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Eccentric domination path decomposition of Dumbbell graph

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Abstract--- A decomposition $(G_1, G_2, \dots, G_n)$ of G is said to be Eccentric Domination Path Decomposition(EDPD) if i) G admits EDD ii) Each G_i is a path ($1 \leq i \leq n$) iii) $q(G_1) = 1$ and $q(G_2) = 2$ or 3 iv) $q(G_i) = 3i - 5$ or $3i - 4$ or $3i - 3$, $i = 3, 4, \dots, n$.

Keywords---decomposition, domination, eccentric domination decomposition.

Introduction

In the article, all the terminologies from the graph theory are used in the case of Frank Harary[3]. A simple Undirected graph without loops or Multiple edges are Considered here. As usual p, q denote the number of vertices and edges of a graph G respectively. A path on p vertices is denoted by P_p . The graph obtained by joining two disjoint cycles $u_1 u_2 \dots u_p u_1$ and $v_1 v_2 \dots v_p v_1$ with edge $u_1 v_1$ is called dumbbell graph and it is denoted by Dh_p .

The theory of domination is the one of the fastest growing fields of graph theory which has been investigated by S.T.Hedetniemi. A set $D \subseteq V(G)$ of vertices in a graph G is a dominating set if every vertex v in $V - D$ is adjacent to a vertex in D . The Minimum Cardinality of a dominating set of G is called the domination number of G and is denoted by $\gamma(G)$.

In 2010, Janakiraman, Bhanumathi and Muthammai introduced and studied the concepts of eccentric dominating sets[5]. A set $D \subseteq V(G)$ is an eccentric dominating set if D is a dominating set of G and for every $v \in V - D$, there exists

atleast one eccentric point of v in D . If D is an eccentric dominating set, then every superset $D' \supseteq D$ is also an eccentric dominating set. But $D'' \subseteq D$ is not necessarily an eccentric dominating set. A eccentric dominating set D is a minimal eccentric dominating set if no proper subset $D'' \subseteq D$ is an eccentric dominating set. The eccentric domination number $\gamma_{ed}(G)$ of a graph G equals the minimum Cardinality of an eccentric dominating set, That is, $\gamma_{ed}(G) = \min |D|$, where, the minimum is taken over D in $\mathcal{D}$, Where $\mathcal{D}$ is the set of all Minimal eccentric dominating sets of G .

The decomposition of Graphs is another important field of graph theory.

Let $G = (V, E)$ be a simple connected graph with p vertices and q edges. If $G_1, G_2, \dots, G_n$ are connected edge disjoint subgraphs of G with $E(G_1) \cup E(G_2) \dots E(G_n)$ then $(G_1, G_2, \dots, G_n)$ is said to be a Decomposition of G .

We introduce a new concept Eccentric domination Decomposition of Graphs[6] and also introduce Eccentric Domination Path decomposition of Graphs which is motivated by the concepts of Ascending Domination Decomposition[8], Connected Domination path Decomposition[1] and Continuous Monotonic Decomposition[2].

A decomposition $(G_1, G_2, \dots, G_n)$ of G is said to be Eccentric Domination Decomposition if

- i) $E(G) = E(G_1) \cup E(G_2) \dots E(G_n)$
- ii) Each G_i is connected
- iii) $\gamma_{ed}(G_i) = i, i = 1, 2 \dots n$

In this paper, we have studied the Eccentric Domination Path decomposition of Dumbbell Graph.

Eccentric Domination Path Decomposition

Definition

A decomposition $(G_1, G_2, \dots, G_n)$ of G is said to be Eccentric Domination Path Decomposition(EDPD) if

- i) G admits EDD
- ii) Each G_i is a path ($1 \leq i \leq n$)
- iii) $q(G_1) = 1$ and $q(G_2) = 2$ or 3
- iv) $q(G_i) = 3i - 5$ or $3i - 4$ or $3i - 3, i = 3, 4 \dots n$.

Eccentric Domination Path Decomposition of Dumbbell graph

Theorem

A Dumbbell Db_p admits EDPD if and only if Db_p has $3j^2 + 3j + 2, j \in N$ vertices.

Proof:

The dumbbell graph Db_p has $2p$ vertices and $2p + 1$ edges.

Assume that Db_p has $3j^2 + 3j + 2, j \in N$

To prove Db_p admits EDPD.

We have the following possibilities for Db_p to admit EDPD Which Shown in table 1.

no.of decomposition n	no.of edges in P_p
1	1
2	2 3
3	4 5 6
4	7 8 9

Table 1.

Let G_1 be the edge e_1 . $\{v_1\}$ and $\{v_2\}$ are eccentric dominating set with cardinality 1. $\gamma_{ed}(G_1) = 1$ Since $\{v_1\}$ and $\{v_2\}$ are the dominating sets and v_1 and v_2 are eccentric to each other. Therefore G_1 must contain one edge. Therefore $q(G_1) = 1$

- Let G_2 be the edge incident in the vertices v_3v_4 . $\{v_2, v_5\}$ are eccentric dominating set with cardinality 2. $\gamma_{ed}(G_2) = 2$ Since $\{v_2, v_4\}$, $\{v_3, v_5\}$, $\{v_3, v_4\}$ and $\{v_2, v_5\}$ are the dominating sets and v_2 and v_5 are eccentric points. Therefore G_2 must contain three edges. Therefore $q(G_2) = 3$.
- We find the upper and lower bound of $q(G_n)$ of P_p such that Db_p admits EDPD using Newton's forward difference table.

