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On radio Heronian Dd-distance mean of some standard graphs

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***Abstract*---**A Radio Heronian Mean Dd-distance Labeling of a connected graph G is an injective map f from the vertex set $V(G) \to \mathrm{N}$ such that for two distinct vertices u and v of G, $D^{\mathrm{Dd}}(u,v) + \left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil \geq 1 + diam^{\mathrm{Dd}}(G)$ where $D^{\mathrm{Dd}}(u,v)$ denotes the Dd-distance between u and v and $diam^{\mathrm{Dd}}(G)$ denotes the Dd-diameter of G.

***Keywords*--** radio heronian mean, dd-distance of splitting star, flower graph, shadow graph, switching wheel, switching helm.

Introduction

A graph $G = (V, E)$ we mean a finite undirected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. Graph labeling was introduced by Alexander Rosa in 1967. In 2004 S. Somasundaram and R. Ponraj introduced Radio mean labeling. Harmonic mean labeling, S. Somasundaram and S S Sandhya introduced in 2012. The concept D-distance introduced D. Reddy Babu et al. The concept radio D-distance introduced T. Nicholas and K. John Bosco in 2017. The concept radio mean D-distance introduced T. Nicholas and K. John Bosco in 2017. In 2017 S S Sandhya intoduced concept of heronian mean labeling. The Dd-distance introduced A. Anto kinsely and P. Siva Ananthi in 2017. The concept radio heronian mean Dd-distance labeling of some basic graphs introduced K. John Bosco and Dinesh M in 2021. For a connected graph G, $u - v$ path is defined as $D^{Dd}(u,v) = D(u,v) + \deg(u) + deg(v)$. we are introducing the concept of radio heronian Dd-distance mean of some standard graphs.

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Main results

Theorem 2.1

The radio heronian mean Dd-distance number of a degree splitting star,

$$rhmn^{Dd}(Spl(B_{n,n})) = \begin{cases} 7, & n = 2 \\ 10, & n = 3 \\ 14, & n = 4 \\ 3n+3, & n \geq 5 \end{cases}.$$

Proof

Let $V(S(B_{n,n})) = \{v\} \cup \{u\} \cup \{u_i/i = 1,2,3 \dots \dots \dots n\}$ and $E = \{u_i v,\ u_i u/i = 1,2 \dots . n\}$..
Then $D^{Dd}(v_o, v_i) = 2n + 3,\ D^{Dd}(v_0, v_j) = 3n + 2, D^{Dd}(v_i, v_{i+1}) = 5, 1 \leq i \leq n$
So $diam^{\mathrm{Dd}}(S(B_{n,n}) = 3n + 2$
By radio heronian mean Dd-distance condition,

$$D^{\mathrm{Dd}}(u,v) + \left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil \geq 1 + diam^{\mathrm{Dd}}(G)$$

If v_0 and v_i are adjacent,

$$D^{Dd}(v_0, v_i) + \left\lceil \frac{f(v_0) + \sqrt{f(v_0)f(v_i)} + f(v_i)}{3} \right\rceil \geq 3n + 3$$

$$2n + 3 + \left\lceil \frac{f(v_0) + \sqrt{f(v_0)f(v_i)} + f(v_i)}{3} \right\rceil \geq 3n + 3$$

$$\left\lceil \frac{f(v_0) + \sqrt{f(v_0)f(v_i)} + f(v_i)}{3} \right\rceil \geq n$$

If v_i and v_j are non adjacent,

$$D^{Dd}(v_i, v_j) + \left\lceil \frac{f(v_i) + \sqrt{f(v_i)f(v_j)} + f(v_j)}{3} \right\rceil \geq 3n + 3$$

$$3n + 2 + \left\lceil \frac{f(v_i) + \sqrt{f(v_i)f(v_j)} + f(v_j)}{3} \right\rceil \geq 3n + 3$$

$$\left\lceil \frac{f(v_0) + \sqrt{f(v_0)f(v_i)} + f(v_i)}{3} \right\rceil \geq 1$$

