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Wiener index of Lehmer three mean graphs

M. J. Abisha
Department of Mathematics, St Jude's College, Thoothoor-629176, Tamil Nadu, India | Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamil Nadu, India
Email: abisharejeesh@gmail.com

K. Rubin Mary
Assistant professor, Department of Mathematics, St Jude's College, Thoothoor-629176, Tamil Nadu, India | Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamil Nadu, India
Email: rubinjude@yahoo.com

***Abstract*---**The Wiener index W(G) of G is equal to the sum of distances between all pairs of vertices of G.The Wiener index W,denoted w (wiener 1947) and also known as the path number or wiener number, is a graph index defined by W(G)=∑_({x,y}∈V(G))▒〖d_G (x,y)〗. In this paper we investigate wiener index for caterpillar, twig and arrow graph.

***Keywords*---**Wiener index, Lehmer three, mean graphs.

Introduction

The Wiener topological index (W), introduced around 1947 by Harry Wiener, is the representation of data through a network of vertices (nodes) and edges (connections) which construct shapes to interpret patterns and relationship properties." In graph theory, we define a simple connected graph, G, with vertices, V and edges, E as $G = (V, E)$. For $, y \in V$, the length of the shortest edge from u to v is represented as the distance, $d(x, y)$. The Wiener Index, $W(G)$, is the sum with respect to (x, y) of the subsets of G.

Definition 1.1

Let G be a (r, s) graph.A function h is called Lehmer three mean labeling of graph G, if it is possible to label the vertices $v \in V$ with distinct labels $h(x)$ from $1,2,3, \dots\dots\dots, s+1$ in such a way that when each edge $e = xy$ is labeled with $h(e) =$

Corresponding author: *Abisha, M. J.; Email: abisharejeesh@gmail.com*
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$\left[\frac{h(x)^3+h(y)^3}{h(x)^2+h(y)^2}\right]$ (or) $\left\lfloor\frac{h(x)^3+h(y)^3}{h(x)^2+h(y)^2}\right\rfloor$ then the edge labels are distinct.In this case "h" is called Lehmer-3 mean labeling of G.

Definition 1.2

Let G be a Lehmer three Mean graph. The Wiener Index W(G) of G is defined by $W(G) = \sum_{\{x,y\}\in V(G)} d_G(,v)$

Definition 1.3

The average distance $\mu(G)$ between the vertices of G by:

$$\mu(G) = \frac{W(G)}{\binom{|V(G)|}{2}}$$

Main Results

Theorem 2.1

Twig $Tw(m)$ be a Lehmer three mean graph, then Wiener index of $Tw(m)$ is $W(G) = (3m-5)\times 1 + 6\,(m-2)\times 2 + \sum_{k=3}^{m-1}\sum_{j=3}^{m-1} 9(m-k)j$ .Average distance is $\mu(G) = \frac{W(G)}{\binom{|V(G)|}{2}}$

Proof:

Let Twig Tw(m) be a Lehmer three mean graph.
Then $W(G) = \sum_{\{u,v\}\in V(G)} d_G(u,v)$
$= \{[d(v_1,v_2) + d(v_1,u_1) + d(v_1,w_1) + d(v_1,v_3) + \cdots + d(v_1,v_m) + d(v_1,u_{m-2}) + d(v_1,w_{m-2})] + [d(u_1,w_1) + d(u_1,v_3) + d(u_1,u_2) + \cdots + d(u_1,u_{m-2}) + d(u_1,w_{m-2}) + d(u_1,v_m)] + [d(w_1,u_2) + d(w_1,w_2) + d(w_1,v_3) + \cdots + d(w_1,w_{m-2}) + d(w_1,u_{m-2}) + d(w_1,v_m)] + \cdots + [d(v_{m-1},u_{m-2}) + d(v_{m-1},w_{m-2}) + d(v_{m-1},v_m)] + [d(u_{m-2},v_m) + d(u_{m-2},u_{m-2})] + d(w_{m-2},v_m)\}$
$=(3m-5)\times 1 + 6\,(m-2)\times 2 + \sum_{k=3}^{m-1}\sum_{j=3}^{m-1} 9(m-k)j$

$$W(G) = (3m-5)\times 1 + 6\,(m-2)\times 2 + \sum_{k=3}^{m-1}\sum_{j=3}^{m-1} 9(m-k)j$$

Average distance is $\mu(G) = \frac{W(G)}{\binom{|V(G)|}{2}}$
$= \frac{(3m-5)\times 1+6\,(m-2)\times 2+\sum_{k=3}^{m-1}\sum_{j=3}^{m-1} 9(m-k)j}{\binom{|V(G)|}{2}}$

Example 2.2

Wiener Index of Twig graph Tw(4) is given below

W(G) = $\sum_{\{u,v\in V(G)\}} d_G(u,v)$

$$W(G) = (1+2+2+2+3+3+3+4+4+4) + (1+1+1+2+2+2+3+3+3) + (2+2+3+3+3+4+4+4) + (2+3+3+3+4+4+4) + (1+1+1+2+2+2) + (2+2+3+3+3) + (2+3+3+3) + (1+1+1) + (2+2) + 2$$
$$= (10\times 1) + (18\times 2) + (18\times 3) + (9\times 4)$$
$$\mu(G) = \frac{136}{11C_2} = \frac{136}{55} \simeq 2.5$$

Theorem 2.3

Let Arrow graph $A^2(m)$ be a Lehmer three mean graph, then Wiener index of $A^2(m)$ is $W(G) = 3m \times 1 + \sum_{k=1}^{m-1}\sum_{j=2}^{m} 4(m-k)j$ .Average distance is $\mu(G) = \frac{W(G)}{\binom{|V(G)|}{2}}$

Proof:

