How to Cite:
Lohgheswary, N., Diana, A. S. F. N. ., Preethy, A., & Lun, A. W. (2022). Analysis on numerical analysis final exam questions. *International Journal of Health Sciences*, *6*(S2), 1165–1174. https://doi.org/10.53730/ijhs.v6nS2.5076

Analysis on Numerical Analysis Final Exam Questions

N. Lohgheswary
Department of Electrical and Electronics Engineering, XIAMEN University Malaysia, Sepang, 43900 Selangor, Malaysia

A. S. Fatin Nur Diana
Centre of Engineering Education, Faculty of Engineering and Built Environment, SEGi University, Kota Damanasara, 47810 Selangor, Malaysia

A. Preethy
Centre of Engineering Education Research, Faculty of Engineering and Built Environment, SEGi University, Kota Damanasara, 47810 Selangor, Malaysia

A. Wei Lun
Centre for Sustainable Process Technology (CESPRO), Faculty of Engineering and Built Environment, Universiti Kebangsaan Malaysia, Bangi, 43600 Selangor, Malaysia

***Abstract*---**Numerical Analysis is one of the key topics for Computational Mathematics in Engineering Mathematics. Students required to learn different methods of analysis to solve a given problem. The objective of this paper is to analyze the final exam questions of Numerical Analysis subject. There are four course outcomes for the Numerical Analysis subject. Five questions were set up for final and each question carries 20 marks. The Bloom Taxonomy for the questions are from application and analysis level. A total of 32 students from Civil Engineering department took the final examination. To analyze this subject, the results of the final examination of students from Civil Engineering department are tabulated in EXCEL and transformed into WINSTEPS. The Rasch model enables one to differentiate the difficulties of the exam questions according to different levels of the group. The ability of the Civil Engineering students cannot be divided into any group. A misfit item identified from Point-Measure Correlation, Outfit MNSQ and Outfit *z*-Standard. Since no item is out of the three measures, therefore there is no misfit question for the Numerical Analysis final examination. The person-item distribution map showed the questions which belong to difficult, easy and very easy group. Course Outcomes

International Journal of Health Sciences ISSN 2550-6978 E-ISSN 2550-696X © 2022.
***Corresponding author**: Lun, A. W.; Email: wl_ang@ukm.edu.my*
Manuscript submitted: 27 Nov 2021, Manuscript revised: 09 Feb 2022, Accepted for publication: 18 March 2022

2, 3 and 4 are considered difficult for this batch of students. The Rasch model able to classify the difficulty level of questions versus the Course Outcomes of Numerical Analysis subject.

Keywords---analysis, difficult topics, final exam, numerical analysis, Rasch model.

Introduction

Students at Faculty of Engineering and the Built Environment, SEGi University compulsory to take four Engineering Mathematics module, namely Engineering Mathematics I (EEM3113), Engineering Mathematics II (EEM3213), Numerical Analysis (ECS3223) and Engineering Statistics (EEM3223). There are 4 engineering departments at the Faculty of Engineering and the Built Environment at SEGi University, Malaysia. They are Department of Electronic and Electrical Engineering, Department of Mechanical Engineering, Department of Chemical Engineering and Department of Civil Engineering. Engineering Mathematics I is taken by students in the first semester followed by Engineering Mathematics II taken in the second semester. In the third semestser students take Engineering Statistics and in the fourth semester students take Numerical Analysis.

The objective of this paper is to use the Rasch model to identify the difficult topics in Numerical Analysis. Rasch model has been used extensively to examine the exam questions. An exit strategy which consist of exit survey, exit test and exit interview was used to assess the students' Programme Outcomes. The exit strategy achieved all the twelve Programme Outcomes. Only the exit survey and exit test are used to assess the Programme Outcomes. Results from the exit survey and exit test agreed well with each other by representing the students' achievement of all Programme Outcomes [1].

Rasch model is useful in measuring course learning outcomes whereby students can be classified exactly according to their performance [2]. Rasch model can be used to form valid measurements on certain dimensions of learning achievement. The development of learning capabilities can be tracked throughout their learning period and the teaching method can be improved for further development. Rasch model is similar to the basic elemental unit SI unit of measurement, which Rasch model acts as an instrument with unit definitions [3]. Logistic measurement has been developed to measure the ability of a student to the learning outcome of a particular course. The Rasch model is able to analyze the test results accurately and is easy to understand. It is also able to classify the results of course learning outcomes for a small sample size.

