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Soft b Derived Set in Soft Tritopological Spaces

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***Abstract*---**This paper deals with the concept and properties of trio soft limit point, trio soft b limit point and trio soft b derived set in soft tritopological spaces.

***Keywords*---**derived set, limit point, soft b, tritopological.

Introduction

In 1999, D.Molodtsov[9] introduced soft set theory in which he discussed about the basic notions of soft set theory and present its first results. In 2003, P.K.Maji, R.Biswas and A.R.Roy[8] define equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set and absolute soft set with examples. In 2011, NaimCagman, Serkan Karatas, Serdar Enginoglu[11] introduced a soft topology on a soft set and define a soft topological space. In 2011, Muhammad Shabir, Munnazza Naz [10] established soft topological spaces which are defined over an initial universe with a fixed set of parameters. In 2014, Basavaraj,M.Ittanagi [2] explained soft bitopological spaces in subtle manner. In 2017, AsmhanFlieh Hassan [1] set forth some new definitions of soft open sets in soft tritopological spaces.

Trio soft b derived set

Definition 2.1 Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a soft tritopological space over X and $A_E \in SS_E(X)$. Then the soft point $l_e \in X$ is said to be trio soft limit point of a soft set A_E if every trio soft neighbourhood containing l_e contains a soft point of A_E other than l_e . That is for every $\tau_{1,2,3}$ open set D_E containing $l_e, D_E \cap A_E \backslash \{l_e\} \neq \emptyset$ .The collection of all trio soft limit points of a soft set A_E is called the trio soft derived set of A_E and is denoted by trio sD(A_E).

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Example 2.2 Consider the soft tritopological space $(X,\tau_1,\tau_2,\tau_3,E)$ where $\tau_1 = \{\bar{X},\emptyset,A_{E_1},A_{E_4}\}$,$\tau_2 = \{\bar{X},\emptyset,A_{E_5},A_{E_{13}}\}$ and $\tau_3 = \{\bar{X},\emptyset,A_{E_9}\}$ and the soft subsets are as in the example 4.2.2. Then trio open sets are $\{\bar{X},\emptyset,A_{E_1},A_{E_4},A_{E_5},A_{E_9},A_{E_{13}}\}$. Now consider the soft subset $A_{E_7} = \{(e_1,\{m\}),(e_2,\{l,m\})\}$. We find the trio soft limit points ofA_{E_7}.

The soft point $(e_1,\{l\})$ is not a trio soft limit point of A_{E_7}. Since A_{E_4} is a trio soft neighbourhood of $(e_1,\{l\})$ which contains no other soft point of A_{E_7} . Again $(e_2,\{l\})$ is not a trio soft limit point of A_{E_7}because A_{E_4} is a trio soft neighbourhood of $(e_1,\{l\})$, which contains no other soft point of A_{E_7} . Since trio soft neighbourhood of $(e_1,\{m\})$ are A_{E_5},A_{E_9} and $\bar{X}$ each of which contains a soft point A_{E_7}other than $(e_1,\{m\})$ and so $(e_1,\{m\})$is a trio soft limit point of A_{E_7}.Also $(e_2,\{m\})$ is a trio soft limit point of A_{E_7}because trio soft neighbourhood of $(e_1,\{m\})$are A_{E_7} and $\bar{X}$ alone each of which contains a soft point of A_{E_7} other than $(e_2,\{m\})$. Hence trio soft limit points of $A_{E_7} = trio - sD(A_{E_7}) = \{(e_1,\{m\}),(e_2,\{m\})\}$.

Definition 2.3 Let $(X,\tau_1,\tau_2,\tau_3,E)$ be a soft tritopological space over X and $A_E \in SS_E(X)$. Then the soft point $l_e \in X$ is said to be trio soft b limit point of a soft set A_E if every trio soft b neighbourhood containing l_e contains a soft point of A_E other than l_e. The collection of all trio soft b limit points of a soft set A_E is called the trio soft b derived set of $A_E$and is denoted by trio sbD(A_E). Example 2.4 For the above example 4.4.2 the trio soft b limit point of a soft set A_{E_7} can be calculated as follows.

