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The SEA Integral Transform and its Application on Differential Equations

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***Abstract*---**The subject of integral transformations' suggestions, properties studying and testing their efficiency via their application into real-life scientific applications, is a never-perishing subject. This work proposed a new integral transform called the Sherifa-Emad-Ali) SEA integral transform that manipulates the kernel function of the ZZ transform via the insertion of the complex parameter into the kernel to produce a new integral transform with a different domain than the ZZ transform. The properties and the application of the proposed SEA integral transform are studied and proved. The applicability of the transform to solve some problems represented by differential equations, including Newton's law of cooling problem and the deflecting of a hinged beam under a uniform load, is also discussed.

***Keywords*---**SEA integral transform, ZZ transform, complex kernel function, newton's law of cooling, beam deflection, differential equations.

Introduction

To solve any problem, it is necessary to figure out how to put it into a format that can be broken down into more minor problems that can be analyzed and solved [1]. Differential equations are the ideal tool in the mathematics world for transforming various scientific systems into comprehensive, analyzable, and, as a result, solvable formats in the form of equations. The functions that appear in the differential equation represent the physical attributes of the system that are required to be represented. In contrast, the derivatives of these functions represent the changing rate at which these attributes vary. The relationship

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between the system attributes and their rate of change is represented via differential equations [2,3]. Integral transforms are one of the most efficient methods that are proved their efficiency in handling and solving various problems through their ability to transform the problem from its current domain where the solution of the problem is complicated into another domain where the problem can be solved in more manageable methods [4-6]. The domain changing capability of integral transform made them a valued tool in handling many disciplines of engineering and natural sciences [7-10]. The main difference between integral transforms lies in their kernel functions, where the integration is performed between the said kernel and the desired function [11].

The intensive study of integral transforms yields many integral transform suggestions [12]. The majority of these transforms have a kernel function that does not include complex parameters. The lack of integral transforms with complex parameters in their kernels has fired the curiosity of mathematicians to study and suggest more integral transforms with that quality in their kernels [13,14]. That's what this work does, too. It proposes a new integral transform with complex parameters in its kernel. The proposed integral transform named (Sherifa-Emad-Ali) SEA integral transform inserts the complex parameter into the kernel of the well-known ZZ transform proposed by Zain UlAbadin Zafar in 2016 [15]. The complex parameter's insertion resulted in a new integral transform that processes the transformation in a different way and within a different domain than the ZZ transform. The fundamental properties of the SEA transform and its application to the basic functions will be discussed. Its applicability in solving some practical differential equation applications will also be demonstrated.

The (Sherifa-Emad-Ali) sea integral transform
Definition of the SEA Transform

The set A of functions for exponential order is defined as:

$$A = \{f(t): \exists N, \lambda_1, \lambda_2 > 0, |f(t)| > Me^{i\lambda_j|t|}, t \in (-1)^{j-1} \times (0, \infty], i \in \mathbb{C}\,, j = 1,2\}. \tag{1}$$

For set A in equation (1), M is a finite random constant, λ_1, λ_2 could be finite or infinite and $f(t)$ is a function that is defined for all $t \geq 0$.

The new SEA integral transform is defined as:

$$H^c\{f(t)\} = \frac{s}{v}\int_{t=0}^{\infty} f(t)e^{-i\left(\frac{s}{v}\right)t}dt\ . \tag{2}$$

Existence of the SEA Transform

For an exponential order ρ where $t \geq k$, of a piecewise and continuous function $f(t)$ in the finite interval $0 \leq t \leq k$. The SEA integral transform $H^c\{f(t)\}$ exists for all $s > \rho, v > \rho^2$.

SEA Integral Transform Properties

The basic properties of the SEA transform are going to be presented and proved through the application of the transform to some fundamental functions.