A measurement model was developed to measure students' achievement in which students' scores were combined with the Rasch model to measure the course learning outcomes [4]. Furthermore Rasch model can be a guide to lecturers to monitor students performance for each course learning outcome which highlights the effectiveness of teaching and learning. Comparative analysis is done on the conventional scoring distribution and Rasch model gives the same results for students' achievement.

The Rasch model is widely used to examine the reliability of exam questions and its impact on students' performance [5]. Students' performance not only depends on the ability of students to answer the exam questions but also the relevance of those questions. Test questions should be arranged from the simplest order to the hardest so that students have more time to answer difficult questions [6]. Although small sample sizes are used, Rasch model is proven to be an effective tool in evaluating the quality of the final exam papers accurately [7].

Methodology

The final examination for Numerical Analysis was conducted in session 2018 / 2019. The students undergo a teaching of 14 weeks in a semester. Students will have a mid-term test and an assignment to be submitted as the coursework components. The weightage for the test is 20%, the weighatge for the assignment is 20% whereas the weightage for final examination is 60%. The total percentage of the test, assignment and final examination will determines the grade of a student for the Numerical Analysis subject.

A total of 5 subjective questions were designed and validated by a lecturer. The duration of the final examination is 3 hours and the total marks are 100. A total of 32 students from Civil Engineering department sat for the final exam of Numerical Analysis. As a first step, grades were compiled in the EXCEL *prn format. The grades were transferred using Bond and Fox [8] known as WINSTEPS. It is a Rasch analysis software used to obtain the logit values. Table 1 list the Course Outcome for the Numerical Analysis subject.

Table 1
Course outcome for numerical analysis subject

Course Outcome	Description
1	Solve numerical models using numerical methods in linear system of equations and nonlinear equations.
2	Solve matrix algebra including solving eigenvalues and eigenvectors by using power method.
3	Apply numerical methods in ordinary differential equation.
4	Solve partial differentiation equation in hyperbolic, parabolic and elliptic equations.

Table 2 shows the Programme Outcome for Numerical Analysis subject.

Table 2
Programme outcome for numerical analysis subject

Programme Outcome	Description
1	Engineering knowledge

Programme Outcome	Description
2	Problem analysis
3	Design / development of solutions
4	Investigation
5	Modern tool usage
6	The engineer and society
7	Environment and sustainability
8	Ethics
9	Communication
10	Individual and team work
11	Lifelong learning
12	Project management and finance

Table 3 illustrates the distribution of final questions together with the marks.

Table 3
Final questions

Q	Description	Marks
1	Given the following non-linear equation $f(x) = 3.2x^2 - \cos x - 0.95$ By using the Bisection method, estimate the root in radians that employs initial guesses of $x_a = 0.25$ and $x_b = 1.38$ and accurate to within $\varepsilon = 0.005$.	20
2	Use Gauss Elimination method with partial pivoting to solve the following system of linear equations. (Calculate in 3 decimal places) $0.189x_1 - 0.598x_2 + 2.379x_3 = -4.224$ $-2.711x_1 + 5.392x_2 - 3.364x_3 = 1.571$ $1.924x_1 - 0.394x_2 + 4.178x_3 = 5.122$	20
3	Solve the following initial value problem $xy + 4\frac{dy}{dx} - x^3\frac{dy}{dx} = 2x^2$ and $y(0.25) = -1.78$ by using Heun's method with $h = 0.2$ and $0.25 \le x \le 1.05$.	20
4	Determine the highest eigenvalue and corresponding eigenvector of the following matrix A by using Power method that employs initial guesses of $v_o = [-1 \quad 2 \quad 1]$ and is accurate to within the absolute error $\varepsilon = 0.03$. $A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 4 & -2 \\ 0 & -1 & 2 \end{bmatrix}$	20
5	Solve the following Heat equation $u_1 = u_{xx} \qquad 0 < x < 2 \qquad t > 0$ with boundary conditions $u(0,t) = u(2,t) = 0 \qquad 0 < t < 0.4$ and initial conditions	20

Q	Description	Marks
	$u(x,0) = -\frac{99}{74}\sin(\pi x) \quad 0 \le x \le 2$, in radians given that $h = 0.4$ and $k = 0.2$. (Calculate in 5 decimal places)	

The details entry of each final question is given in Table 4.

