125	50000

As Table (6) shows, z* value must be obtained between two values (5215.5 and 248380) that is in line with the standardized value of z, 2.028, in Table (5); with this approach to Solve this bi-objective Cell Formation problem, compiling time is reduced to one third. Therefore, one can conclude that the performance of the Benders solution approach is confirmed for the proposed model. Based on the questions raised in the study, one has to state that necessary to mention the reason for using a precise solution algorithm such as Benders to solve CMS problems as proposed in this study to provide an innovative approach to conventional innovative approaches like a genetic, ant colony and so on. Moreover, the way of solving Benders Decomposition has been explained in detail in this chapter. The advantages of the current solution approach are its strong mathematical base, which has become very prevalent in recent years for solving MIP models in engineering and mathematics, as well as the optimality of the expected answers to NP-hard problems.

Conclusion

The paper presented a new model of cell formation under dynamic conditions, which is finally solved using the Benders decomposition approach. In the current model, items like real-world costs like human resources, cell formation, inter-cell, and intra-cell movements, machine purchase and revenue costs, reconfiguration costs, machinery fixed/variables costs, and the costs related to machinery over time were taken into account. Then objective function linearization and the constraints were performed as described in Chapter 3. In the next step, standardization was performed with a multi-objective approach and the bi-objective function was transformed into a single objective. Then the model in question has been validated with GAMS software and its implementation was carried out with specific conditions in Benders exact solution approach, then the answer and its results were shown.

A review of the literature of CMS research in most studies revealed that because of the complexities of solving integrated models, it tends to integrate some of the design-related features. However, the features are interdependent, and reaching optimization in designing a system calls for a holistic solution approach. Indeed, one can state that in this type of system, only one part of the design features are included; however, the study tried to include more design features compared to the previous studies. For instance, the proposed model tries to consider more design features: a cost was considered for cell formation, = not previously considered in this type of model. Besides the cases stated in this model, it has been tried to reduce the inter-cell working hours deviation in the objective function.

Concerning the size of the formed cell, the lower bound is considered too, which is the real meaning of cell formation in the desired period, as at least two machines have to be allocated to form each cell. As a recommendation commonly seen in CMS problems in papers, one can state the problem of better and more comprehensive consideration of design features in CMS.

- Integrating goals closer to the real world more and comprehensively for optimization and solution by optimal multi-objective solving approaches.
- More significance to the human force sphere (like working hours, training, hiring/firing, skills, and so on) to reduce the associated costs in CMS models.
- Solving CMS models with precise algorithms and metaheuristics with larger aspects if possible.
- Modification and improvement of Benders exact solution and similar approaches to reach more accurate values for solving CMS models better could be useful.

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