Define $\tau_1 = \{\bar{X},\emptyset,A_{E_1},A_{E_4}\}$,$\tau_2 = \{\bar{X},\emptyset,A_{E_5},A_{E_{13}}\}$ and $\tau_3 = \{\bar{X},\emptyset,A_{E_9}\}$ and the soft subsets are as in the example 4.2.2. Then trio open sets are $\{\bar{X},\emptyset,A_{E_1},A_{E_4},A_{E_5},A_{E_9},A_{E_{13}}\}$ and the collection of soft b open sets of X are $trio - SbO = \{\bar{X},\emptyset,A_{E_1},A_{E_2},A_{E_3},A_{E_4},A_{E_5},A_{E_7},A_{E_9},A_{E_{13}},A_{E_{15}}\}$. The soft point $(e_1,\{l\})$ is not a trio soft b limit point of A_{E_7}. Since A_{E_4} is a trio soft b neighborhood of $(e_1,\{l\})$ which contains no other soft point of A_{E_7}. Again $(e_2,\{l\})$ is not a trio soft b limit point of A_{E_7},because $A_{E_{13}}$ is a trio soft neighbourhood of $(e_1,\{l\})$ which contains no other soft point of A_{E_7}. The soft point $(e_1,\{m\})$ is a trio a soft b limit point of A_{E_7} ,because trio soft b neighbourhood of $(e_1,\{m\})$ are A_{E_5},A_{E_7},A_{E_9} and $\bar{X}$ each of which contains a soft point of A_{E_7} other than $(e_1,\{m\})$ and $(e_2,\{m\})$ is not a trio trio soft b limit point of A_{E_7} ,because trio soft b neighbourhoodA_{E_2} of $(e_2,\{m\})$ contains no other soft point of A_{E_7}. The trio soft b limit points of $A_{E_7} = trio - sbD(A_{E_7}) = \{(e_1,\{m\})\}$.

Proposition 2.5 Every trio soft b limit point is a trio soft limit point

Proof: Let $(X,\tau_1,\tau_2,\tau_3,E)$ be a soft tritopological space over X. Let $l_e \in X$. Let $l_e \in X$ be a trio soft b limit point of a soft set A_E . Suppose l_e is not trio soft limit point of a soft set A_E , then there exists a $\tau_{1,2,3}$ open set D_E containing l_e such that $D_E \cap A_E \backslash \{l_e\} = \emptyset$. But every $\tau_{1,2,3}$ open set is trio soft b open set . Thus D_E is a trio soft b open set containing $l_e$and $D_E \cap A_E \backslash \{l_e\} = \emptyset$. Therefore l_e is not a trio soft b limit point of a soft set A_E .This contradicts the hypothesis. Therefore l_e is a trio soft limit point of a soft set A_E.

Note 2.6 A trio soft limit point need not be a trio soft b limit point as shown in the following example

Example 2.7

Theorem 4.4.8Let A_E and D_E be soft subsets of $(X, \tau_1, \tau_2, \tau_3, E)$. Then the trio soft b derived sets trio sbD(A_E) and trio sbD(D_E) satisfies the following conditions:

- trio sbD$(\emptyset) = \emptyset$
- If $A_E \subseteq D_E$ then trio sbD$(A_E) \subseteq$trio sbD(D_E)
- $l_e \in$ trio sbD(A_E) implies $l_e \in$ trio sbD$(A_E \setminus \{l_e\})$
- trio sbD$(A_E) \cup$ trio sbD$(D_E) =$ trio sbD$(A_E \cup D_E)$
- trio sbD$(A_E \cap D_E) \subseteq$ trio sbD$(A_E) \cap$ trio sbD(D_E)

Proof:

