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A New Complex SEL Integral Transform and its Applications on Ordinary Differential Equations

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***Abstract*---**This paper introduces a new complex integral transformation obtained by inserting a complex parameter into the well-known Rangaig integral transform kernel function. The new integral transform is denoted by the acronym SEL and is called the Complex (Serifenur-Emad-Luay) integral transform. The proposed SEL integral transform features are explained and shown to correspond to some fundamental functions. The application of the SEL transform to finding the solution of some differential equations, including those arising in some real-world practical applications, is discussed as an illustration of the actual fields that could benefit from this novel transform.

***Keywords*---**complex kernel, integral transform, rangaig transform, sel integral transform, differential equations, uniformly loaded beam, newton's law of cooling.

Introduction

Since the introduction of integral transforms by Euler in 1763 [1], many integral transforms have been proposed [2]. Integral transforms were given prominence in mathematicians' minds because of their ability to convert a problem from one domain where solving it is pretty difficult into another where the problem's solution is more manageable. The basic function of an integral transform is to produce a new function by integrating the function desired to be transformed from one domain to another with the kernel function of the transform within a specific limit. From this prospect, the main difference between integral transforms lies in their kernel functions and the limit of integration [3-5].

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The standard kernel function of the integral transforms does not contain complex parameters [6-10]. However, many mathematicians have recently directed their efforts to insert complex parameters into the kernel of the integral transforms to gain more generality in the series of these transforms [11,12]. This work follows the recent direction of inserting the complex parameter into yet another well-known Rangaig integral transform kernel function. The insertion of complex parameters into the kernel of the Rangaig integral transform produces a new integral transform with a different domain named after the people who proposed it and is called the Complex (Serifenur-Emad-Luay) SEL integral transform. The SEL transform's basic properties and applications to solving differential equations in some physical applications will be discussed in this paper.

The complex (Serifenur-Emad-Luay) sel integral transform

For an exponential order function H, that is defined as:

$$H = \{h(t): \exists N, \lambda_1, \lambda_2 > 0, |h(t)| > Ne^{i\lambda_j|t|}, t \in (-1)^{j-1} \times (-\infty, 0], i \in \mathbb{C}\} \quad (1)$$

For the set in equation (1), the arbitrary constant N must be finite, and the constants λ_1, λ_2 can be finite or infinite. It is possible to define the new complex (Serifenur-Emad-Luay) SEL integral transform as:

$$\eta^c\{h(t)\} = \frac{1}{\mu}\int_{t=-\infty}^{0} e^{i\mu t} h(t) dt \ , \frac{1}{\lambda_1} \le \mu \le \frac{1}{\lambda_2} \quad (2)$$

In the Complex SEL integral transform, the complex variable $(i\mu)$ factorizes the h's function variable t. In other words, the function $h(t)$ is remapped into the function $L(i\mu)$ of the $(i\mu)$-space.

The complex Sel transform for some fundamental functions

For any existing $h(t)$ function where $h(t) \in H$. The Complex SEL transform is satisfied if the conditions $t \ge 0$ and the function $h(t)$ is a continuous piecewise with decreasing exponential order are fulfilled.

Theorem (1)
Let $L(i\mu)$ denotes the transformation of the Complex SEL of $h(t) \in H$. Then the following theorems are applied.

- If $h(t) = 1$, then the Complex SEL Transform to $h(t)$ is:
  $\eta^c\{1\} = \frac{1}{\mu}\int_{t=-\infty}^{0} e^{i\mu t} dt = \frac{1}{i\mu^2}\int_{t=-\infty}^{0} e^{i\mu t} \cdot i\mu \, dt$,
  $\eta^c\{1\} = \frac{1}{i\mu^2}\left[e^{i\mu t}\Big|_{t=-\infty}^{0}\right] = \frac{1}{i\mu^2}[1-0] = \frac{-i}{\mu^2}$.
- If $h(t) = t$, then the Complex SEL Transform to $h(t)$ is:
  $\eta^c\{t\} = \frac{1}{\mu}\int_{t=-\infty}^{0} e^{i\mu t} t \, dt$,
  $\eta^c\{t\} = \frac{1}{\mu}\left[\frac{t}{i\mu} + \frac{1}{\mu^2}\right] e^{i\mu t}\Big|_{t=-\infty}^{0} = \frac{1}{\mu^3}$.
- If $h(t) = t^2$, then the Complex SEL Transform to $h(t)$ is:
  $\eta^c\{t^2\} = \frac{1}{\mu}\int_{t=-\infty}^{0} e^{i\mu t} t^2 \, dt$,

$$\eta^c\{t^2\} = \frac{1}{\mu}\left[\frac{t^2}{i\mu} + \frac{2t}{\mu^2} + \frac{2}{i^3\mu^3}\right] e^{i\mu t}\Big|_{t=-\infty}^{0} = \frac{2i}{\mu^4}.$$

