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The Complex EFG Integral Transform and its Applications

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Abstract---In recent years, many integral transforms have been proposed to fulfill the vast number of fields that have benefited from them. The "Complex (Emad-Faruk-Ghaith) EFG" transform is introduced in this paper as a novel general complex integral transformation. The complex EFG transform properties, its application, and the inverse complex FEG transform's application to various fundamental functions are discussed. Using the complex EFG transform to solve high-order ordinary differential equations and other miscellaneous scientific and engineering problems demonstrates the efficacy of the transform in converting the core of some problems into simple, solvable algebraic equations. The EFG transform can be beneficial as a competent new transform in numerous scientific fields.

Keywords---complex EFG transform, ordinary differential equations, inverse complex EFG transform, beam, pharmacokinetics.

Introduction

Starting from the definition of integral transform, in which a new domain remapped function is generated from performing the integration on the original function with another function called the kernel function within appropriate limits, where the resulted function properties could be more malleable to characterization and manipulation [1-3], many scientific fields have exploited the

integral transforms to solve the most complicated problems of their applications, those fields include but not limited to nuclear physics, engineering, Biomedical, signal processing [4-13]. The vast importance of integral transforms led the mathematicians to increase their effort in suggesting and testing the properties of new integral transforms, where each transform proved its worth and shone in specific situations. Laplace, Aboodh, Smudu, Elzaki, Alzighair, SEE, and many other integral transforms are appropriate examples of integral transforms [14].

This work introduces yet another integral transform, under the name "Complex (Emad-Faruk-Ghaith) EFG" transform. The properties and applicability of the EFG transform on some fundamental functions are going to be demonstrated, and the efficiency of the transform in solving high order ordinary differential equations and the equations of a selection of physical, engineering and pharmacokinetics problems is also presented in the paper.

The complex EFG integral transform

For the function of exponential order in set B, which defined as:

$$B = \{f(t): \text{there exists } M, L_1, L_2 > 0, |f(t)| < Me^{-iL_j|t|}, \text{ if } t \in (-1)^j \times [0, \infty), j = 1, 2.\} \quad (1)$$

Where i is a complex number and $i^2 = -1$, the M constant must be a finite value for a particular function in the set B, while L_1 and L_2 may be finite or infinite. The complex EFG integral transform denoted by the operator $G^c\{\cdot\}$ is defined as:

$$G^c\{f(t)\} = F(iv) = \lim_{p \rightarrow \infty} \int_{t=0}^p f(t)e^{-iq(v)t} dt, \quad t \geq 0, L_1 \leq Q(v) \leq L_2, \quad (2)$$

$q(v)$ is a function of the parameter v .

The variable iv in the complex EFG transform is used to factor the variable t in the argument of the function $f(t)$.

The complex EFG integral transform for some basic functions

For any piecewise, exponential, and continuous function $f(t)$ for $t \geq 0$, the complex EFG integral for that function $G^c\{f(t)\}$ may or may not exists.

For $f(t) = k$, where k is an arbitrary constant, then:

$$G^c\{k\} = F(iv) = \lim_{p \rightarrow \infty} \int_{t=0}^p e^{-iq(v)t} k dt = \lim_{p \rightarrow \infty} \frac{-k}{iq(v)} [e^{-iq(v)t}]_0^p = \frac{-k}{iq(v)} [0 - 1] = \frac{-ik}{q(v)}.$$

$$G^c\{k\} = \frac{-ik}{q(v)}, \quad q(v) \neq 0, \operatorname{Re}(q(v)) > 0.$$

For $f(t) = t$, then:

$$G^c\{t\} = \lim_{p \rightarrow \infty} \int_{t=0}^p e^{-iq(v)t} t dt,$$

Performing integration by parts, gives: $G^c\{t\} = -\frac{1}{(q(v))^2}$, $q(v) \neq 0$, $Re(q(v)) > 0$. Similarly,

$$G^c\{t^2\} = \frac{(2!)i}{(q(v))^3}, \quad q(v) \neq 0, Re(q(v)) > 0.$$

$$G^c\{t^3\} = \frac{(3!)}{(q(v))^4}, \quad q(v) \neq 0, Re(q(v)) > 0.$$

In general: $G^c\{t^n\} = \frac{(-1)^n(i)^{n-1}n!}{(q(v))^{n+1}}$, $q(v) \neq 0$, $Re(q(v)) > 0$, n is a positive integer number.