- For $f(t) = e^{\mathrm{at}}$, where $t \geqslant 0$ and a is a constant, then the SEA transform to $f(t)$ is: $H^c\{e^{at}\} = -\left[\frac{asv}{a^2v^2+s^2} + i\,\frac{s^2}{a^2v^2+s^2}\right]$.
  Proof
  From the SEA transform definition:
  $H^c\{e^{at}\} = \frac{s}{v}\int_{t=0}^{\infty} e^{-i\left(\frac{s}{v}\right)t} e^{at} dt$,
  $H^c\{e^{at}\} = \frac{s}{v}\int_{t=0}^{\infty} e^{-\left(i\left(\frac{s}{v}\right)-a\right)t}\ dt = \frac{-s}{v\left(i\left(\frac{s}{v}\right)-a\right)}\left[e^{-(i(s))t}\right|_0^{\infty}$,
  $H^c\{e^{at}\} = \frac{-s}{is-av}[0-1] = \frac{s}{-is-av}\cdot\frac{-is-av}{-is-av} = -\left[\frac{asv}{s^2+a^2v^2} + i\,\frac{s^2}{s^2+a^2v^2}\right]$.
  Or
  $H^c\{e^{at}\} = -\left[\frac{asv}{s^2+a^2v^2} + i\,\frac{s^2}{s^2+a^2v^2}\right]$.
- For $f(t) = k$, where k is a constant, then the SEA transform to $f(t)$ is: $H^c\{k\} = -ik,\ i \in \mathbb{C}$.
  Proof
  From the SEA transform definition:
  $H^C\{k\} = \frac{s}{v}\int_{t=0}^{\infty} e^{-i\frac{s}{v}t}\cdot k\,dt$,
  $H^C\{k\} = \frac{-ks}{vi\left(\frac{s}{v}\right)}\left[e^{-i\,\frac{s}{v}\,t}\right|_0^{\infty}$,
  $H^C\{k\} = \frac{ks}{is} = \frac{k}{i} = -ik$.
- $H^C\{t^n\} = (-1)^n(i)^{n-1}(n!)\left(\frac{v}{s}\right)^n$, $n \in \mathbb{N}$.
  - If $n = 1$ then: $f(t) = t$, and the SEA transform to $f(t)$ is: $H^C\{t\} = \frac{-v}{s}$.
    Proof
    From the SEA transform definition:
    $H^C\{t\} = \frac{s}{v}\int_0^{\infty} e^{-i\,\frac{s}{v}\,t}\ t\,dt = \frac{s}{v}\left[\frac{iv}{s}t + \frac{v^2}{s^2}\right] e^{-i\,\frac{s}{v}\,t}\Big|_0^{\infty} = \frac{s}{v}\left[0 - \left(\frac{v^2}{s^2}\right)\right]$.
  - If $n = 2$ then: $f(t) = t^2$, and the SEA transform to $f(t)$ is: $H^C\{t^2\} = 2i\left(\frac{v}{s}\right)^2$.
    Proof
    From the SEA transform definition:
    $H^C\{t^2\} = \frac{s}{v}\int_{t=0}^{\infty} e^{-i\frac{s}{v}t}\ t^2 dt$,
    Performing integration by parts, gives:
    $H^C\{t^2\} = \frac{s}{v}\left[\left(\frac{it^2v}{s} + \frac{2tv^2}{s^2} - 2\,\frac{iv^3}{s^3}\right) e^{-i\frac{s}{v}t}\right|_0^{\infty}$,
    $H^C\{t^2\} = \frac{s}{v}\left[0 - \left(-2i\,\frac{v^3}{s^3}\right)\right] = \frac{iv^2}{s^2} = 2i\left(\frac{v}{s}\right)^2$.
  - If $n = 3$ then: $f(t) = t^3$, and the SEA transform to $f(t)$ is: $H^C\{t^3\} = (3!)\left(\frac{v}{s}\right)^3$.
    Proof
    From the SEA transform definition:

$H^C\{t^3\} = \frac{s}{v}\int_{t=0}^{\infty} e^{-i\frac{s}{v}t}\, t^3 dt$,

Performing integration by parts, gives:

$H^C\{t^3\} = \frac{s}{v}\left[\left(\frac{it^3v}{s} + \frac{3t^2v^2}{s^2} - \frac{i6tv^3}{s^3} - \frac{6v^4}{s^4}\right) e^{-i\frac{s}{v}t}\right]_0^{\infty}$,

$H^C\{t^3\} = \frac{s}{v}\left[0 - \left(0 + 0 - 0 - \frac{6v^4}{s^4}\right]\right. = \frac{s}{v}\left[6\,\frac{v^4}{s^4}\right] = 6\,\frac{v^3}{s^3} = 3!\left(\frac{v}{s}\right)^3$.