- If $h(t) = t^3$, then the Complex SEL Transform to $h(t)$ is:

$$\eta^c\{t^3\} = \frac{1}{\mu}\int_{t=-\infty}^{0} e^{i\mu t} t^3 \, dt,$$

$$\eta^c\{t^3\} = \frac{1}{\mu}\left[\frac{t^3}{i\mu} - \frac{3t^2}{i^2\mu^2} + \frac{6t}{i^3\mu^3} - \frac{6}{i^4\mu^4}\right] e^{i\mu t}\Big|_{t=-\infty}^{0} = \frac{1}{\mu}\left[\frac{-6}{i^4\mu^4}\right] = \frac{(-1)(3!)}{\mu^5}.$$

In general, if $h(t) = \eta^c\{t^n\}$, then the Complex SEL transform to $h(t)$ is: $\eta^c\{t^n\} = \frac{(i)^{n-1}n!}{\mu^{n+2}}$, n is a positive integer.

Theorem (2): The Complex SEL Transform to an Exponential Function

The Complex SEL Transform to the function e^{at} is: $\eta^c\{e^{at}\} = \frac{-1}{\mu}\left[\frac{-a}{a^2+\mu^2} + i\,\frac{\mu}{a^2+\mu^2}\right]$.

Proof:

$$\eta^c\{e^{at}\} = \frac{1}{\mu}\int_{t=-\infty}^{0} e^{i\mu t} e^{at}\, dt = \frac{1}{\mu}\int_{t=-\infty}^{0} e^{(i\mu+a)t}\, dt,$$

$$\eta^c\{e^{at}\} = \frac{1}{\mu}\left(\frac{1}{i\mu+a}\left[e^{(i\mu+a)t}\right]\Big|_{t=-\infty}^{0}\right) = \frac{1}{\mu}\left(\frac{1}{i\mu+a}[1-0]\right) = \frac{1}{\mu}\left(\frac{1}{a+i\mu}\cdot\frac{a-i\mu}{a-i\mu}\right),$$

$$\eta^c\{e^{at}\} = \frac{1}{\mu}\left[\frac{a}{a^2+\mu^2} - i\,\frac{\mu}{a^2+\mu^2}\right].$$

OR

$$\eta^c\{e^{at}\} = \frac{-1}{\mu}\left[\frac{-a}{a^2+\mu^2} + i\,\frac{\mu}{a^2+\mu^2}\right].$$

Theorem (3): The Complex SEL Transform to the Trigonometric Functions

The Complex SEL Transform to the trigonometric functions $sin(t)$ and $cos(t)$ respectively is:

$$\eta^c\{\sin(t)\} = \frac{1}{\mu}\left(\frac{1}{1-\mu^2}\right) \text{ and } \eta^c\{\cos(t)\} = \frac{i}{1-\mu^2}.$$

Proof:

From the Complex SEL definition

$$\eta^c\{\sin(t)\} = \frac{1}{\mu}\int_{t=-\infty}^{0} e^{i\mu t}\sin(t)dt,$$

$$= \frac{1}{\mu}\int_{t=-\infty}^{0} e^{i\mu t}\left(\frac{e^{it}-e^{-it}}{2i}\right)dt = \frac{1}{2i\mu}\left[\int_{t=-\infty}^{0} e^{i(\mu+1)t}dt - \int_{t=-\infty}^{0} e^{i(1-\mu)t}dt\right],$$

$$= \frac{1}{2i\mu}\cdot\left[\frac{1}{i(\mu+1)} e^{i(\mu+1)t}\Big|_{t=-\infty}^{0} + \frac{1}{2i\mu i(1-\mu)}\left[e^{-i(1-\mu)t}\Big|_{-\infty}^{0}\right.\right.,$$