For $f(t) = e^{at}$, then:

$$G^c\{e^{at}\} = \lim_{p \rightarrow \infty} \int_{t=0}^p e^{at} \cdot e^{-iq(v)t} dt,$$

After simple computations,

$$G^c\{e^{at}\} = -\left[\frac{a}{a^2 + (q(v))^2} + i \frac{q(v)}{a^2 + (q(v))^2} \right], \quad a - iq(v) \neq 0, Re(a - iq(v)) > 0,$$

0, a is an arbitrary constant.

The above outcome is useful in determining the complex EFG transform for the following functions:

$$G^c\{\sin(at)\} = \frac{-a}{(q(v))^2 - a^2}, \quad q(v) > |a|.$$

$$G^c\{\cos(at)\} = \frac{-iq(v)}{(q(v))^2 - a^2}, \quad q(v) > |a|.$$

$$G^c\{\sinh(at)\} = \frac{-a}{(q(v))^2 + a^2}, \quad Re(q(v)) > 0, q(v) \neq \pm ia.$$

$$G^c\{\cosh(at)\} = \frac{-iq(v)}{(q(v))^2 + a^2}, \quad Re(q(v)) > 0, q(v) \neq \pm ia.$$

Theorem (I)

Let $F(iv)$ be the complex EFG transform of the function $f(t)$ as ($F(iv) = G^c\{f(t)\}$), then:

$$G^c\{\dot{f}(t)\} = -f(0) + iq(v)F(iv).$$

Proof: from the complex EFG transform definition, then:

$$G^c\{\dot{f}(t)\} = \lim_{p \rightarrow \infty} \int_0^p e^{-iq(v)t} \dot{f}(t) dt.$$

Integration by parts, gives: $u = e^{-iq(v)t}$, $dv = \dot{f}(t)dt$, $du = -iq(v)e^{-iq(v)t}dt$, $v = f(t)$.

$$G^c\{\dot{f}(t)\} = \lim_{p \rightarrow \infty} f(t)e^{-iq(v)t} \Big|_0^p + \lim_{p \rightarrow \infty} \int_0^p f(t) iq(v) e^{-iq(v)t} dt,$$

$$G^c\{\dot{f}(t)\} = [-f(0)] + iq(v)G^c\{f(t)\}.$$

$$\therefore G^c\{\dot{f}(t)\} = -f(0) + iq(v)F(iv).$$

$$G^c\{\ddot{f}(t)\} = -\dot{f}(0) - iqvf(0) - (q(v))^2F(iv).$$

Proof: $G^c\{\dot{f}(t)\} = \lim_{p \rightarrow \infty} \int_0^p e^{-iq(v)t} \dot{f}(t) dt$, integration by parts, provides:

$$u = e^{-iq(v)t}, \quad dv = \dot{f}(t) dt, \quad du = -iq(v)e^{-iq(v)t}, \quad v = f(t).$$

$$G^c\{\dot{f}(t)\} = \lim_{p \rightarrow \infty} [f(t)e^{-iq(v)t}]_0^p + \lim_{p \rightarrow \infty} \int_0^p \dot{f}(t) iq(v) e^{-iq(v)t} dt,$$

$$G^c\{\dot{f}(t)\} = -f(0) + iqv [-f(0) + iqv G^c\{f(t)\}],$$

$$\therefore G^c\{\dot{f}(t)\} = -f(0) - iqv f(0) - (q(v))^2 G^c\{f(t)\}.$$

The general case can be proved through mathematical induction, as:

$$G^c\{f^{(n)}(t)\} = -f^{(n-1)}(0) - iqvf^{(n-2)}(0) - (iq(v))^2 f^{(n-3)}(0) - \dots - (iq(v))^{n-1} f(0) + (iq(v))^n F(iv).$$

Proposition: The Complex EFG transform Shifting Property

If $\{F(t)\} = F(iq(v))$, then: $G^c\{e^{at}f(t)\} = F(iq(v) - a)$, where a is a constant.