- For $f(t) = \sin(at)$, then the SEA transform to $f(t)$ is: $H^C\{\sin(at)\} = \frac{-asv}{s^2-a^2v^2}$, where a is a constant.
  Proof
  From the SEA transform definition:
  $H^C\{\sin(at)\} = \frac{s}{v}\int_{t=0}^{\infty} e^{-i\frac{s}{v}t} \sin(at)dt$,
  $H^C\{\sin(at)\} = \frac{s}{v}\int_{t=0}^{\infty} e^{-i\frac{s}{v}t} \cdot \left(\frac{e^{iat}-e^{-iat}}{2i}\right) dt = \left(\frac{s}{2iv}\right)\int_{t=0}^{\infty} \mathrm{e}^{-i\left(\frac{s}{v}-a\right)t} dt -$
  $\left(\frac{s}{2iv}\right)\int_{t=0}^{\infty} e^{-i\left(\frac{s}{v}+a\right)t} dt$,
  $H^C\{\sin(at)\} = \frac{-s}{2iv\cdot i\left(\frac{s}{v}-a\right)} e^{-i\left(\frac{s}{v}-a\right)t}\Big|_0^{\infty} + \frac{s}{2iv\,.i\left(\frac{s}{v}+a\right)} e^{-i\left(\frac{s}{v}+a\right)t}$,
  $H^C\{\sin(at)\} = \frac{s}{2v\left(\frac{s}{v}-a\right)}[0-1] - \frac{s}{2v\left(\frac{s}{v}+a\right)}[0-1]$,
  $H^C\{\sin(at)\} = \frac{-s}{2v}\left[\frac{1}{\left(\frac{s}{v}-a\right)} - \frac{1}{\left(\frac{s}{v}+a\right)}\right] = \frac{-s}{2v}\left[\frac{\frac{s}{v}+a-\frac{s}{v}+a}{\left(\frac{s}{v}-a\right)\left(\frac{s}{v}+a\right)}\right]$,
  $H^C\{\sin(at)\} = \frac{-as}{v\left(\left(\frac{s}{v}\right)^2-a^2\right)} = \frac{-asv}{s^2-a^2v^2}$.
- For $f(t) = \cos(at)$, then the SEA transform to $f(t)$ is: $H^C\{\cos(at)\} = \frac{-is^2}{s^2-a^2v^2}$, where a is a constant.
  Proof
  From the SEA transform definition:
  $H^C\{\cos(at)\} = \frac{s}{v}\int_{t=0}^{\infty} e^{-i\frac{s}{v}t} \cos(at)dt$,
  $H^C\{\cos(at)\} = \frac{s}{v}\int_{t=0}^{\infty} e^{-i\frac{s}{v}t} \cdot \left(\frac{e^{iat}+e^{-iat}}{2}\right) dt = \left(\frac{s}{2v}\right)\int_{t=0}^{\infty} \mathrm{e}^{-i\left(\frac{s}{v}-a\right)t} dt +$
  $\left(\frac{s}{2v}\right)\int_{t=0}^{\infty} e^{-i\left(\frac{s}{v}+a\right)t} dt$,
  $H^C\{\cos(at)\} = \frac{s}{2vi\left(\frac{s}{v}-a\right)} + \frac{s}{2vi\left(\frac{s}{v}+a\right)}$,
  $H^C\{\cos(at)\} = \frac{s}{2iv}\left[\frac{\left(\frac{s}{v}+a\right)+\left(\frac{s}{v}-a\right)}{\left(\frac{s}{v}-a\right)\left(\frac{s}{v}+a\right)}\right]$,
  $H^C\{\cos(at)\} = \frac{s}{2iv}\left[\frac{\frac{s}{v}+a+\frac{s}{v}-a}{\frac{s^2}{v^2}-a^2}\right] = \frac{s}{iv}\left[\frac{\frac{s}{v}}{\frac{s^2}{v^2}-a^2}\right]$,
  $H^C\{\cos(at)\} = -i\,\frac{s}{v^2}\left[\frac{v^2s}{s^2-a^2v^2}\right]$,
  $H^C\{\cos(at)\} = \frac{-is^2}{s^2-a^2v^2}$.

SEA Transform Linearity Property

Considering the functions $f(t)$ and $g(t)$ and the constants a and b, then:
$H^c\{af(t) + bg(t)\} = aH^c\{f(t)\} + bH^c\{g(t)\}$.
Proof:
From the SEA transform definition:

$H^C\{f(t)\} = \frac{s}{v}\int_{t=0}^{\infty} e^{-i\frac{s}{v}t} f(t)\,dt$,
$H^c\{af(t) + bg(t)\} = \frac{s}{v}\int_{t=0}^{\infty} e^{-i\frac{s}{v}t}(af(t) + bg(t))dt$,
$H^c\{af(t) + bg(t)\} = a\frac{s}{v}\int_{t=0}^{\infty} e^{-i\frac{s}{v}t}f(t)dt + b\frac{s}{v}\int_{t=0}^{\infty} e^{-i\frac{s}{v}t}g(t)dt$,
$H^c\{af(t) + bg(t)\} = aH^c\{f(t)\} + bH^C\{g(t)\}$.