Table 4
Entry number for final questions

Q	Course Outcome	Programme Outcome	Bloom Taxonomy Level	Description
1	1	1	3	Application
2	1	1	3	Application
3	3	2	4	Analysis
4	2	1	3	Application
5	4	2	4	Analysis

Results and Discussion

The Rasch analysis for this paper is divided into three parts, where the first part is called the "Person Measure", the second part is the "Item Measure", next is the "Item Statistics" and the last is "Person Item Distribution Map". These outputs are extracted from Bond & Fox Steps [8]. Figure 1 shows the summary statistics for individuals. Individuals represent the students who took Numerical Analysis course. The person summary reveals a strong reliability of Cronbach Alpha = 0.68 and person reliability = 0.40. The result of separation was 0.82 indicating that the students cannot be divided into any groups.

```
SUMMARY OF 32 MEASURED (EXTREME AND NON-EXTREME) Person
-------------------------------------------------------------------------------
|          TOTAL                          MODEL          INFIT        OUTFIT    |
|          SCORE     COUNT     MEASURE    S.E.       MNSQ   ZSTD   MNSQ   ZSTD |
|-----------------------------------------------------------------------------|
| MEAN      18.0       5.0         .58     .57                                 |
|  SEM        .9        .0         .17     .08                                 |
| P.SD       5.1        .0         .94     .43                                 |
| S.SD       5.2        .0         .96     .44                                 |
| MAX.      25.0       5.0        2.33    1.56                                 |
| MIN.       7.0       5.0       -1.26     .33                                 |
|-----------------------------------------------------------------------------|
| REAL RMSE    .73 TRUE SD     .60  SEPARATION   .82  Person RELIABILITY  .40 |
|MODEL RMSE    .72 TRUE SD     .61  SEPARATION   .85  Person RELIABILITY  .42 |
| S.E. OF Person MEAN = .17                                                   |
-------------------------------------------------------------------------------
Person RAW SCORE-TO-MEASURE CORRELATION = .95
CRONBACH ALPHA (KR-20) Person RAW SCORE "TEST" RELIABILITY = .68  SEM = 2.90
```

Figure 1. Summary statistics for individual

Figure 2 shows the summary statistics for the 5 items involved in this study. "Items" represents the questions tested on the final test. The item summary summarises very high reliability of 0.86 and item separation = 2.45. The value of the item separation indicates that there are three groups classifiable from the questions as "difficult", "easy" and "very easy". The value for the mean item is 0.

SUMMARY OF 5 MEASURED (NON-EXTREME) Item

	TOTAL SCORE	COUNT	MEASURE	MODEL S.E.	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD
MEAN	115.4	32.0	.00	.16	1.06	.18	.98	-.05
SEM	8.7	.0	.24	.01	.12	.40	.11	.27
P.SD	17.3	.0	.48	.03	.24	.80	.21	.54
S.SD	19.4	.0	.54	.03	.26	.89	.24	.60
MAX.	145.0	32.0	.55	.22	1.42	1.08	1.33	.67
MIN.	93.0	32.0	-.87	.15	.79	-.88	.67	-1.00

REAL RMSE	.18	TRUE SD	.44	SEPARATION 2.45	Item RELIABILITY .86
MODEL RMSE	.17	TRUE SD	.45	SEPARATION 2.71	Item RELIABILITY .88

S.E. OF Item MEAN = .24

Figure 2. Summary statistics for items

Figure 3 shows the item statistics for the exam questions. Item statistics enable to check if there is any misfit item or unfit questions. If there is any misfit question, the question should be deleted or rephrase. To identify whether an item fits the instrument, a three step calculation is examined. They are Point-Measure Correlation value, Outfit Mean Square (MNSQ) and Outfit z-Standard (ZSTD). An item is classified as non fit if it fails to meet all three criteria. The standard range for Point-Measure Correlation is [9].

From Figure 3, item 1 is out of the Point-Measure Correlation. The range for Outfit MNSQ is [10]. All the items are within the Outtfit MNSQ range. The range for Outfit z-Standard is [8]. All the items are within the range of Outfit z-Standard. Since no item is out of range of Point-Measure Correlation, Outfit MNSQ and Outfit z-Standard therefore there is no misfit item.