Proof: from complex EFG transform definition,

$$G^c\{f(t)\} = \lim_{p \rightarrow \infty} \int_0^p f(t) e^{-iq(v)t} dt,$$

$$G^c\{e^{at}f(t)\} = \lim_{p \rightarrow \infty} \int_{t=0}^p e^{at} f(t) e^{-iq(v)t} dt = \lim_{p \rightarrow \infty} \int_{t=0}^p e^{-[iq(v)-a]t} f(t) dt,$$

Or,

$$G^c\{e^{at}f(t)\} = F(iq(v) - a).$$

$$G^c\{e^{-at}f(t)\} = F(iq(v) + a).$$

Proposition: The Complex EFG Transform Convolution Property

If $G^c\{f(t)\} = F_1(iv)$ and $G^c\{h(t)\} = F_2(iv)$, then: $G^c\{f(t) * h(t)\} = G^c\{f(t)\} * G^c\{h(t)\} = F_1(iv) \cdot F_2(iv)$.

Proposition (3.4): The Complex EFG Transform Linearity Property

$$G^c[\alpha f(t) + \beta g(t)] = \alpha G^c\{f(t)\} + \beta G^c\{g(t)\},$$

Where α and β arbitrary constants.

Proposition: The Complex EFG Transform Change of Scale Property

If $G^c\{f(t)\} = F(iv)$, then: $G^c\{f(at)\} = \lim_{p \rightarrow \infty} \int_0^p f(at)e^{-iq(v)t} dt$.

Substituting $at = p \Rightarrow adt = dp$ into the above equation, gives:

$$G^c\{f(at)\} = \frac{1}{a} \lim_{p \rightarrow \infty} \int_0^p f(p)e^{-iq(v)\frac{p}{a}} dp = \frac{1}{a} F(iav).$$

Thus, if $G^c\{f(at)\} = F(iv)$, then: $G^c\{f(at)\} = \frac{1}{a} F(iav)$.

The inverse complex EFG integral transform

If $G^c\{f(t)\} = F(iv)$, then $f(t)$ is said to be the inverse of the complex EFG integral transform of $F(iv)$, denoted by $G^{c^{-1}}\{.\}$ operator, and can be defined mathematically by: $f(t) = G^{c^{-1}}\{F(iv)\}$.

The inverse complex EFG transform for some elementary functions

- $G^{c^{-1}}\{K\} = \frac{-ik}{q(v)}$, $K \equiv \text{Constan.}$
- $G^{c^{-1}}\{t\} = \frac{-1}{(q(v))^2}$.
- $G^{c^{-1}}\{t^n\} = \frac{(-1)^n (i)^{n-1} n!}{(q(v))^{n+1}}$, n is a positive integer number.
- $G^{c^{-1}}\{e^{at}\} = -\left[\frac{a}{a^2 + (q(v))^2} + i \frac{q(v)}{a^2 + (q(v))^2} \right]$, a is an arbitrary constant.
- $G^{c^{-1}}\left\{\frac{\sin(at)}{a}\right\} = \frac{-1}{(q(v))^2 - a^2}$.
- $G^{c^{-1}}\{\cos(at)\} = \frac{-iq(v)}{(q(v))^2 - a^2}$.
- $G^{c^{-1}}\left\{\frac{\sinh(at)}{a}\right\} = \frac{-1}{(q(v))^2 + a^2}$.
- $G^{c^{-1}}\{\cosh(at)\} = \frac{-iq(v)}{(q(v))^2 + a^2}$.

Applying the complex EFG integral transform into some applications

The complex FEG transform is used in this section to solve and evaluate several applications, including systems of ordinary differential equations governed by specific initial conditions and some practical applications, including an electric circuit, nuclear physics, pharmacokinetics, and beam deflection problems.

Application (I)

Consider the 1st order differential equation: $\frac{dy}{dx} + y = 0$, with $y(0) = 1$ (3).