$$= \frac{-1}{2\mu(\mu+1)} + \frac{-1}{2\mu(1-\mu)},$$

$$= -\frac{1}{2\mu}\left[\frac{1}{\mu+1} + \frac{1}{1-\mu}\right] = \frac{-1}{2\mu}\left[\frac{1-\mu+\mu+1}{\mu^2-1}\right] = \frac{-1}{\mu}\left[\frac{1}{\mu^2-1}\right].$$

$$\therefore \eta^c\{\sin(t)\} = \frac{-1}{\mu}\left(\frac{1}{\mu^2-1}\right) = \frac{1}{\mu}\left(\frac{1}{1-\mu^2}\right).$$

Similarly,

$$\eta^c\{\cos(t)\} = \frac{1}{\mu}\int_{t=-\infty}^{0} e^{i\mu t}\cos(t)dt = \frac{1}{\mu}\int_{t=-\infty}^{0} e^{i\mu t}\left(\frac{e^{it}+e^{-it}}{2}\right)dt,$$

$$= \frac{1}{2\mu}\left[\int_{t=-\infty}^{0} e^{i(\mu+1)t}dt + \int_{t=-\infty}^{0} e^{i(\mu-1)t}dt\right],$$

$$= \frac{1}{2\mu}\left[\frac{1}{i(\mu+1)} e^{i(\mu+1)t}\Big|_{t=-\infty}^{0} + \frac{1}{2\mu}\left[\frac{1}{i(\mu-1)} e^{i(\mu-1)t}\Big|_{t=-\infty}^{0}\right.\right.,$$

$= \frac{1}{2\mu}\left[\frac{1}{i(\mu+1)}\right] + \frac{1}{2\mu}\left[\frac{1}{i(\mu-1)}\Big)\right]$,
$= \frac{1}{2i\mu}\left[\frac{1}{\mu+1} + \frac{1}{\mu-1}\right] = \frac{1}{2i\mu}\left[\frac{\mu-1+\mu+1}{\mu^2-1}\right]$,
$= \frac{1}{i}\left[\frac{1}{\mu^2-1}\right] = \frac{-i}{\mu^2-1} = \frac{i}{1-\mu^2}$,
$\therefore \eta^c\{\cos(t)\} = \frac{i}{1-\mu^2}$.

Theorem (4): The Complex SEL Transform of Derivatives

If the nth derivative of the function $h(t) \in H$ exists in the set of equation (1), then the Complex SEL transform of the nth derivative of the $h(t)$ function denoted by $\eta^c\{\mathrm{h}^{(n)}(t)\}$ is defined as follows.

If $h(t), h'(t), \cdots, h^{(n)}(t) \in H$, then:

i. $\eta^c\{h'(t)\} = \frac{1}{\mu}h(0) - i\mu\,\eta^c\{h(t)\}$.
Proof:
From the Complex SEL transform definition: $\eta^c\{h'(t)\} = \frac{1}{\mu}\int_{t=-\infty}^{0} e^{i\mu t}h'(t)dt$,
Performing integration by parts.
Let: $u = e^{i\mu t}$, $dv = h'(t)dt$, $du = i\mu e^{i\mu t}dt$, $v = h(t)$.
Then:
$\eta^c\{h'(t)\} = \frac{1}{\mu}\int_{t=-\infty}^{0} e^{i\mu t}h'(t)dt = \frac{1}{\mu}\left[e^{i\mu t}h(t)\Big|_{t=-\infty}^{0} - i\mu\int_{t=0}^{\infty} e^{i\mu t}h(t)dt\right]$,
$\eta^c\{h'(t)\} = \frac{1}{\mu}[(h(0) - 0)] - i\mu\frac{1}{\mu}\int_{t=-\infty}^{0} e^{i\mu t}h(t)dt$,
$\eta^c\{h'(t)\} = \frac{1}{\mu}h(0) - i\mu\,\eta^c\{h(t)\}$.