SEA Integral Transform for Derivatives

- If $H^C\{f(t)\} = Z(v,s)$, then: $H^C\{f'(t)\} = \frac{-sf(0)}{v} + i\frac{s}{v}Z(v,s)$.
  Proof
  From the SEA transform definition:
  $H^C\{f'(t)\} = \frac{s}{v}\int_{t=0}^{\infty} e^{-i\frac{s}{v}t} f'(t)\,dt$,
  Performing integration by parts,
  $u = e^{-i\frac{s}{v}t}$, $dv = f'(t)dt$.
  $du = -i\frac{s}{v}e^{-i\frac{s}{v}t}dt$, $v = f(t)$ ·
  $H^C\{f'(t)\} = \frac{s}{v}\int_{t=0}^{\infty} e^{-i\left(\frac{s}{v}\right)t}f'(t)dt = \frac{s}{v}\left[e^{-i\frac{s}{v}t}f(t)\Big|_{t=0}^{\infty} + i\frac{s}{v}\int_{t=0}^{\infty} f(t)e^{-i\left(\frac{s}{v}\right)t}dt\right]$,
  $H^C\{f'(t)\} = \frac{s}{v}[0 - f(0)] + i\frac{s}{v}H^C\{f(t)\}$,
  $\therefore H^C\{f'(t)\} = \frac{-sf(0)}{v} + i\frac{s}{v}Z(v,s)$.
- $H^C\{f''(t)\} = \frac{-s}{v}f'(0) - i\frac{s^2}{v^2}f(0) - \frac{s^2}{v^2}H^c\{f(t)\}$.
  Proof
  From the SEA transform definition:
  $H^C\{f''(t)\} = \frac{s}{v}\int_{t=0}^{\infty} e^{-i\left(\frac{s}{v}\right)t} f''(t)\,dt$,
  Performing integration by parts,
  $u = e^{-i\left(\frac{s}{v}\right)t}$, $dv = f''(t)dt$.
  $du = -i\left(\frac{s}{v}\right)e^{-i\left(\frac{s}{v}\right)t}dt$, $v = f'(t)$.
  $H^C\{f''(t)\} = \frac{s}{v}\left[e^{-i\left(\frac{s}{v}\right)t}f'(t)\Big|_{0}^{\infty} + i\left(\frac{s}{v}\right)\int_{t=0}^{\infty} f'(t)e^{-i\left(\frac{s}{v}\right)t}dt\right.$,
  $H^C\{f''(t)\} = \frac{s}{v}[-f'(0)] + i\frac{s}{v}H^C\{f'(t)\}$,
  $H^C\{f''(t)\} = \frac{-sf'(0)}{v} + i\frac{s}{v}\left[\frac{-s}{v}f(0) + i\frac{s}{v}H^c\{f(t)\}\right]$,
  $H^C\{f''(t)\} = \frac{-sf'(0)}{v} - i\frac{s^2}{v^2}f(0) - \frac{s^2}{v^2}H^c\{f(t)\}$.
- $H^C\{f'''(t)\} = -\frac{s}{v}f''(0) - i\frac{s^2}{v^2}f'(0) + \frac{s^3}{v^3}f(0) - i\frac{s^3}{v^3}H^c\{f(t)\}$.
  Proof
  From the SEA transform definition:
  $H^C\{f'''(t)\} = \frac{s}{v}\int_{t=0}^{\infty} e^{-i\left(\frac{s}{v}\right)t} f'''(t)\,dt$,
  Performing integration by parts,
  $u = e^{-i\left(\frac{s}{v}\right)t}$, $dv^* = f'''(t)dt$,
  $du = -i\left(\frac{s}{v}\right)e^{-i\left(\frac{s}{v}\right)t}dt$, $v^* = f''(t)$.
  $H^C\{f'''(t)\} = \frac{s}{v}\left(u \cdot v^* - \int v^* du\right)$,

$H^C\{f'''(t)\} = \frac{s}{v} e^{-i\left(\frac{s}{v}\right)t} f''(t)\Big|_{t=0}^{\infty} + i\frac{s^2}{v^2}\int_{t=0}^{\infty} f''(t)e^{-i\left(\frac{s}{v}\right)t}dt$,

$H^C\{f'''(t)\} = \left(0 - \frac{s}{v}f''(0)\right) + i\frac{s}{v}\left[\frac{-s}{v}f'(0) - i\frac{s^2}{v^2}f(0) - \frac{s^2}{v^2}H^C\{f(t)\}\right]$,

$\therefore H^c\{f'''(t)\} = -\frac{s}{v}f''(0) - i\frac{s^2}{v^2}f'(0) + \frac{s^3}{v^3}f(0) - i\frac{s^3}{v^3}H^c\{f(t)\}$.