Solution

To solve Equation (3), the complex EFG transform is applied to both sides of the equation as:

$$\begin{aligned}
G^c\{\dot{y}\} + G^c\{y\} &= 0, \\
-y(0) + iq(v)G^c\{y\} + G^c\{y\} &= 0, \\
-1 + iq(v)G^c\{y\} + G^c\{y\} &= 0, \\
(1 + iq(v))G^c\{y\} &= 1 \Rightarrow G^c\{y\} = \frac{1}{1+iq(v)}, \\
\text{Then, } G^c\{y\} &= \frac{1}{1+iq(v)} \cdot \frac{1-iq(v)}{1-iq(v)} = \frac{1-iq(v)}{1+(q(v))^2}, \\
G^c\{y\} &= -\left[\frac{-1}{1+(q(v))^2} + i \frac{q(v)}{1+(q(v))^2} \right].
\end{aligned}$$

Applying the inverse complex EFG transform, obtains: $y(x) = e^{-x}$.
The concluded result is the exact solution to Equation (3).

Application (II)

Consider the initial value problem (I.V.P): $\dot{y} + 2y = x$, with $y(0) = 1$.
(4)

Solution

To solve Equation (4), the complex EFG transform is applied to both sides of the equation as:

$$\begin{aligned}
G^c\{\dot{y}\} + 2G^c\{y\} &= G^c\{x\}, \\
-y(0) + iq(v)G^c\{y\} + 2G^c\{y\} &= \frac{-1}{(q(v))^2}, \\
(2 + iq(v))G^c\{y\} &= \frac{-1}{(q(v))^2} + 1, \quad \Rightarrow G^c\{y\} = \frac{-1}{(q(v))^2 + (2+iq(v))} + \frac{1}{(2+iq(v))}, \\
\text{Now, by taking: } \frac{-1}{(q(v))^2 + (2+iq(v))} &= \frac{A}{(q(v))^2} + \frac{B}{q(v)} + \frac{C}{(2+iq(v))}, \\
\text{After simple computation: } A &= -\frac{1}{2}, B = \frac{i}{4}, C = \frac{1}{4}. \\
\text{Then, } G^c\{y(x)\} &= \frac{-1}{2(q(v))^2} + \frac{i}{4q(v)} + \frac{1}{4(2+iq(v))} + \frac{1}{(2+iq(v))}, \\
G^c\{y(x)\} &= \frac{-1}{2(q(v))^2} + \frac{i}{4q(v)} + \frac{5}{4} \left[\frac{1}{(2+iq(v))} \cdot \frac{(2-iq(v))}{(2-iq(v))} \right] = \frac{-1}{2(q(v))^2} + \frac{i}{4q(v)} + \frac{-5}{4} \left[\frac{2}{4+(q(v))^2} + \frac{iq(v)}{4+(q(v))^2} \right].
\end{aligned}$$

Applying the inverse complex EFG transform, obtains: $y(x) = \frac{1}{2}x - \frac{1}{4} + \frac{5}{4}e^{-2x}$.
The concluded result is the exact solution to Equation (4).

Application (III)

Consider the following (I.V.P): $\frac{d^2y}{dx^2} + 9y = \cos(2x)$ with $y(0) = 1, y\left(\frac{\pi}{2}\right) = -1$.
(5)

Solution

Since $y'(0)$ is unknown, let $\dot{y}(0) = b$.

To solve Equation (5), the complex EFG transform is applied to both sides of the equation as:

$$G^c\{\dot{y}\} + 9G^c\{y\} = G^c\{\cos(2x)\},$$

$$-\dot{y}(0) - iq(v)y(0) - (q(v))^2 G^c\{y(x)\} + 9S^c\{y(x)\} = \frac{-iq(v)}{(q(v))^2 - 4},$$

$$-b - iq(v) - (q(v))^2 G^c\{y(x)\} + 9G^c\{y(x)\} = \frac{-iq(v)}{(q(v))^2 - 4}.$$

$$\left[9 - (q(v))^2\right] G^c\{y(x)\} = b + iq(v) - \frac{-iq(v)}{(q(v))^2 - 4}.$$

$$\therefore G^c\{y(x)\} = \frac{b}{9 - (q(v))^2} + \frac{iq(v)}{9 - (q(v))^2} - \frac{iq(v)}{((q(v))^2 - 4)(9 - (q(v))^2)}.$$

$$\text{Now, by taking: } \frac{-iq(v)}{[(q(v))^2 - 4][9 - (q(v))^2]} = \frac{Aq(v) + B}{(q(v))^2 - 4} + \frac{Cq(v) + D}{9 - (q(v))^2}.$$

After simple computations: $A = C = \frac{-i}{5}$ and $B = D = 0$.

$$\text{Then, } G^c\{y(x)\} = F(iv) = \frac{-3b}{3[9 - (q(v))^2]} + \frac{-iq(v)}{(q(v))^2 - 9} + \frac{-iq(v)}{5[(q(v))^2 - 4]} + \frac{iq(v)}{5[(q(v))^2 - 9]}.$$

Applying the inverse complex EFG transform, obtains: $y(x) = \frac{b}{3} \sin(3x) + \frac{4}{5} \cos(3x) + \frac{1}{5} \cos(2x)$.