ii. $\eta^c\{h''(t)\} = -ih(0) + \frac{1}{\mu}h'(0) - \mu^2\,\eta^c\{h(t)\}$.
Proof:
From the Complex SEL transform definition: $\eta^c\{h''(t)\} = \frac{1}{\mu}\int_{t=-\infty}^{0} e^{i\mu t}h''(t)dt$,
Performing integration by parts.
Let: $u = e^{i\mu t}$, $dv = h''(t)$, $du = i\mu e^{i\mu t}dt$, $v = h'(t)$.
$\eta^c\{h''(t)\} = \frac{1}{\mu}\left[e^{i\mu t}h'(t)\Big|_{t=-\infty}^{0} - i\mu\int_{t=-\infty}^{0} e^{i\mu t}h'(t)dt\right]$,
$\eta^c\{h''(t)\} = \frac{1}{\mu}h'(0) - i\mu\left[\frac{1}{\mu}h(0) - i\mu\,\eta^c\{h(t)\}\right]$,
$\eta^c\{h''(t)\} = -ih(0) + \frac{1}{\mu}h'(0) - \mu^2\eta^c\{h(t)\}$.

iii. $\eta^c\{h'''(t)\} = -\mu h(0) - ih'(0) + \frac{1}{\mu}h''(0) + i\mu^3\,\eta^c\{h(t)\}$.
Proof:
From the Complex SEL transform definition: $\eta^c\{h'''(t)\} = \frac{1}{\mu}\int_{t=-\infty}^{0} e^{i\mu t}h'''(t)dt$,
Performing integration by parts.

Let: $u = e^{i\mu t}, dv = h'''(t)dt$, $du = i\mu e^{i\mu t}dt$, $v = h''(t)$.

$\eta^c\{h'''(t)\} = \frac{1}{\mu}\left[e^{i\mu t}h''(t)\Big|_{t=-\infty}^{0} - i\mu\int_{t=-\infty}^{0} e^{i\mu t}h''(t)dt\right]$,

$\eta^c\{h'''(t)\} = \frac{1}{\mu}h''(0) - i\mu\,\eta^c\{h''(t)\}$,

$\eta^c\{h'''(t)\} = \frac{1}{\mu}h''(0) - i\mu\left[-ih(0) + \frac{1}{\mu}h'(0) - \mu^2\,\eta^c\{h(t)\}\right]$,

$\eta^c\{h'''(t)\} = \frac{1}{\mu}h''(0) - \mu h(0) - ih'(0) + i\mu^3\,\eta^c\{h(t)\}$,

$\eta^c\{h'''(t)\} = -\mu h(0) - ih'(0) + \frac{1}{\mu}h''(0) + i\mu^3\,\eta^c\{h(t)\}$.

iv. $\eta^c\{h^{(4)}(t)\} = \frac{1}{\mu}h'''(0) + i\mu^2 h(0) - \mu h'(0) - ih''(0) + \mu^4\,\eta^c\{h(t)\}$.

Proof:

From the Complex SEL transform definition: $\eta^c\{h^{(4)}(t)\} = \frac{1}{\mu}\int_{t=-\infty}^{0} e^{i\mu t}h^{(4)}(t)dt$,

Performing integration by parts.

Let: $u = e^{i\mu t}, dv = h^{(4)}(t)dt$, $du = i\mu e^{int}dt$, $v = h'''(t)$.

$\eta^c\{h^{(4)}(t)\} = \frac{1}{\mu}\left[e^{i\mu t}h'''(t)\Big|_{-\infty}^{0} - i\mu\int_{-\infty}^{0} h'''(t)e^{i\mu t}dt\right]$,

$\eta^c\{h^{(4)}(t)\} = \frac{1}{\mu}\left[(h'''(0) - 0) - i\mu\int_{-\infty}^{0} h'''(t)e^{i\mu t}dt\right]$,

$\eta^c\{h^{(4)}(t)\} = \frac{1}{\mu}h'''(0) - i\frac{\mu}{\mu}\int_{-\infty}^{0} h'''(t)e^{i\mu t}dt$,

$\eta^c\{h^{(4)}(t)\} = \frac{1}{\mu}h'''(0) - i\mu\left[-\mu h(0) - ih'(0) + \frac{1}{\mu}h''(0) + i\mu^3\eta^c\{h(t)\}\right]$,

$\eta^c\{h^{(4)}(t)\} = \frac{1}{\mu}h'''(0) + i\mu^2 h(0) - \mu h'(0) - ih''(0) + \mu^4\eta^c\{h(t)\}$.