Item STATISTICS: MEASURE ORDER

ENTRY NUMBER	TOTAL SCORE	TOTAL COUNT	MEASURE	MODEL S.E.	INFIT MNSQ	INFIT ZSTD	OUTFIT MNSQ	OUTFIT ZSTD	PTMEASUR-AL CORR.	PTMEASUR-AL EXP.	EXACT MATCH OBS%	EXACT MATCH EXP%	Item
4	93	32	.55	.15	1.10	.49	.95	-.03	.71	.71	33.3	34.1	4
5	107	32	.24	.15	1.18	.85	.99	.07	.67	.67	14.8	25.6	5
3	111	32	.16	.15	.79	-.88	.67	-1.00	.70	.66	33.3	25.7	3
2	121	32	-.08	.16	.81	-.65	.96	.02	.64	.63	22.2	25.3	2
1	145	32	-.87	.22	1.42	1.08	1.33	.67	.38	.48	63.0	70.0	1
MEAN	115.4	32.0	.00	.16	1.06	.2	.98	-.1			33.3	36.1	
P.SD	17.3	.0	.48	.03	.24	.8	.21	.5			16.4	17.2	

Figure 3. Item statistics

Person problem-solving skills and item difficulty were mapped side by side on the same vertical line with the logit unit. Figure 3 refers to the Person-Item Distribution Map (PIDM). The discussion aims at the performance of the item with all of the 5 items spread on the logit scale. The scale for the items is made up samples ranging from 0.55 to -0.87 where the most difficult item and the most able exam takers were laid out on top of the scale.

On the left side, each student was represented by their department number for example X representing a student who took Numerical Analysis final from Civil Engineering Department. The right hand side illustrates the test item which was

represented by numbers. PIDM shows that the final exam questions can be divided into three categories, namely difficult, easy and very easy. Questions 3, 4 and 5 are categorized in the difficult group. Question 3 and 5 are analysis level of question. Question 3 is application level. This batch of students find difficult to answer the ordinary differential equation question. This could be mainly due to ordinary differential equation belongs to the higher level in Calculus syllabus.

Question 4 needs to find the eigenvalue and eigenvector. Power method is iteration method where needs to find eigenvalue until it meets tolerance error. Once find the eigenvalue then corresponding eigenvector to can be found. Students find difficulty on the iteration method. Question 5 belongs to heat equation. Students face difficulty in answering higher level of differentiation. They need to analyze the initial value problem.

Question 2 belongs to the easy group. Question 2 is application level in Bloom Taxonomy. This question is easy for the students as the students have studied inverse matrix method or the Gaussian elimination method in their Engineering Mathematics I. Since the students knew the basic of matrix, therefore they just need to apply the partial pivoting into higher order of group. Question 1 categorizes in very easy group. Question 1 is also from application level from Bloom Taxonomy. This is a straight forward question. Students need to find the root of the nonlinear equation given initial quess by using bisection method.

Table 5
summarizes the results of the person-item distribution map.

Question	Course Outcome	Level of Bloom's Taxonomy	Category
1	1	Application	Very Easy
2	1	Application	Easy
3	3	Analysis	Difficult
4	2	Application	Difficult
5	4	Analysis	Difficult