By taking into account that $y\left(\frac{\pi}{2}\right) = -1$, It is possible to determine the value of the variable b as: $b = \frac{12}{5}$.

Then the exact solution to Equation (5) is: $y(x) = \frac{1}{5} \cos(2x) + \frac{4}{5} \cos(3x) + \frac{4}{5} \sin(3x)$.

Application (IV): Utilizing the Complex EFG Transform in Electric Circuits

A simple electric circuit is shown in Figure 1, in which a resistance R , inductance L , a capacitive condenser C , and a voltage power supply E are connected in series [15]. Applying Kirchhoff's law on the circuit, gives: $L \frac{di}{dt} + RI + \frac{Q}{c} = E$.

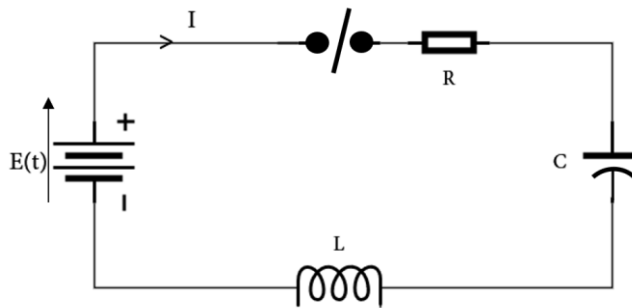


Figure 1. Simple electric circuit

Assuming that:

The inductance value is 2 Henrys.

The resistor value is 16 Ohms.

The capacitor value is 0.02 Farad.

The emf value is 300 Volts.

At $t=0$ the change in the circuit's capacitor and current are both zero. The charges and the current are changed accordingly at $t > 0$.

Q is the prompt charge and I is the current change at the time t .

From Kirchhoff's law: $L \frac{di}{dt} + RI + \frac{Q}{c} = E$,

$$2\dot{Q}(t) + 16Q(t) + 50Q(t) = E \text{ where } I = \dot{Q}(t),$$

$$\text{Then, } \dot{Q}(t) + 8Q(t) + 25Q(t) = 150$$

$$\text{Similarly, } \frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 25y = 150.$$

Applying the complex EFG integral transform to both sides of the above equation, obtains:

$$-\dot{y}(0) - iq(v)y(0) - (q(v))^2 G^c\{y(t)\} + 8(-y(0) + iq(v)G^c\{y(t)\} + 25G^c\{y(t)\}) = 150G^c\{1\},$$

Using the initial conditions, gives: $F(iv) = G^c\{y(t)\} = \frac{-150i}{q(v)[-(q(v))^2 + 8iq(v) + 25]}$, $G^c\{y(t)\} = 150i \left[\frac{A}{q(v)} + \frac{Bq(v)+D}{(q(v))^2 - 8iq(v) - 25} \right]$.

After simple computations: $A = \frac{-1}{25}$, $B = \frac{1}{25}$ and $D = \frac{-8}{25}i$.

$$\text{So, } G^c\{y(t)\} = \frac{-6i}{q(v)} - \frac{6(iq(v)+4)-24}{[(iq(v)+4)^2+9]} - \frac{24}{[(iq(v)+4)^2+9]},$$

$$F(iv) = G^c\{y(t)\} = \frac{-6i}{q(v)} - \left[\frac{-6i(q(v)-4i)}{(q(v)-4i)^2-9} \right] - \left[\frac{-(8)(3)}{(q(v)-4i)^2-9} \right].$$

Applying the inverse complex EFG integral transform and shifting property, gives: $y(t) = 6 - 6e^{-4t} \cos(3t) - 8e^{-4t} \sin(3t)$ or $Q(t) = 6 - 6e^{-4t} \cos(3t) - 8e^{-4t} \sin(3t)$, And $I = \dot{Q} = \frac{dQ}{dt} = 50e^{-4t} \sin(3t)$.