- $H^c\{f^{(4)}(t)\} = -\frac{s}{v}f'''(0) - i\frac{s^2}{v^2}f''(0) + \frac{s^3}{v^3}f'(0) + i\frac{s^4}{v^4}f(0) + \frac{s^4}{v^4}H^c\{f(t)$.

  Proof

  From the SEA transform definition:

  $H^C\{f^{(4)}(t)\} = \frac{s}{v}\int_{t=0}^{\infty} e^{-i\left(\frac{s}{v}\right)t} f^{(4)}(t)\, dt$,

  Performing integration by parts,

  $u = e^{-i\left(\frac{s}{v}\right)t}$, $dv^* = f^{(4)}(t)dt$.

  $du = -i\left(\frac{s}{v}\right)e^{-i\left(\frac{s}{v}\right)t}dt$, $v^* = f'''(t)$.

  $H^C\{f^{(4)}(t)\} = \frac{s}{v}\left(u \cdot v^* - \int v^* du\right)$,

  $H^C\{f^{(4)}(t)\} = \frac{s}{v}\left[e^{-i\left(\frac{s}{v}\right)t}f''(t)\Big|_{t=0}^{\infty} + i\frac{s}{v}\int_{t=0}^{\infty} f'''(t)e^{-i\left(\frac{s}{v}\right)t}dt\right]$,

  $H^C\{f^{(4)}(t)\} = \frac{s}{v}[0 - f''(0)] + i\frac{s^2}{v^2}\int_{t=0}^{\infty} f'''(t)e^{-i\left(\frac{s}{v}\right)t}dt$,

  $H^C\{f^{(4)}(t)\}= -\frac{s}{v}f'''(0) + i\frac{s}{v}[-\frac{s}{v}f''(0) - i\frac{s^2}{v^2}f'(0) + \frac{s^3}{v^3}f(0) - i\frac{s^3}{v^3}H^c\{f(t)\}$,

  $\therefore H^C\{f^{(4)}(t)\} = -\frac{s}{V}f'''(0) - i\frac{s^2}{v^2}f''(0) + \frac{s^3}{v^3}f'(0) + i\frac{s^4}{v^4}f(0) + \frac{s^4}{v^4}H^c\{f(t)\}$.

- In general: $H^c\{f^{(n)}(t)\} = \sum_{m=1}^{n} - f^{(n-m)}(0)\frac{s^m}{v^m}(i)^{m-1} + \frac{i^n s^n}{v^n}H^c\{f(t)\}$.

  Proof: By mathematical induction.

  For n=1, $H^c\{f'(t)\} = -f(0)\frac{s}{v} + i\frac{s}{v}H^c\{f(t)\}$.

  Thus, it is true for $n = 1$.

  Assuming that it is also true for $n = k$, which it means:

  $H^c\{f^{(k)}(t)\} = \sum_{m=1}^{k} - f^{(k-m)}(0)\frac{s^m}{v^m}(i)^{m-1} + \frac{i^k s^k}{v^k}H^c\{f(t)\}$.

  It is possible to prove that for $n = k + 1$, as:

  $H^c\{f^{(k+1)}(t)\} = H^c\left\{\left(f^{(k)}(t)\right)'\right\} = -f^{(k)}(0)\frac{s}{v} + i\frac{s}{v}H^c\{f^{(k)}(t)\}$,

  $= -f^{(k)}(0)\frac{s}{v} + i\frac{s}{v}\left[\sum_{m=1}^{k} -f^{(k-m)}(0)\frac{s^m}{v^m}(i)^{m-1} + i^k\frac{s^k}{v^k}H^c\{f(t)\}\right]$,

  $= \sum_{m=1}^{k+1} -f^{(k-m+1)}(0)\frac{s^m}{v^m}(i)^{m-1} + i^{k+1}\frac{s^{k+1}}{v^{k+1}}H^c\{f(t)\}$.

  $= H^c\{f^{(k+1)}(t)\}$.

  That proves the theorem is true for $\in \mathbb{N}$.