First we find the lower bound of $q(G_n)$ using table 1.

n	q	Δq	$\Delta^2 q$
3	4		
4	7	3	0
5	10	3	0
6	13	3	0
7	16	3	

$$n = n_0 + xh$$

$$= x + 3$$

$$x = n - 3$$

$$q = q_0 + x\Delta q_0 + \frac{n(n-1)}{2!}\Delta q_0$$

$$= 4 + (n-3)3 + \frac{(n-3)(n-4)}{2!} \times 0$$

$$= 4 + 3n - 9$$

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$$= 3n - 5$$

Next we find the upper bound of $q(G_n)$ using table 1.

n	q	Δq	$\Delta^2 q$
3	6		
4	9	3	0
5	12	3	0
6	15	3	0
7	18	3	

$$n = n_0 + xh$$

$$= x + 3$$

$$x = n - 3$$

$$q = q_0 + x\Delta q_0 + \frac{n(n-1)}{2!}\Delta q_0$$

$$= 6 + (n-3)3 + \frac{(n-3)(n-4)}{2!} \times 0$$

$$= 6 + 3n - 9$$

$$= 3n - 3$$

Conversely Suppose Db_p has an EDPD.

To prove that Db_p has $3j^2 + 3j + 2$, $j \in N$ vertices.

Suppose Db_p has no $3j^2 + 3j + 2$, $j \in N$ vertices.

The following are the two possibilities.

- If we add the vertices $1, 2, \dots, n$ in Db_p in the above construction $G_1, G_2, \dots, G_n$ then there will be remaining 1 to n vertices and we cannot adjust them to satisfy the minimum eccentric dominating sets of G_i . Therefore the resulting decomposition does not admit EDPD. Therefore $\gamma_{ed}(G_i) \neq i$. We get contradiction for our assumption.
- If we remove the vertices $1, 2, \dots, n$ in Db_p in the above construction $G_1, G_2, \dots, G_n$, then there will be remaining 1 to $n-1$ vertices and we cannot adjust them to satisfy the minimum eccentric dominating sets of G_i . Therefore the resulting decomposition does not admit EDPD. Therefore $\gamma_{ed}(G_i) \neq i$. We get contradiction for our assumption.

Theorem

If Db_p , $p = 3j^2 + 3j + 2$, $j \in N$ admits EDPD then $\gamma_{ed}\{Db_p\} = 2j^2 + 2j + 2$, $j \in N$

proof:

Let D be a dominating set in Db_p . By theorem 3.1 Db_p admits EDPD.

Let $u_1, u_2, \dots, u_{3j^2+3j+2}, v_1, v_2, \dots, v_{3j^2+3j+2}$ be the vertices of Db_p .

Let $S = \{u_2, u_5, \dots, u_{3j-2}, v_2, v_5, \dots, v_{3j-2}\}$. Then S is a minimum Eccentric dominating Set of Db_p .

Therefore $|s| = |D| = 3j^2 + 3j + 2 - (j + 1)$

$$= 2j^2 + 2j + 2$$

That is, $\gamma_{ed}\{Db_p\} = 2j^2 + 2j + 2, j \in N$

Theorem

If $Db_p, p = 3j^2 + 3j + 2, j \in N$ admits EDPD and $\gamma(Db_p) = |D|$ then $\gamma(Db_p) = \gamma_{ed}\{Db_p\}$.

Proof:

Given $\gamma(Db_p) = |D|$. Therefore D is a minimum dominating set of Db_p .

D consists of $2j^2 + 2j + 2$ vertices.

Therefore $\gamma(Db_p) = 2j^2 + 2j + 2$ (1)

Let S be a minimum Eccentric dominating Set of Db_p . By theorem 3.2 S consists of $2j^2 + 2j + 2$ vertices.

Therefore $\gamma(Db_p) = 2j^2 + 2j + 2, j \in N$ (2)

From equation (1) and (2)

$$\gamma(Db_p) = \gamma_{ed}\{Db_p\}.$$

Theorem

If $Db_p, p = 3j^2 + 3j + 2, j \in N$ admits EDPD then $\gamma_{ed}\{Db_p\} = \sum_{i=1}^{2j} \gamma_{ed}(G_i) + \frac{2j+4}{2}, j \in N$.

Proof:

By our assumption $\gamma_{ed}(G_1) = 1$,

$\gamma_{ed}(G_2) = 2, \dots, \gamma_{ed}(G_{2j}) = 2j, j \in N$

Then

$$\begin{aligned} \sum_{i=1}^{2j} \gamma_{ed}(G_i) + \frac{2j+4}{2} &= [\gamma_{ed}(G_1) + \gamma_{ed}(G_2) + \dots + \gamma_{ed}(G_{2j})] + \frac{2j+4}{2} \\ &= [1 + 2 + \dots + 2j] + \frac{2j+4}{2} \\ &= \frac{2j(2j+1)}{2} + \frac{2j+4}{2} \\ &= 2j^2 + 2j + 2 \\ \sum_{i=1}^{2j} \gamma_{ed}(G_i) + \frac{2j+4}{2} &= \gamma_{ed}\{Db_p\}, \quad j \in N. \end{aligned}$$

by theorem 3.2

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