The resultant equations are the expressions for charge and current at any time t , when $t > 0$.

Application (V): Utilizing the Complex EFG Transform in Nuclear Physics

From the fundamentals of nuclear physics [7,15], the first-order linear ordinary differential equation is considered: $\frac{dN(t)}{dt} = -\alpha N(t)$.

The above equation represents the radioactive decay formula, where:

$N(t)$ represents the remaining amount of undecayed atoms in a radioactive isotope sample through the time t .

α represents the constant of decay.

Applying the complex EFG to the radioactive decay formula: $G^c\{\dot{N}(t)\} + \alpha G^c\{N(t)\} = 0$,

Therefore, $-N(0) + iq(v)G^c\{N(t)\} + \alpha G^c\{N(t)\} = 0$,

$$\begin{aligned} (\alpha + iq(v))G^c\{N(t)\} &= N(0), \\ G^c\{N(t)\} &= \frac{N(0)}{\alpha + iq(v)} = \frac{N_0}{\alpha + iq(v)} \cdot \frac{\alpha - iq(v)}{\alpha - iq(v)}, \\ G^c\{N(t)\} &= -N_0 \left[\frac{-\alpha}{\alpha^2 + (q(v))^2} + \frac{iq(v)}{\alpha^2 + (q(v))^2} \right], \end{aligned}$$

Applying the inverse complex EFG integral transform to both sides of the above equation, the equation of radioactive decay obtains: $N(t) = N_0 e^{-\alpha t}$.

Application (VI): Utilizing the Complex EFG Transform in Pharmacokinetics

The problem pharmacokinetics is: $\dot{C}(t) + \lambda C(t) = \frac{\beta}{\text{volume}}$, $t > 0$, with $C(0) = 0$, where:

$C(t)$ represents the concentration of the medication in the blood through the time t .

λ represents the elimination constant velocity.

β represents the infusion proportion measured in ($^{mg}/_{min}$).

Volume represents the volume of the drug distribution.

Applying the complex EFG integral transform to both sides of pharmacokinetics problem:

$$-C(0) + iq(v)G^c\{C(t)\} + \lambda G^c\{C(t)\} = \frac{\beta}{\text{volume}} G^c\{1\},$$

$$(iq(v) + \lambda)G^c\{C(t)\} = \frac{\beta}{\text{volume}} \cdot \frac{-i}{q(v)},$$

$$G^c\{C(t)\} = F(iv) = \frac{-i\beta}{\text{volume } q(v)(iq(v) + \lambda)},$$

$$F(iv) = \frac{\beta}{\text{volume}} \left[\frac{1}{iq(v)(iq(v) + \lambda)} \right],$$

$$F(iv) = \frac{\beta}{\text{volume}} \left[\frac{A}{iq(v)} + \frac{D}{iq(v) + \lambda} \right].$$

After simple computations: $A = \frac{1}{\lambda}$ and $D = \frac{-1}{\lambda}$.

$$\text{So, } G^c\{C(t)\} = F(iv) = \frac{\beta}{\text{volume}} \left[\frac{\frac{1}{\lambda}}{iq(v)} + \frac{\frac{-1}{\lambda}}{iq(v) + \lambda} \right],$$

$$\text{Then, } G^c\{C(t)\} = F(iv) = \frac{\beta}{\lambda \cdot \text{volume}} \left[\frac{-i}{q(v)} + \frac{-\lambda}{(q(v))^2 + \lambda^2} + \frac{iq(v)}{\lambda^2 + (q(v))^2} \right].$$

Applying the inverse complex EFG integral to the above equation, the equation of pharmacokinetics obtains: $C(t) = \frac{\beta}{\lambda \cdot \text{volume}} [1 - e^{-\lambda t}]$.

Application (VII): Utilizing the Complex EFG Transform in the Deflection of the Beams

It is possible to find any point deflection of a beam shown in Figure 2, with $x = 0$ and $x = L$ hinges at its ends, and support w_0 (per unit of length) of uniform load [8].