Applications

The implementation of the Complex SEL integral transform into solving some mathematical applications is discussed in this section.

Application (1)

Solve the following differential equation with the given initial condition using the Complex SEL transform:

$\frac{dy}{dx} + 2y = x,\ y(0) = 1.$

Solution:

$\eta^c\left\{\frac{dy}{dx}\right\} + 2\eta^c\{y\} = \eta^c\{x\}$,

$\frac{1}{\mu}y(0) - i\mu\eta^c\{y\} + 2\eta^c\{y\} = \frac{1}{\mu^3}$,

$(2 - i\mu)\eta^c\{y\} = \frac{1}{\mu^3} - \frac{1}{\mu}$,

$\eta^c\{y\} = \frac{1}{\mu^3(2-i\mu)} - \frac{1}{\mu(2-i\mu)}$,

$\eta^c\{y\} = \frac{1}{\mu}\left[\frac{1}{\mu^2(2-i\mu)} - \frac{1}{(2-i\mu)}\right]$,

$\eta^c\{y\} = \frac{1}{\mu}\left[\frac{A}{\mu} + \frac{B}{\mu^2} + \frac{C}{(2-i\mu)}\right] - \frac{1}{\mu}\frac{1}{(2-i\mu)}$,

After simple computations, $A = \frac{i}{4}, B = \frac{1}{2}$ and $C = -\frac{1}{4}$,

$\eta^c\{y(x)\} = \frac{1}{\mu}\left[\frac{i}{4\mu} + \frac{1}{2\mu^2} - \frac{1}{4(2-i\mu)} - \frac{1}{2-i\mu}\right]$,

$\eta^c\{y(x)\} = \frac{i}{4\mu^2} + \frac{1}{2\mu^3} - \frac{5}{4\mu(2-i\mu)}$,

$\eta^c\{y(x)\} = \frac{i}{4\mu^2} + \frac{1}{2\mu^3} - \frac{5}{4\mu}\left[\frac{1}{2-i\mu} \cdot \frac{2+i\mu}{2+i\mu}\right]$,

$\eta^c\{y(x)\} = \frac{i}{4\mu^2} + \frac{1}{2\mu^3} - \frac{5}{4\mu}\left[\frac{2}{4+\mu^2} + i\frac{\mu}{4+\mu^2}\right]$,

Taking the inverse of the Complex SEL transform, the solution is obtained as: $y(x) = -\frac{1}{4}(1) + \frac{1}{2}x + \frac{5}{4}e^{-2x}$.

Application (2): Using the Complex SEL Transform to Solve Newton's Law of Cooling Problem

Based on Newton's law of cooling, an object surrounded by an environment with a different temperature than the object itself changes proportionally to the difference between the object's temperature and its surroundings.

Let:

θ be a body temperature at time t,

θ_0 be the surrounding environment temperature,

Then, the Newton's law of cooling relation would be:

$\frac{d\theta}{dt} \propto [\theta(t) - \theta_0(t)] \Rightarrow \frac{d\theta}{dt} = -\beta[\theta(t) - \theta_0(t)]$, (3)

Where β is proportionality constant ($\beta > 0$).

The problem: if a body temperature drops from 80°C to 60°C in 20 minutes, when placed in the air, and the surrounding air temperature is 40°C.

i. What will be the body temperature after 40 minutes?
ii. When will the body temperature reach 55°C?

Solution:

Considering:

The surrounding air temperature is: $\theta_0 = 40$°C,

The initial body temperature $\theta(0) = 80$°C at time $t = 0$,

After time $t = 20$ min., the body temperature becomes $\theta(20) = 60$°C.

Applying Newton's law of cooling in equation (3):

$\frac{d\theta}{dt} \propto [\theta(t) - \theta_0(t)]$,

$\Rightarrow \frac{d\theta}{dt} = -\beta[\theta(t) - \theta_0(t)]$, where β is a constant,

$\Rightarrow \frac{d\theta}{dt} = -\beta\theta(t) + \beta\theta_0(t)$,

$\Rightarrow \frac{d\theta}{dt} + \beta\theta(t) = \beta\theta_0(t)$ or $\theta'(t) + \beta\theta(t) = 40\beta$, (4)

Taking the Complex SEL transform to both sides of equation (4) obtains:

$\eta^c\{\theta'(t)\} + \beta\eta^c\{\theta(t)\} = 40\beta\eta^c\{1\}$.