Practical applications of sea integral transform

Solving and Analyzing Newton's Law of Cooling Problem Using the SEA Integral Transform

In Newton's law of cooling, the temperature of an object that is surrounded by a temperature-different environment changes relative to the temperature difference between the object and the environment. The mathematical formula of Newton's law of cooling is written as:

$$\frac{d\theta}{dt} \propto [\theta(t) - \theta_0(t)] \Rightarrow \frac{d\theta}{dt} = -\mu[\theta(t) - \theta_0(t)] \ , \tag{3}$$

Where, θ is the body temperature at time t, θ_0 is the surrounding environment temperature and μ is proportionality constant ($\mu > 0$). It is possible to solve the following Newton's law cooling problem using the SEA integral transform. The problem: if the surrounding air temperature is 40 degrees Celsius and the body temperature drops from 80 degrees Celsius to 60 degrees Celsius in 20 minutes.

(i) What would the temperature of the body be after 40 minutes?
(ii) When the temperature of the body reaches 55 degrees Celsius?

Solution
By considering the temperature of the environment air being $\theta_0 = 40°\text{C}$ and the temperature at $t = 0$ being $\theta(0) = 80°\text{C}$, and after passing 20 minutes the temperature would be $\theta(20) = 60°\text{C}$.

Newton's law of cooling in equation (3), obtains:

$\frac{d\theta}{dt} \alpha[\theta(t) - \theta_0(t)]$,

$\Rightarrow \frac{d\theta}{dt} = -\mu[\theta(t) - \theta_0(t)]$, where µ is a constant.

$\Rightarrow \frac{d\theta}{dt} = -\mu\theta(t) + \mu\theta_0(t)$,

$\Rightarrow \frac{d\theta}{dt} + \mu\theta(t) = \mu\theta_0(t)$ or $\theta'(t) + \mu\theta(t) = 40\mu.$ (4)

Applying the SEA transform to both sides of equation (4), gives:

$H^c\{\theta'(t)\} + \mu H^c\{\theta(t)\} = 40\mu H^c\{1\}$.

By using the SEA transform of derivatives property:

$-\frac{s}{v}\theta(0) + i\frac{s}{v}H^c\{\theta(t)\} + \mu H^c\{\theta(t)\} = 40\mu \text{i}, i \in \mathbb{C}$ (5)

Substituting $\theta(0) = 80°\text{C}$ into equation (5), gives:

$-\frac{s}{v}(80) + i\frac{s}{v}H^c\{\theta(t)\} + \mu H^c\{\theta(t)\}\} = -40i\mu$,

$\Rightarrow \left(i\frac{s}{v} + \mu\right)H^C\{\theta(t)\} = -40i\mu + 80\frac{s}{v}$,

$\Rightarrow \left(i\frac{s}{v} + \mu\right)\frac{1}{v}(is + v\mu)H^C\{\theta(t)\} = \frac{-40i\mu v+80s}{v}$,

$\Rightarrow H^C\{\theta(t)\} = \frac{-40i\mu v+80s}{v} \cdot \frac{v}{is+v\mu}$,

$\Rightarrow H^C\{\theta(t)\} = 40\left(\frac{-i\mu v+2s}{is+v\mu}\right),$ (6)

$H^C\{\theta(t)\} = 40\left[\frac{-i\mu v+2s}{v\mu+is} \cdot \frac{v\mu-is}{v\mu-is}\right]$,

$H^C\{\theta(t)\} = 40\left[-i\frac{(v^2\mu^2+s^2)}{(v^2\mu^2+s^2)} + \frac{\mu vs-is^2}{(v^2\mu^2+s^2)}\right]$,

$H^C\{\theta(t)\} = 40\left[-i - \left(\frac{-\mu vs}{v^2\mu^2+s^2} + i\frac{s^2}{v^2\mu^2+s^2}\right)\right].$ (7)

Applying the inverse of the SEA integral transform to both sides of equation (7) gives:

$\Rightarrow H^{C^{-1}}\{H^C\{\theta(t)\}\} = H^{C^{-1}}\left\{40\left[-i - \left(\frac{-\mu sv}{v^2\mu^2+s^2} + i\frac{s^2}{v^2\mu^2+s^2}\right)\right]\right\}$,

The resultant equation satisfies the linear property,

$\Rightarrow \theta(t) = 40\left[H^{C^{-1}}\{-i\} - H^{C^{-1}}\left\{\frac{-\mu sv}{v^2\mu^2+s^2} + i\frac{s^2}{v^2\mu^2+s^2}\right\}\right], \Rightarrow \theta(t) = 40[1 + e^{-\mu t}],$ (8)