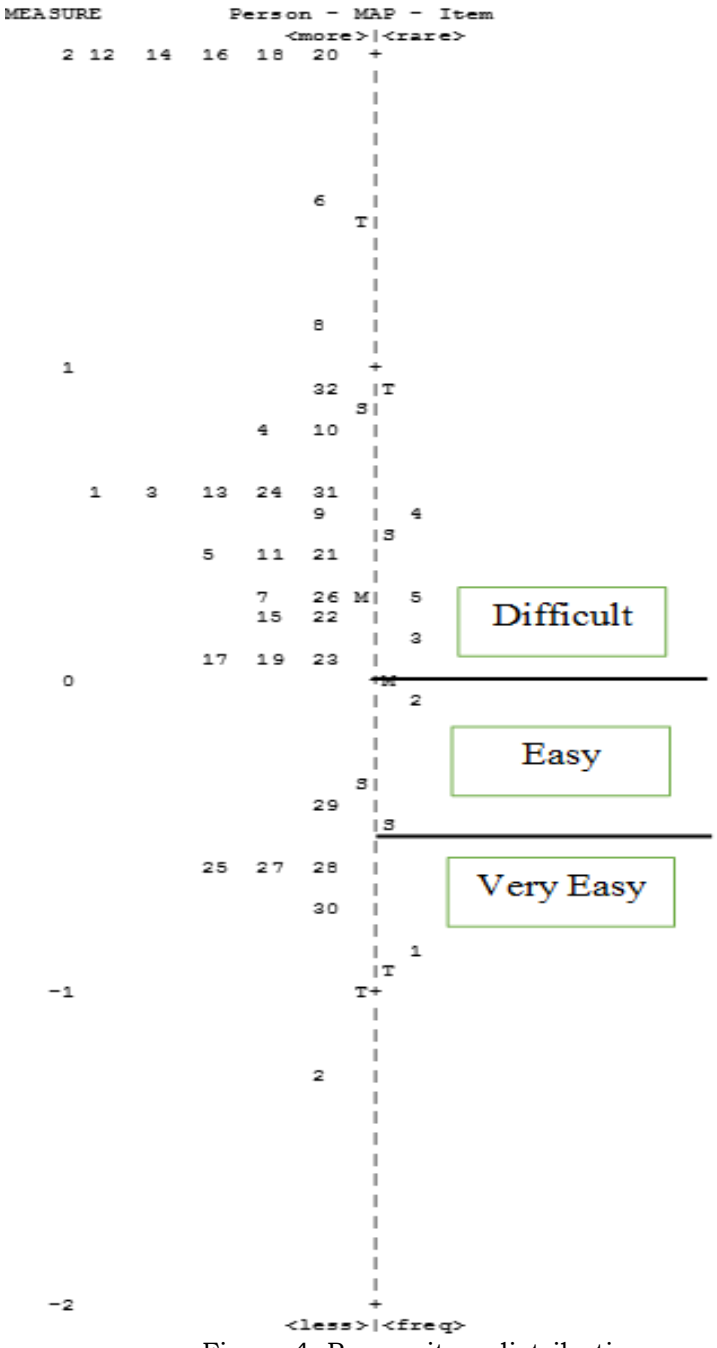

Figure 4. Person-item distribution map

Conclusion

The present study brings an important conclusion where the final test questions (items) for Numerical Analysis course was well designed and constructed. Rasch model categories the exam questions to the difficulty level. The summary

statistics for person unable to group the students into any group as they all are of the same ability. The summary statistics for items able to separate the question into 3 levels. This level can be clearly seen in person-item distribution map where questions grouped into difficult, easy and very easy level. The item statistic shows there is no misfit item for the test. This means all the questions set up according to the syllabus and able to test the students.

Acknowledgment

The authors wish to express gratitude towards XIAMEN University Malaysia (XMUMRF/2022-C9/IECE/0026), Universiti Kebangsaan Malaysia (GUP-2018-151) and SEGi University (SEGiIRF/2018-14/FoEBE-21/84) for supporting the research.

References

1. A. Ayob, H. F. A. Wahab, N. arshad, A. A. A. Bakar and H. Husain, "Assessment of student Programme Outcomes through a comprehensive exit strategy", Procedia Social and Behavioral Sciences, vol 18, pp. 33-38, 2011.
2. M. Azlinah, A. A. Azrilah, Z. Sohaimi and S. M. Mohd, "Appraisal of course learning outcomes using Rasch measurement: A case study in Information Technologies Education", 7th WSEAS International Conference on Artificia; Intelligence, Knowledge Engineering and Data Base, pp. 482-488, 2008.
3. M. Saifuddin, A. A. Azrilah, N. A. Rodzo'an, N. A. Omar, A. Zaharim and H. Basri, "Easier learning outcomes analysis using Rasch model in engineering education research", Proceedings of the 7th WSEAS International Conference on Engineering Education, pp. 442-447, 2010.
4. S. A. Osman, S. I. Naam, O. Jaafar, W. H. W. Badaruzzaman and R. A. O. O. K. Rahmat, "Application of Rasch model in measuring students' performance in Civil Engineering Design II course", International Conference on Teaching and Learning in Higher Education", pp. 59-66, 2012.
5. M. N. Zulkifli, H. O. Mohd, R. Noorhelyna, H. M. Fadiah and A. Izamarlina, "How good was the test set up? From Rasch analysis perspective", Regional Conference on Engineering Education & Research in Higher Education, 2010.
6. M. N. Zulkifi, M. H. Jamalluddin, I. Nur Arzilah, O. Haliza and M. H. Osman, "Reliability analysis on examination questions in a Mathematics course using Rasch measurement model", Sains Malaysiana, 41(9), pp. 1171-1176, 2012.
7. Z. M. Zain, "Evaluating the reliability and qualitu of examination paper for multi-tier application development course using Rasch measurement model", International Journal of Applied Engineering Research, 12(3), pp. 3887-3893, 2017.
8. T. G. Bond and C.M. Fox,"Applying the Rach model: Fundamental measurement in the human science," Mahwah, New Jersey: Lawrence Erlbaum Associates, 2007.
9. A. A. Aziz, M. Azlinah, H. A. Hamzah, A. G. Hamzah, Z. Sohaimi and M. Saifudin, "Application of Rasch Model in validating the construct

measurement instrument", International Journal of Education and Information Technologies, 2, pp. 105-112.
10. J. M. Linacre, WINSTEPS, Computer Program, Chicago, IL: http://www.winsteps.com, 2008.