- Suppose l_e is any soft point of X and trio sbD($\emptyset$). That implies l_e is a trio soft b limit point of $\emptyset$. This means that every trio soft b neighbourhood of l_e contains atleast one soft point of $\emptyset$ other than l_e. But $\emptyset$ contains no soft point ,which contradicts $l_e \in$ trio sbD($\emptyset$). Hence trio sbD$(\emptyset) = \emptyset$.
- Let $l_e \in$ trio sbD(A_E) . Then l_e is a trio soft b limit point of A_E. That is every trio soft b neighbourhood of $l_e$contains a soft point of A_E other than l_e . Since $A_E \subseteq D_E$, every trio soft b neighbourhood of l_e contains a soft point of $D_E$other thanl_e. Therefore l_e is a trio soft b limit point of D_E. That is $l_e \in$ trio sbD(D_E) . Hence trio sbD$(A_E) \subseteq$ trio sbD(D_E).
- Let $l_e \in$ trio sbD(A_E) . This implies that l_e is a trio soft b limit point of A_E. Then every trio trio soft b neighbourhood of $l_e$contains atleast one soft point of A_E other than l_e . It means that every trio soft b neighbourhood of $l_e$contains atleast one soft point of A_E other than l_e of $(A_E \setminus \{l_e\})$. Hence l_e is a trio soft b limit point of $(A_E \setminus \{l_e\})$. T herefore $l_e \in$ trio sbD$(A_E \setminus \{l_e\})$.
- We have $A_E \subseteq A_E \cup D_E$ and $D_E \subseteq A_E \cup D_E$. Then by ii) trio sbD$(A_E)\subseteq$ trio sbD$(A_E \cup D_E)$ and trio sbD$(D_E)\subseteq$ trio sbD$(A_E \cup D_E)$. Therefore trio sbD$(A_E) \cup$ trio sbD$(D_E) \subseteq$ trio sbD$(A_E \cup D_E)$. Again we have to prove, trio sbD$(A_E \cup D_E) \subseteq$ trio sbD$(A_E) \cup$ trio sbD(D_E) . Suppose $l_e \in$ trio sbD$(A_E \cup D_E)$. Then l_e is a trio soft b limit point of A_E or l_e is a trio soft b limit point of D_E. That implies $l_e \in$ trio sbD$(A_E) \cup$ trio sbD(D_E) . Therefore trio sbD$(A_E \cup D_E) \subseteq$ trio sbD$(A_E) \cup$ trio sbD(D_E). Therefore trio sbD$(A_E \cup D_E) \subseteq$ trio sbD$(A_E) \cup$ trio sbD(D_E). Hence trio sbD$(A_E) \cup$ trio sbD$(D_E) =$ trio sbD$(A_E \cup D_E)$.
- Now $A_E \cap D_E \subseteq A_E$ and $A_E \cap D_E \subseteq D_E$. Then by ii) trio sbD$(A_E \cup D_E) \subseteq$trio sbD(A_E) and trio sbD$(A_E \cap D_E) \subseteq$ trio sbD(A_E) and trio sbD$(A_E \cap D_E) \subseteq$ trio sbD(D_E). Therefore trio sbD$(A_E \cap D_E) \subseteq$ trio sbD$(A_E) \cap$ trio sbD(D_E).

Theorem 2.8

Let $(X, \tau_1, \tau_2, \tau_3, E)$ be a soft tritopological space over X and $A_E \in SS_E(X)$. Then A_E is trio soft b closed if and only if trio sbD$(A_E) \subseteq A_E$.

Proof: Let A_E be trio soft b closed set. Then ${A_E}^C$ is a trio soft b open set and so to each $l_e \in {A_E}^C$,there exists a trio soft b neighbourhood D_E of l_e such that $l_e \in D_E \subseteq {A_E}^C$. Since $A_E \cap {A_E}^C = \emptyset$, the trio soft b neighbourhoodD_E contains no soft point of A_E and so l_e is not a trio soft b limit point of A_E . Thus no soft point of ${A_E}^C$ can be a trio soft b limit point of A_E . That is $A_E$contains all its trio soft b limit points. Hence trio $\mathrm{sbD}(A_E) \subseteq A_E$.

Conversely let trio $\mathrm{sbD}(A_E) \subseteq A_E$. To prove A_E be trio soft b closed set,it is enough to prove that ${A_E}^C$ is trio soft b open set. Let $l_e \in {A_E}^C$. Then $l_e \notin A_E$. Since trio $\mathrm{sbD}(A_E) \subseteq A_E, l_e \notin$ trio $\mathrm{sbD}(A_E)$. Hence there exists a trio soft b neighbourhoodD_E of l_e ,which does not contain any soft point of A_E. Therefore there exists a trio soft b neighbourhood D_E of l_e ,which consists of only soft points of ${A_E}^C$. That is,$l_e \in D_E \subseteq {A_E}^C$. Therefore ${A_E}^C$ is a trio soft b open set and so A_E is trio soft b closed set.

Theorem 2.9 In any soft tritopological space $(X, \tau_1, \tau_2, \tau_3, E)$,every trio soft b derived set is a trio soft b closed set.