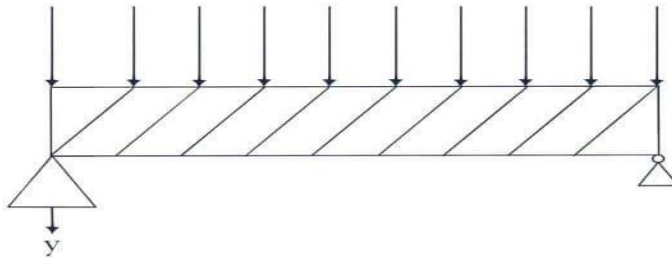


Figure 2. Hinged beam under uniformed load

The ordinary differential equation and the boundary conditions for the beam are:

$$y^{(4)} = \frac{W_0}{EI}, \text{ Or } \frac{d^4 y}{dx^4} = \frac{W_0}{EI}, \quad 0 < x < L, \quad \text{with conditions } y(0) = y_0 = 0, \dot{y}(0) = \dot{y}_0 = 0, \\ y(L) = 0, \dot{y}(L) = \dot{y}_L = 0.$$

Where:

E represents the young's modulus,

I represents the cross-section's moment of inertia about an axis normal to the plane of bending.

EI represents the flexural rigidity of the beam.

$\dot{y}(x), M(x) = EI\dot{y}(x)$ and $S(x) = \dot{M}(x) = EI\dot{\dot{y}}(x)$, are the slope, bending moment, and shear at a point respectively.

Applying the complex EFG transforms to both sides of the given ordinary differential equation, obtains: $G^c \left\{ \frac{d^4 y}{dx^4} \right\} = \frac{W_0}{EI} G^c \{1\}$,

$$-\dot{\dot{y}}(0) - iqv\dot{y}(0) - (iq(v))^2 \dot{y}(0) - (iq(v))^3 y(0) + (iq(v))^4 F(iv) = \frac{W_0}{EI} \left(\frac{-i}{q(v)} \right),$$

Using the provided and the following unknown conditions, $\dot{y}(0) = c_1, \dot{\dot{y}}(0) = c_2$, gives: $-c_2 - (iq(v))^2 c_1 + (iq(v))^4 F(iv) = \frac{-W_0 i}{EI q(v)}$,

$$(iq(v))^4 F(iv) = c_2 + (iq(v))^2 c_1 - \frac{W_0 i}{EI q(v)},$$

$$F(iv) = \frac{c_2}{i^4 (q(v))^4} + \frac{i^2 (q(v))^2 c_1}{i^4 (q(v))^4} - \frac{W_0 i}{EI i^4 (q(v))^5},$$

$$F(iv) = \frac{c_2}{(q(v))^4} + \frac{-c_1}{(q(v))^2} + \frac{W_0 i}{EI i^4 (q(v))^5},$$

$$F(iv) = \frac{-c_1}{(q(v))^2} + \frac{c_2}{(q(v))^4} + \frac{W_0 i}{EI i^4 (q(v))^5}.$$

Applying the inverse complex EFG integral to find: $y(x) = c_1 x + c_2 \cdot \frac{x^3}{6} + \frac{W_0}{EI} \cdot \frac{x^4}{24}$,

Applying the provided conditions to the beam deflecting equation, obtains: $c_1 = \frac{W_0 L^3}{24EI}$, $c_2 = \frac{W_0 L}{2EI}$.

Thus, the required deflection equation is: $y(x) = \frac{W_0}{24EI} x(L-x)(L^2 - Lx - x^2)$.

The acquired equation made it possible to calculate the exact moment in which bending happened and the shearing beam point, particularly at the ends.

Discussion and Conclusions

This work introduces a novel integral transform called the "Complex (Emad-Faruk-Ghaith) EFG" integral transform. The basic properties and the application of the complex EFG and inverse complex EFG transform on some fundamental functions, proved the extraordinary capability of this brand-new transform in handling these functions with such efficiency and with as little computation as possible. The complex EFG transform has proved its worthiness as a powerful tool in the integral transforms world when utilized in solving ordinary differential equations and various applications, including nuclear physics, electrical circuits, civil engineering beam deflection, and pharmacokinetics. These applications represent a portion in which this new integral transform can be exploited in.

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