Using the Complex SEL transform of derivatives property, gives:

$\frac{1}{\mu}\theta(0) - i\mu\eta^c\{\theta(t)\} + \beta\eta^c\{\theta(t)\} = \frac{-40i\beta}{\mu^2}$, $i \in \mathbb{C}$ (5)

Since it is given that $\theta(0) = 80°$ at time $t = 0$, then substituting $\theta(0)$ into equation (5) gives:

$\frac{80}{\mu} - i\mu\eta^c\{\theta(t)\} + \beta\eta^c\{\theta(t)\} = \frac{-40i\beta}{\mu^2}$,

$\Rightarrow (\beta - i\mu)\eta^c\{\theta(t)\} = \frac{-40i\beta}{\mu^2} - \frac{80}{\mu}$,

$\Rightarrow (\beta - i\mu)\eta^c\{\theta(t)\} = \frac{-40i\beta-80\mu}{\mu^2}$,

$\Rightarrow \eta^c\{\theta(t)\} = \frac{-40(i\beta+2\mu)}{\mu^2(\beta-i\mu)}$, (6)

$= -40\left(\frac{A}{\mu} + \frac{\beta}{\mu^2} + \frac{C}{\beta-i\mu}\right)$,

$= -40\left(\frac{A\mu(\beta-i\mu)+B(\beta-i\mu)+C\mu^2}{\mu^2(\beta-i\mu)}\right)$,

$= -40\left(\frac{A\mu\beta-iA\mu^2+B\beta-iB\mu+C\mu^2}{\mu^2(\beta-i\mu)}\right)$,

$C - iA = 0 \Rightarrow C = \frac{i}{\beta}$,

$A\beta - iB = 2 \Rightarrow A\beta = 2 - 1 \Rightarrow A = \frac{1}{\beta}$,

$B\beta = i\beta \Rightarrow B = i$,

$\therefore \eta^c\{\theta(t)\} = -40\left(\frac{\frac{1}{\beta}}{\mu} + \frac{i}{\mu^2} + \frac{\frac{i}{\beta}}{\beta-i\mu}\right)$,

$\eta^c\{\theta(t)\} = -40\left(\frac{i}{\mu^2}\right) + (-40)\left[\frac{1}{\beta\mu} + \frac{i}{\beta(\beta-\mathrm{i}\mu)}\right]$,

$= -40\left(\frac{i}{\mu^2} + \frac{-40}{\beta}\left[\frac{1}{\mu} + \frac{i}{\beta-i\mu}\right]\right)$,

$= -40\left[\frac{i}{\mu^2}\right] - \frac{40}{\beta}\left[\frac{\beta-i\mu+i\mu}{\mu(\beta-i\mu)}\right]$,

$\eta^c\{\theta(t)\} = 40\left[\frac{-i}{\mu^2}\right] - \frac{40}{\beta}\left[\frac{\beta}{\mu(\beta-i\mu)}\right]$,

$= 40\left[\frac{-i}{\mu^2}\right] - \frac{40}{\mu}\left[\frac{1}{\beta-i\mu}\cdot\frac{\beta+i\mu}{\beta+i\mu}\right]$,

$= 40\left[\frac{-i}{\mu^2}\right] - \frac{40}{\mu}\left[\frac{\beta}{\beta^2+\mu^2} + i\,\frac{\mu}{\beta^2+\mu^2}\right]^2$, (7)

Taking inverse to the Complex SEL transform to equation (7) gives:

$\theta(t) = 40\eta^{\mathrm{c}^{-1}}\left\{\frac{-i}{\mu^2}\right\} + 40\eta^{\mathrm{c}^{-1}}\left\{\frac{-1}{\mu}\left[\frac{\beta}{\beta^2+\mu^2} + i\,\frac{\mu}{\beta^2+\mu^2}\right]\right\}$,

$\theta(t) = 40(1) + 40e^{-\beta t}$, (8)

Since it is given that $\theta(20) = 60°\mathrm{C}$ at $t = 20$, then:

$60 = 40 + 40e^{-20\beta} \Rightarrow 20 = 40e^{-20\beta}$,

$\frac{1}{2} = e^{-20\beta} \Rightarrow \frac{1}{2} = \left(e^{-\beta}\right)^{20} \Rightarrow \left(\frac{1}{2}\right)^{\frac{1}{20}} = e^{-\beta}$. (9)

i. It is possible to find the body temperature at time $t = 40$ using equation (9) as:

$\theta(t) = 40 + 40e^{-40\mu}$,

$\theta(t) = 40 + 40(e^{-\mu})^{40} \Rightarrow \theta(t) = 40 + 40\left(\frac{1}{2}\right)^{\frac{40}{20}}$,

$\theta(t) = 40 + 40\left(\frac{1}{2}\right)^{20}$,

$\theta(t) = 50°\mathrm{C}$.