Proof: Let $A_E \in SS_E(X)$ and trio $\mathrm{sbD}(A_E)$ be the trio soft b derived set of A_E. Then trio $\mathrm{sbD}(A_E)$ is trio soft b closed. By theorem 3.9 trio sbD[trio sbD(A_E)]⊆ trio $\mathrm{sbD}(A_E)$. It is enough to prove that every trio soft b limit point of trio $\mathrm{sbD}(A_E)$ belongs to trio $\mathrm{sbD}(A_E)$.Let l_e be a trio soft b limit point of trio $\mathrm{sbD}(A_E)$. Then every trio soft b neighbourhood D_E of l_e contains a soft point of trio $\mathrm{sbD}(A_E)$ other than l_e . That is $(D_E \backslash \{x_e\} \cap$ trio $\mathrm{sbD}(A_E) \neq \emptyset$.

Let $m_e \in (D_E \backslash \{x_e\} \cap$ trio $\mathrm{sbD}(A_E)$ as $m_e \neq l_e$. Then $y_e \in (D_E \backslash \{x_e\})$ and $y_e \in$ trio $\mathrm{sbD}(A_E)$. Then m_e is a trio soft b limit point of A_E . Therefore every trio soft b neighbourhood containing m_e contains a soft point of A_E other than m_e. Now D_E is every trio soft b neighbourhood of m_e. Therefore $D_E \cap A_E \neq \emptyset$. Since D_E is an arbitrary soft b neighbourhood of l_e , l_e is a trio soft b limit point of A_E . That is $l_e \in$ trio $\mathrm{sbD}(A_E)$. Thus $l_e \in$ trio sbD[trio $\mathrm{sbD}(A_E)$] implies $l_e \in$ trio $\mathrm{sbD}(A_E)$. Therefore trio soft b derived set is a trio soft b closed set.

Theorem 2.10 If A_E is a soft subset of a softtritopological space $(X, \tau_1, \tau_2, \tau_3, E)$ over X. Then $A_E \cup$ trio $\mathrm{sbD}(A_E)$is a trio soft b closed set.

Proof: We have to prove $A_E \cup$ trio $\mathrm{sbD}(A_E)$ is a trio soft b closed set. It is enough to prove that the soft set $(A_E \cup \text{trio } \mathrm{sbD}(A_E))^C$ is trio soft b open set. By Demorgan's law, $(A_E \cup \text{trio } \mathrm{sbD}(A_E))^C = {A_E}^C \cap (\text{trio } \mathrm{sbD}(A_E))^C$. Let $l_e \in {A_E}^C \cap (\text{trio } \mathrm{sbD}(A_E))^C$. Then $l_e \in {A_E}^C$and $l_e \in (\text{trio } \mathrm{sbD}(A_E))^C$ or $l_e \notin {A_E}^C$and $l_e \notin (\text{trio } \mathrm{sbD}(A_E))^C$. If $l_e \notin (\text{trio } \mathrm{sbD}(A_E))^C$,then l_e is not a trio soft b limit point of A_E. It follows that there exist a trio soft b neighbourhood D_E of l_e which contains no soft point of $A_E$other than l_e . But $l_e \notin A_E$ so that D_E contains no soft point of A_E and so $D_E \subseteq {A_E}^C$. Again D_E is trio soft b open and is a trio soft b neighbourhood of each of its soft points. But as D_E does not contain any soft point of A_E , no soft point of D_E can be a soft limit point of A_E. That is no soft point of D_E can belong to trio $\mathrm{sbD}(A_E)$. Thus D_E is a trio soft b open subset of $(\text{trio } \mathrm{sbD}(A_E))^C$. That is $D_E \subseteq (\text{trio } \mathrm{sbD}(A_E))^C$. Thus for all $l_e \in {A_E}^C \cap (\text{trio } \mathrm{sbD}(A_E))^C$ there exist trio soft b neighbourhood D_E of l_e such that $D_E \subseteq {A_E}^C \cap (\text{trio } \mathrm{sbD}(A_E))^C = (A_E \cup \text{trio } \mathrm{sbD}(A_E))^C$. That implies $(A_E \cup \text{trio } \mathrm{sbD}(A_E))^C$

contains a trio soft b neighbourhood D_E of each of its soft points. Therefore it is trio soft b open set. Hence $A_E \cup \mathrm{trio\ sbD}(A_E)$ is trio soft b closed.

Conclusion

This paper gives a glimpse of properties of trio soft limit point, trio soft b limit point and trio soft b derived set using trio soft b open and trio soft b closed sets in soft tritopological spaces.

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