Let Arrow graph $A^2(m)$ be a Lehmer three mean graph.
Then $W(G) = \sum_{\{u,v\}\in V(G)} d_G(u,v)$
$= \{[d(u_1,v_1) + d(u_1,w_1) + d(u_1,v_2) + \cdots + d(u_1,v_m) + d(v_1,w_m)] + [d(v_1,v_2) + d(v_1,w_1) + \cdots + d(v_1,v_m) + d(v_1,w_m)] + [d(w_1,v_2) + d(w_1,w_2) + \cdots + d(w_1,v_m) + d(w_1,u_{m-2}) + d(w_1,v_m)] + \cdots + +[d(w_{m-1},v_m) + d(w_{m-1},w_m)] + d(v_m,w_m)\}$
$= 3m \times 1 + \sum_{k=1}^{m-1}\sum_{j=2}^{m} 4(m-k)j$

$$W(G) = 3m \times 1 + \sum_{k=1}^{m-1}\sum_{j=2}^{m} 4(m-k)j$$

Average distance is $\mu(G) = \frac{W(G)}{\binom{|V(G)|}{2}}$
$= \frac{3m\times 1 + \sum_{k=1}^{m-1}\sum_{j=2}^{m} 4(m-k)j}{\binom{|V(G)|}{2}}$

Example 2.4

Wiener Index of Arrow graph $A^2(4)$ is shown below

$$W(G) = \sum_{\{u,v\in V(G)\}} d_G(u,v)$$

$$W(G) = (1+1+2+2+3+3+4+4) + (1+1+2+2+3+3+4) + (2+1+3+2+4+3) + (1+1+2+2+3) + (2+1+3+2) + (1+1+2) + (2+1) + 1$$
$$= (12\times 1) + (12\times 2) + (8\times 3) + (4\times 4)$$
$$\mu(G) = \frac{76}{9C_2} = \frac{136}{36} \simeq 3.7$$

Theorem 2.3

Let Caterpilar graph be a Lehmer three mean graph, then Wiener index of $A^2(m)$ is $W(G) = (3m-1)\times 1 + 2(3m-3)\times 2 + \sum_{k=2}^{m-1}\sum_{j=3}^{m} 3\big(3m-(3k-1)\big)j$ + 4(m+1)

Proof:

Let Caterpilar graph be a Lehmer three mean graph.

Then $W(G) = \sum_{\{u,v\}\in V(G)} d_G(u,v)$

$= \{[d(u_1,v_1) + d(u_1,w_1) + d(u_1,u_2) + \cdots + d(u_1,u_m) + d(u_1,v_m) + d(u_1,w_m)] + [d(v_1,w_1) + d(v_1,u_2) + d(v_1,v_2) \ldots + d(v_1,u_m) + d(v_1,v_m) + d(v_1,w_m)] + \cdots + +[d(u_m,v_m) + d(u_m,w_m)] + d(v_m,w_m)\}$

$=(3m-1)\times 1 + 2(3m-3)\times 2 + \sum_{k=2}^{m-1}\sum_{j=3}^{m} 3\big(3m-(3k-1)\big)j + 4(m+1)$

$$W(G) = (3m-1)\times 1 + 2(3m-3)\times 2 + \sum_{k=2}^{m-1}\sum_{j=3}^{m} 3\big(3m-(3k-1)\big)j + 4(m+1)$$

Example 2.4

Wiener Index of Caterpilar graph is shown below

$$\mathrm{W(G)} = \sum_{\{u,v\in V(G)\}} d_G(u,v)$$

$$\begin{aligned} W(G) = {} & (1+1+1+2+2+2+3+3+3+4+4+4+5+5+5+6+6) \\ & + (2+2+3+3+3+4+4+4+5+5+5+6+6+6+7+7) \\ & + (2+3+3+3+4+4+4+5+5+5+6+6+6+7+7) \\ & +(1+1+1+2+2+2+3+3+3+4+4+4+5+5) \\ & + (2+2+3+3+3+4+4+4+5+5+5+6+6) \\ & + (2+3+3+3+4+4+4+5+5+5+6+6) \\ & + (1+1+1+2+2+2+3+3+3+4+4) \\ & + (2+2+3+3+3+4+4+4+5+5) \\ & +(2+3+3+3+4+4+4+5+5) + (1+1+1+2+2+2+3+3) \\ & + (2+2+3+3+3+4+4) + (2+3+3+3+4+4) \\ & + (1+1+1+2+2) + (2+2+3+3) + (2+3+3) + (1+1) + 2 \end{aligned}$$

$= (17\times 1) + (30\times 2) + (39\times 3) + (30\times 4) + (21\times 5) + (12\times 6) + (4\times 7)$

$= 519$

Average distance is $\mu(G) = \frac{W(G)}{\binom{|V(G)|}{2}}$

$= \frac{519}{18c_2}$

$\mu(G) = \frac{519}{153} \simeq 3.39$

References

1. Y.E.A. Ban, S. Berg and N.H. Mustafa. ”A Conjecture on Wiener Index in Combinatorial chemistry” Algorithm 40(2): 99-118, 2004.
2. D. Bonchev and D.J. Klein,” On the Wiener number of thron trees, stars, rings and rods, croatica chemical ”Acta 75:613-620, 2002.
3. A.Dobriyan and I Gutman,” on a graph invariant related to sum of all distances in a graph” publ. Inst. Math. (Beograd) 56: 18-22 1994
4. S. Somasundaram, S.S. Sandhya and T.S. Pavithra,” Lehmer-3 mean Labeling of graphs” “International Journal of Mathematical Forum,12(17)(2017),819-825.
5. M.J.Abisha and K.Rubin Mary, “K-Lehmer three mean labeling of some graphs”, Malaya Journal of Mathematik, Vol 8,No.3,1219-1221,2020.