Since it has been given in the problem that at $t = 20\text{min.}, \theta(20) = 60°\text{C}$, then:

$60 = 40 + 40e^{-20\mu} \Rightarrow 20 = 40e^{-20\mu}$,

$\Rightarrow \frac{1}{2} = e^{-20\mu} \Rightarrow (e^{-\mu})^{20} = \frac{1}{2} \Rightarrow e^{-\mu} = \left(\frac{1}{2}\right)^{\frac{1}{2}}$ (9)

(i) It is possible to find the temperature of the body be after 40 minutes from equation (9) as:

$\theta(t) = 40 + 40e^{-40\mu}$,

$\theta(t) = 40 + 40(e^{-\mu})^{40} = 40 + 40\left(\frac{1}{2}\right)^{\frac{40}{20}}$,

$\theta(t) = 40 + 40\left(\frac{1}{2}\right)^{20}$,

$\theta(\mathrm{t}) = 50°\mathrm{C}$.

The concluded result indicates that the body temperature would be 50 degrees Celsius after 40 minutes.

(ii) To find the required time for the body temperature to reach 50 degrees Celsius.

$55 = 40 + 40^{-\mu t} \Rightarrow 15 = 40(e^{-\mu})^{t}$,

$\Rightarrow \frac{15}{40} = (e^{-\mu})^{t}$.

(10)

Substituting equation (9) in equation (10) provides:

$\frac{15}{40} = \left(\frac{1}{2}\right)^{\frac{t}{20}}$.

Taking the natural logarithms to both sides of the resultant equation gives:

$\ln\left(\frac{15}{40}\right) = \frac{t}{20}\ln\left(\frac{1}{2}\right) \Rightarrow t = \frac{20\ln\left(\frac{15}{40}\right)}{\ln\left(\frac{1}{2}\right)}$,

$\therefore t = 28.3$ minutes.

The resultant time indicates the required time for the body to reach the desired temperature, which is 50°C.

Solving and Analyzing the Beam Deflection Problem Using the SEA Integral Transform

It is possible to express the deflection problem of a beam hinged at its ends $x = 0$ and $x = L$ that supports a uniform load w_0 (per length unit) shown in figure (1) via the following ordinary differential equation with boundary conditions as [1]:

$y^{(4)} = \frac{\omega_0}{EI}$ or $\frac{d^4y}{dx^4} = \frac{\omega_0}{EI}$, $0 < x < L$, (11)

$y(0) = y_0 = 0$, $y''(0) = y_0'' = 0$, $y(L) = 0$, $y''(L) = y_L'' = 0$ (12)

Where:

E is the modulus of Young's.

I is the normal bending plain moment of inertia for a cross-section about an axis.

EI is the beam's flexural rigidity.

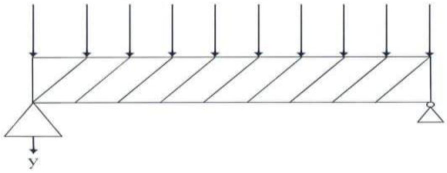

Figure 1: A hinged beam under the effect of a uniform load.

There are some abundances associated with the beam in question that are necessary to be known. These abundances are:

$y'(x)$ represents the slop.

$M(x) = EI\acute{y}(x)$ represents the moment of bending.

$S(x) = \acute{M}(x) = EI\acute{\acute{y}}(x)$ represents the point's shear.

Applying the SEA transform into both sides of equation (11), gets:

$$H^c\left\{\frac{d^4y}{dx^4}\right\} = \frac{\omega_0}{EI}H^c\{1\} \ ,$$

$$-\frac{s}{v}y'''(0) - i\frac{s^2}{v^2}y''(0) + \frac{s^3}{v^3}y'(0) + \frac{is^4}{v^4}y(0) + \frac{s^4}{v^4}H^c\{y(x)\} = \frac{\omega_0}{EI}H^c\{1\} \ (13)$$

Applying the conditions provided in equation (12) with the following unknown conditions $y'(0) = C_1 \,, y'''(0) = C_2$ to equation (13) obtains:

$$-C_2\frac{s}{v} + C_1\frac{s^3}{v^3} + \frac{s^4}{\text{v}}H^c\{y(x)\} = \frac{-w_0}{EI}i \ ,$$

$$\frac{s^4}{v^4}H^c\{y(x)\} = \frac{-w_0 i}{EI} + C_2\frac{s}{v} - C_1\frac{s^3}{v^3} \ ,$$

$$H^C\{y(x)\} = \frac{-w_0 i v^4}{EIs^4} + C_2\frac{sv^4}{vs^4} - C_1\frac{s^3v^4}{v^3s^4} \ ,$$

$$H^C\{y(x)\} = \frac{-w_0 i v^4}{EIs^4} + C_2\frac{v^3}{s^3} - \frac{C_1 v}{s} \ .$$