Therefore $f(v_i) = n + i + 1, 1 \leq i \leq n$

Hence $rhmn^{Dd}(Spl(B_{n,n})) = \begin{cases} 7, & n = 2 \\ 10, & n = 3 \\ 14, & n = 4 \\ 3n+3, & n \geq 5 \end{cases}.$

Theorem 2.2

The radio heronian mean Dd-distance number of a flower graph, $rhmn^{Dd}(Fl_n) \leq \{4n - 1,\ n \geq 3$

Proof

Let $V(Fl_n) = \{v\} \cup \{u\} \cup \{u_i/i = 1,2,3 \dots \dots \dots n\}$ and $E = \{u_i v,\ u_i u, u_i u_j\ /i = 1,2 \dots . n\}$.

Then $D^{Dd}(v_1,u_1)=D^{Dd}(v_2,u_2)=D^{Dd}(v_3,u_3)=n+4, 1\le i\le n$
$D^{Dd}(v_0,v_1)=D^{Dd}(v_0,v_2)=D^{Dd}(v_0,v_3)=n+8, 1\le i\le n$
$D^{Dd}(v_0,u_1)=D^{Dd}(v_0,u_2)=D^{Dd}(v_0,u_3)=n+6, 1\le i\le n$
So $diam^{\mathrm{Dd}}(Fl_n)=3n+5$
By radio heronian mean Dd-distance condition,
$D^{\mathrm{Dd}}(u,v)+\left\lceil\frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3}\right\rceil\ge 1+diam^{\mathrm{Dd}}(G)$
If (v_1,u_2) and (v_0,v_1) are adjacent,

$$D^{Dd}(v_1,u_1)+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(u_1)}+f(u_1)}{3}\right\rceil\ge 3n+6$$

$$n+4+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(u_1)}+f(u_1)}{3}\right\rceil\ge 3n+6$$

$$D^{Dd}(v_0,v_1)+\left\lceil\frac{f(v_0)+\sqrt{f(v_0)f(v_1)}+f(v_1)}{3}\right\rceil\ge 3n+6$$

$$n+8+\left\lceil\frac{f(v_0)+\sqrt{f(v_0)f(v_1)}+f(v_1)}{3}\right\rceil\ge 3n+6$$

If (v_0,u_1) and (v_1,u_2) are non adjacent,

$$D^{Dd}(v_0,u_1)+\left\lceil\frac{f(v_0)+\sqrt{f(v_0)f(u_1)}+f(u_1)}{3}\right\rceil\ge 3n+6$$

$$n+6+\left\lceil\frac{f(v_0)+\sqrt{f(v_0)f(u_1)}+f(u_1)}{3}\right\rceil\ge 3n+6$$

$$D^{Dd}(v_1,u_2)+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(u_2)}+f(u_2)}{3}\right\rceil\ge 3n+6$$

$$n+5+\left\lceil\frac{f(v_1)+\sqrt{f(v_1)f(v_2)}+f(v_2)}{3}\right\rceil\ge 3n+6$$

Therefore $f(v_i)=2n+i-2, 1\le i\le n$
Hence $rhmn^{Dd}(Fl_n)\le\{4n-1,\ n\ge 3$

Theorem: 2.3

The radio heronian mean Dd-distance number of a shadow graph,

$$rhmn^{D}(D_2(K_{1,n}))=\begin{cases}7, & n=2\\ 11, & n=3\\ 6n-6, & n\ge 4.\end{cases}$$

Proof

Let $V(D_2(K_{1,n}))=\{v_i,u_{j,}/i=1,2,3\ldots\ldots\ldots n\ and\ j=1,2,\ldots.n\}$ be the vertex set and
$E=\{v_iu_j,u_iu_{j+1}/i=1,2\ldots.n, j=1,2..n$
Then $D^{Dd}(v_1,u_1)=D^{Dd}(v_1,u_2)=n+11, D^{Dd}(u_1,u_2)=n+4, D^{Dd}(v_1,v_2)=n+16\quad 1\le i\le n$
So $diam^{\mathrm{Dd}}\left(D_2(K_{1,n})\right)=4n+2$
By radio heronian mean Dd-distance condition
$D^{\mathrm{Dd}}(u,v)+\left\lceil\frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3}\right\rceil\ge 1+diam^{\mathrm{Dd}}(G)$