Therefore, after time equals 40 minutes, the body temperature would be 50°C.

ii. To find the required time for the body temperature to reach 55°C, then:

$55 = 40 + 40e^{-\mu t} \Rightarrow 15 = 40(e^{-\mu})^t$,

$\Rightarrow \frac{15}{40} = (e^{-\mu})^t$, (10)

Substituting equation (9) in equation (10), gives:

$\frac{15}{40} = \left(\frac{1}{2}\right)^{\frac{t}{20}}$, applying the natural logarithm to both sides of the result equation obtains:

$\ln\left(\frac{15}{40}\right) = \frac{t}{20}\ln\left(\frac{1}{2}\right) \Rightarrow t = \frac{20\ln\left(\frac{15}{40}\right)}{\ln\left(\frac{1}{2}\right)}$,

$\therefore t = 28.3$ minutes.

Therefore, the body reaches the required temperature after 28.3 minutes.

Application (3): Using the Complex SEL Transform to Solve the Problem of a Uniformly Loaded Beam

The differential equation with the following boundary conditions that represent the deflection of a beam under a uniform load of w_0 per unit with hinges x=0 and x=L at its ends, as shown in figure (1), is:

$$y^{(4)}(x) = \frac{w_0}{EI}, or \frac{d^4y}{dx^4} = \frac{w_0}{EI} \quad , 0 < x < 1 \qquad (11)$$

Where:

E ≡ Young's modulus,

I ≡ the cross-section's moment of inertia about an axis normal to the plane of bending,

EI ≡ the beam's flexural rigidity,

$y'(0)$ ≡ the beam's slop,

$M(x) = EIy''(x)$ is the beam's bending moment,

$S(x) = M'(x) = EIy''(x)$ is the beam's shear at a point.

It is possible to solve the deflection of the uniformly loaded beam using the Complex SEL transform through the application of the transform to both sides of equation (11):

$$\eta^c\left\{\frac{d^4y}{dx^4}\right\} = \frac{w_0}{EI}\eta^c\{1\} ,$$

$$\frac{1}{\mu}y'''(0) + i\mu^2 y(0) - \mu y'(0) - iy''(0) + \mu^4\eta^c\{y(x)\} = \frac{w_0}{EI}\left(\frac{-i}{\mu^2}\right) , \qquad (12)$$

Using the first and second conditions and the unknown conditions $y'(0) = C_1$ and $y'''(0) = C_2$ into equation (12), obtains:

$$\frac{C_2}{\mu} - \mu C_1 + \mu^4\eta^c\{y(x)\} = \frac{-iw_0}{EI\mu^2} ,$$

$$\mu^4\eta^c\{y(x)\} = \frac{-iw_0}{EI\mu^2} - \frac{C_2}{\mu} + \mu C_1 , \qquad (\div \mu^4)$$

$$\eta^c\{y(x)\} = \frac{-iw_0}{EI\mu^6} - \frac{C_2}{\mu^5} + \frac{C_1}{\mu^3} ,$$

Applying the inverse Complex SEL transform to find:

$$y(x) = \frac{w_0}{EI}\frac{x^4}{24} + \frac{C_2x^3}{6} + C_1x .$$

Conclusions

The new Complex SEL integral transform extends the kernel of the Rangaig integral transform to include complex parameter. By incorporating a complex parameter into the kernel function of the Rangaig transform, the new proposed transform gains more generality and a broader range than the original transform. The study of the properties and application of the SEL transform to various essential functions demonstrated the transform's capacity to handle them efficiently. The SEL transform has also been used to solve several differential equations, including some practical applications' differential equations. It has proven to be effective in locating a solution using appropriate mathematical computations.

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