Applying the inverse of the SEA integral transform to both sides of the resultant equation gives:

$$y(x) = \frac{w_0}{EI(24)}x^4 + C_2\frac{x^3}{6} + C_1 x \ ,$$

$$y(x) = \frac{w_0 x^4}{24EI} + \frac{C_2}{6}x^3 + C_1 x \ ,$$

Applying the conditions $y(L) = 0 \,,\ y''(L) = y_L'' = 0$ from equation (12) yields:

$C_1 = \frac{w_2 L^3}{24EI}$ and $C_2 = \frac{w_1 L}{2EL}$.

Thus, the deflection of the beam can be found from the equation:

$$y(x) = \frac{w_0}{24EI}x(L-x)(L^2 - Lx - x^2) \ .$$

The concluded equation made it possible to calculate the hinged beam's shear and bending instant at any point.

Conclusions

The suggested SEA integral transform is a new integral transform that modifies the ZZ transform kernel by inserting the complex parameter into the ZZ kernel. The SEA integral transform properties have been demonstrated by applying them to fundamental functions and derivatives. SEA transform efficiency and capability to handle practical applications have been proven by using the transform to solve some important applications that use differential equations to represent their problems, including Newton's law of cooling and the problem of a hinged beam that deflects under a uniform load.

References

1. Hiebert, James. Making sense: Teaching and learning mathematics with understanding. Heinemann, 361 Hanover Street, Portsmouth, NH 03801-3912., 1997.
2. Hochstadt, Harry. Differential equations. Courier Corporation, 2014.
3. Zwillinger, Daniel, and Vladimir Dobrushkin. Handbook of differential equations. Chapman and Hall/CRC, 1998.
4. Kreyszig, E., Advanced Engineering Mathematics, 9th ed., WILEY, 2014.
5. Pundir S.K., Integral Transform Methods in Science and Engineering, CB, 2017.
6. Saichev, A. I. , Woyczynski, W., Distributions in the Physical and Engineering Sciences, Volume 1: Distributional and Fractal Calculus, Integral Transforms and Wavelets (Applied and Numerical Harmonic Analysis), 1st ed, Birkhäuser, 2018.
7. Gupta, R., Gupta, R. and Verma, D., Solving Schrodinger equation for a quantum mechanical particle by a new integral transform: Rohit Transform,". ASIO Journal of Chemistry, Physics, Mathematics & Applied Sciences (ASIO-JCPMAS), 2020; 4(1),32-36.
8. Aggarwal, S. and Gupta, A.R., Sumudu transform for the solution of Abel's integral equation. Journal of Emerging Technologies and Innovative Research, 2019; 6(4), 423-431.
9. Neamah Albukhuttar, A. Dheyaa Ridha, Z. & Neamah Kadhim, H., Applications of A Shehu Transform to the Heat and Transport Equations, International Journal of Psychological Rehabitation,2020; 24(05), 4254-4263.
10. Abbas ES, Kuffi EA, Hanna E. Al-Zughair integral transformation in solving improved heat and Poisson PDEs. In AIP Conference Proceedings 2022 Jan 11 (Vol. 2386, No. 1, p. 040041). AIP Publishing LLC.
11. Saitoh, S., Sawano, Y., Theory of Reproducing Kernels and Applications. Springer, 2016.
12. Kuffi, Emad A., Noor Kadhim Meftin, Elaf Sabah Abbas, and Eman A. Mansour. A Review on the Integral Transforms. Eurasian Journal of Physics, Chemistry and Mathematics 1, 2021: 20-28.
13. Ali Hassan Mohammed and Sarah Falih Makttoof, A Complex Al-Tememe Transform, International J. of Pure & Eng. Mathematics, 2017; 5(2), 17-30.
14. Eman A. Mansour, Sadiq A. Mehdi, Emad A. Kuffi, The New Integral Transform and its Applications, Int. J. Nonlinear Anal. Appl. 2021; 12(2), 849-856.
15. Zain UlAbadin Zafar, ZZ Transform Method, International Journal of Advanced Engineering and Global Technology, 2016; 4(1), 1605-1611.