If (v_1, u_1) and (v_1, u_2) are adjacent,

$$D^{Dd}(v_1,u_1) + \left\lceil \frac{f(v_1)+\sqrt{f(v_1)f(u_1)}+f(u_1)}{3} \right\rceil \geq n+11+\left\lceil \frac{f(v_1)+\sqrt{f(v_1)f(u_{12})}+f(u_1)}{3} \right\rceil \geq 3n+3$$

$$D^{Dd}(v_1,u_2) + \left\lceil \frac{f(v_1)+\sqrt{f(v_1)f(u_2)}+f(u_2)}{3} \right\rceil \geq n+11+\left\lceil \frac{f(v_1)+\sqrt{f(v_1)f(u_2)}+f(u_2)}{3} \right\rceil \geq 3n+3$$

If (u_1, u_2) are non adjacent,

$$D^{Dd}(u_1,u_2) + \left\lceil \frac{f(u_1)+\sqrt{f(u_1)f(u_2)}+f(u_2)}{3} \right\rceil \geq n+4+\left\lceil \frac{f(u_1)+\sqrt{f(u_1)f(u_2)}+f(u_2)}{3} \right\rceil \geq 3n+3$$

$$D^{Dd}(v_1,v_2) + \left\lceil \frac{f(v_1)+\sqrt{f(v_1)f(v_2)}+f(v_2)}{3} \right\rceil \geq n+16+\left\lceil \frac{f(v_1)+\sqrt{f(v_1)f(v_2)}+f(v_2)}{3} \right\rceil \geq n+6$$

Therefore $f(v_i) = 4n+i-8, 1 \leq i \leq n$

Hence $rhmn^{D}(D_2(K_{1,n})) = \begin{cases} 7, & n=2 \\ 11, & n=3 \\ 6n-6, & n \geq 4. \end{cases}$

Theorem: 2.4

The radio heronian mean Dd-distance number of a switching wheel,

$$rhmn^{Dd}(SW_n) = \begin{cases} 8, & n=4 \\ 2n-1, & n \geq 5 \end{cases}.$$

Proof

Let $V(SW_n) = \{v\} \cup \{u\} \cup \{u_i / i = 1,2,3 \dots \dots \dots n\}$ and $E = \{u_i v,\ u_i u, u_i u_j \ / i = 1,2 \dots . n\}$.
Then $D^{Dd}(v_1,u_1) = n+5, D^{Dd}(v_2,u_1) = D^{Dd}(v_1,v_2) = n+2, 1 \leq i \leq n$
So $diam^{\mathrm{Dd}}(SW_n) = 2n+2$
By radio heronian mean Dd-distance condition

$$D^{\mathrm{Dd}}(u,v) + \left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil \geq 1 + diam^{\mathrm{Dd}}(G)$$

If (v_1, v_2) and (v_2, u_1) are adjacent,

$$D^{Dd}(v_1,v_2) + \left\lceil \frac{f(v_1)+\sqrt{f(v_1)f(v_2)}+f(v_2)}{3} \right\rceil \geq n+2+\left\lceil \frac{f(v_1)+\sqrt{f(v_1)f(v_2)}+f(v_2)}{3} \right\rceil \geq 2n+3$$

$$D^{Dd}(v_2,u_1) + \left\lceil \frac{f(v_2)+\sqrt{f(v_2)f(u_1)}+f(u_1)}{3} \right\rceil \geq n+2+\left\lceil \frac{f(v_2)+\sqrt{f(v_2)f(u_1)}+f(u_1)}{3} \right\rceil \geq 2n+3$$

If v_1 and u_1 are non adjacent,

$$D^{Dd}(v_1,u_1) + \left\lceil \frac{f(v_1)+\sqrt{f(v_1)f(u_1)}+f(u_1)}{3} \right\rceil \geq n+5+\left\lceil \frac{f(v_1)+\sqrt{f(v_1)f(u_{12})}+f(u_1)}{3} \right\rceil \geq 2n+3$$

Therefore $f(v_i) = 2n+i, 1 \leq i \leq n$

Hence $rhmn^{Dd}(DS(P_n)) = \begin{cases} 3, & n = 2 \\ 6, & n = 3 \\ 2n+1, & n \geq 4 \end{cases}$.

Theorem 2.5

The radio heronian mean Dd-distance number of a switching helm,

$$rhmn^{Dd}(SH_n) \leq \begin{cases} 9, & n = 3 \\ 12, & n = 4 \\ 3n-1, & n \geq 5 \end{cases}$$

Proof

Let $V(SH_n) = \{v\} \cup \{u\} \cup \{u_i / i = 1,2,3 \ldots\ldots\ldots n\}$ and $E = \{u_i v,\ u_i u, u_i u_j\ / i = 1,2 \ldots . n\}$.
Then $D^{Dd}(v_1, u_1) = D^{Dd}(v_2, u_2) = D^{Dd}(v_3, u_3) = n + 1, 1 \leq i \leq n$
$D^{Dd}(v_0, v_1) = D^{Dd}(v_0, v_2) = D^{Dd}(v_0, v_3) = n + 5, 1 \leq i \leq n$
$D^{Dd}(v_0, u_1) = D^{Dd}(v_0, u_2) = D^{Dd}(v_0, u_3) = n + 3, 1 \leq i \leq n$
So $diam^{\mathrm{Dd}}(SH_n) = 2n + 4$
By radio heronian mean Dd-distance condition

$$D^{\mathrm{Dd}}(u, v) + \left\lceil \frac{f(u) + \sqrt{f(u)f(v)} + f(v)}{3} \right\rceil \geq 1 + diam^{\mathrm{Dd}}(G)$$

If $(v_1, u_{2)}$ and (v_0, u_1) are adjacent,

$$D^{Dd}(v_1, u_1) + \left\lceil \frac{f(v_1) + \sqrt{f(v_1)f(u_1)} + f(u_1)}{3} \right\rceil \geq n + 1 + \left\lceil \frac{f(v_1) + \sqrt{f(v_1)f(u_1)} + f(u_1)}{3} \right\rceil$$
$$\geq 2n + 5$$
$$D^{Dd}(v_0, u_1) + \left\lceil \frac{f(v_0) + \sqrt{f(v_0)f(u_1)} + f(u_1)}{3} \right\rceil \geq n + 3 + \left\lceil \frac{f(v_0) + \sqrt{f(v_0)f(u_1)} + f(u_1)}{3} \right\rceil$$
$$\geq 2n + 5$$

If (v_0, v_1) and (u_1, u_2) are non adjacent,

$$D^{Dd}(v_0, v_1) + \left\lceil \frac{f(v_0) + \sqrt{f(v_0)f(v_1)} + f v_1)}{3} \right\rceil \geq n + 5 + \left\lceil \frac{f(v_0) + \sqrt{f(v_0)f(v_1)} + f(v_1)}{3} \right\rceil$$
$$\geq 2n + 5$$
$$D^{Dd}(u_1, u_2) + \left\lceil \frac{f(u_1) + \sqrt{f(u_1)f(u_2)} + f(u_2)}{3} \right\rceil \geq n + 2 + \left\lceil \frac{f(u_1) + \sqrt{f(u_1)f(u_2)} + f(u_2)}{3} \right\rceil$$
$$\geq 2n + 5$$

Therefore $f(v_i) = n - i, 1 \leq i \leq n$

Hence $rhmn^{Dd}(SH_n) \leq \begin{cases} 9, & n = 3 \\ 12, & n = 4 \\ 3n-1, & n \geq 5 \end{cases}$

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