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# Equality of total domination and chromatic total domination in graphs

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**Abstract**---Let  $G = (V, E)$  be a simple, finite and undirected graph and without isolated vertex. A subset  $D$  of  $V$  is said to be dominating set if for every  $v$  in  $V - D$  there exist a vertex  $u$  in  $D$  such that  $u$  and  $v$  are adjacent. The minimum cardinality of a dominating set of  $G$  is called the domination number of  $G$  and is denoted by  $\gamma(G)$ .  $D$  is minimal dominating set of a graph  $G$  if no proper subset of  $D$  is a dominating set of  $G$ .  $D$  is a total dominating set of  $G$  if  $\langle D \rangle$  has no isolates. The minimum cardinality of a total dominating set of  $G$  is called the total domination number of  $G$  and is denoted by  $\gamma_t(G)$ . A Subset  $D$  of  $V$  ( $D \subseteq V$ ) is said to be a chromatic total dominating set if  $D$  is a total dominating set and  $\chi(\langle D \rangle) = \chi(G)$ . The minimum cardinality of the chromatic total dominating set is called a chromatic total dominating number  $\gamma_{ch}^t(G)$ . In any graph  $G$ , every chromatic total dominating set is a total dominating set. But converse is not true. In some graphs, every  $\gamma_t$  set is a chromatic total dominating set. In some graphs, some  $\gamma_t$  set is a chromatic total dominating set and some  $\gamma_t$  set is not chromatic total dominating set. While in some graphs no  $\gamma_t$  set is a chromatic total dominating set. This observation motivates us to classify all graphs into three categories.

**Keywords**---total domination, chromatic total domination, chromatic total domination number.

**Introduction**

Let  $G = (V, E)$  be a simple, finite and undirected graph and without isolated vertex. A subset  $D$  of  $V$  is said to be dominating set if for every  $v$  in  $V - D$  there exist a

vertex  $u$  in  $D$  such that  $u$  and  $v$  are adjacent. The minimum cardinality of a dominating set of  $G$  is called the domination number of  $G$  and is denoted by  $\gamma(G)$ .  $D$  is minimal dominating set of a graph  $G$  if no proper subset of  $D$  is a dominating set of  $G$ .  $D$  is a total dominating set of  $G$  if  $\langle D \rangle$  has no isolates. The minimum cardinality of a total dominating set of  $G$  is called the total domination number of  $G$  and is denoted by  $\gamma_t(G)$ . A Subset  $D$  of  $V$  ( $D \subseteq V$ ) is said to be a chromatic total dominating set if  $D$  is a total dominating set and  $\chi(\langle D \rangle) = \chi(G)$ . The minimum cardinality of the chromatic total dominating set is called a chromatic total dominating number  $\gamma_{ch}^t(G)$ . In any graph  $G$ , every chromatic total dominating set is a total dominating set. But converse is not true. In some graphs, every  $\gamma_t$  set is a chromatic total dominating set. In some graphs, some  $\gamma_t$  set is a chromatic total dominating set and some  $\gamma_t$  set is not chromatic total dominating set. While in some graphs no  $\gamma_t$  set is a chromatic total dominating set. This observation motivates us to classify all graphs into three categories.

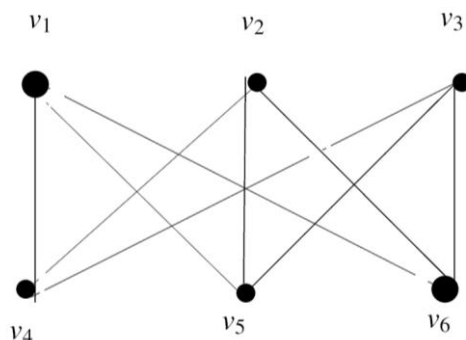
### Equality of Sets

#### Definition 2.1

A graph  $G$  is said to be in  $(\gamma_t(G) \equiv \gamma_{ch}^t(G))$ -class if every  $\gamma_t$  set is a chromatic total dominating set.

#### Examples 2.2

Consider the following graph



$G$

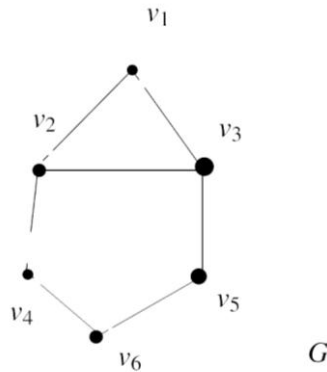
$$D_1 = \{v_1, v_4\}, D_2 = \{v_1, v_5\}, D_3 = \{v_1, v_6\}, D_4 = \{v_2, v_4\}, D_5 = \{v_2, v_5\}, D_6 = \{v_2, v_6\}, \\ D_7 = \{v_3, v_4\}, D_8 = \{v_3, v_5\}, D_9 = \{v_3, v_6\}. \\ D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8, D_9 \text{ are } \gamma_t \text{ sets as well as } \gamma_{ch}^t\text{-sets.}$$

#### Definition 2.3

A graph  $G$  is said to be in  $(\gamma_t(G) \equiv \gamma_{ch}^t(G))$ -class if some  $\gamma_t$  set is a chromatic total dominating set and some  $\gamma_t$  is not chromatic total dominating set

**Examples 2.4**

Consider the following graph:



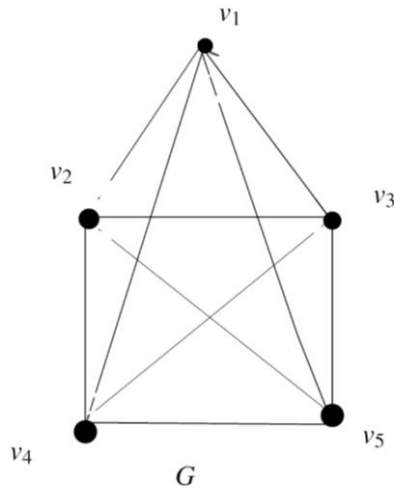
$D_1 = \{v_1, v_2, v_3, v_4\}$  and  $D_2 = \{v_1, v_3, v_5, v_6\}$ .  $D_1$  and  $D_2$  are  $\gamma_t$ -sets but  $D_1$  is  $\gamma_{ch}^t(G)$  set but  $D_2$  is not  $\gamma_{ch}^t(G)$  set

**Definition 2.5**

A graph  $G$  is said to be in  $(\gamma_t(G) \neq \gamma_{ch}^t(G))$ -class if no  $\gamma_t$  is a chromatic total dominating set .

**Examples 2.6**

Consider the following graph:



$D_1 = \{v_1, v_2\}, D_2 = \{v_1, v_4\}, D_3 = \{v_1, v_3\}, D_4 = \{v_1, v_5\}, D_5 = \{v_2, v_3\}$   $D_6 = \{v_2, v_4\}, D_7 = \{v_2, v_5\}, D_8 = \{v_3, v_4\}, D_9 = \{v_3, v_5\}, D_{10} = \{v_4, v_5\}$   
 All these are  $\gamma_t$  set but not  $\gamma_{ch}^t$  set .

**Observation 2.7**

Let  $G$  be a  $\chi$  critical graph with  $|V(G)| > 1$ . Then  $G$  is in  $(\gamma_t(G) \neq \gamma_{ch}^t(G))$ -class. (Since for such graph  $\gamma_t(G) < n$  and  $\gamma_{ch}^t(G) = n$ )

**Theorem 2.8**

Let  $G=K_n$ ,  $n \geq 2$ , then  $G$  is in  $(\gamma_t \neq \gamma_{ch}^t)$ -class

*Proof.* Let  $G$  be complete graph with  $n \geq 2$ . Let  $D$  be the total dominating set where  $\gamma_t(K_n)=2$ . Therefore,  $\chi(< D >) = 2 \neq n = \chi(G)$  for every  $\gamma_t$  sets. Thus,  $D$  is not a chromatic total dominating set. Hence  $G$  is in  $(\gamma_t(G) \neq \gamma_{ch}^t(G))$ -class

**Theorem 2.9**

Let  $G=K_{1,n}$ , then  $G$  is in  $(\gamma_t \equiv \gamma_{ch}^t)$ -class

*Proof.* Let  $G$  be Star graph with  $n+1$  vertices. Let  $D$  be the  $\gamma_t$  set of  $G$ . Therefore  $\chi(< D >)=2=\chi(G)$  for every  $\gamma_t$  sets. Thus,  $D$  is also chromatic total dominating set. Hence,  $G$  is in  $(\gamma_t \equiv \gamma_{ch}^t)$ -class

**Theorem 2.10**

Let  $G=F_n$ , with  $n \geq 3$ , then  $G$  is in  $(\gamma_t(G) \neq \gamma_{ch}^t(G))$ -class

*Proof.* Let  $G$  be a Fan graph with  $n$  vertices. Let  $D$  be the total dominating set. Let  $u$  be the vertex of  $F_n$  with  $\deg(u)=n-1$  and  $\gamma_t(F_n)=2$ . But  $\chi(< D >) \neq \chi(G)$ . Therefore  $D$  is not chromatic total dominating set. Hence  $G$  is in  $(\gamma_t(G) \neq \gamma_{ch}^t(G))$ -class

**Theorem 2.11**

Let  $G=W_n$ , then  $G$  is in  $(\gamma_t(G) \neq \gamma_{ch}^t(G))$ -class

*Proof.* Let  $W_n$  be a wheel with  $n$  vertices and  $V(W_n)=\{x, v_1, v_2, \dots, v_{n-1}\}$ . Let  $x$  be a full degree vertex of  $W_n$ . Let  $D$  be the total dominating set. The full degree vertex dominates all other vertex in the graph. Since  $D$  should not contain isolated vertex we choose additional vertex and  $\chi(< D >)=2 \neq \chi(G)$ . Therefore,  $D$  is total dominating set with  $\gamma_t(G)=2$  and it is not chromatic total dominating set. Hence  $G$  is in  $(\gamma_t(G) \neq \gamma_{ch}^t(G))$ -class

**Theorem 2.12**

Let  $G$  be a complete binary tree, then  $G$  is in  $(\gamma_t \equiv \gamma_{ch}^t)$ -class

*Proof.* Let  $G$  be complete binary tree and  $B_k$  denote a complete binary tree formed at depth  $k$ . Let  $D_k$  be a  $\gamma_t$  set of  $B_k$ .  $D_k$  includes the set of vertices in the  $i$ th level and  $(i-1)$ th level should also be included in  $D_k$ . It is also found that the number of set of vertices in the  $i$ th level is twice than that of vertices in the  $(i-1)$ th level. To dominate the 0th level any one vertex at 1st level also include in  $D_k$ . It is observed that  $D_k$  includes the minimum number of vertices. Hence the total dominating set

and  $\chi(< D >) = \chi(G)$  at depth k. Therefore, D is a chromatic total dominating set. Hence G is in  $(\gamma_t \equiv \gamma_{ch}^t)$ -class

### Theorem 2.13

Let  $G = K_{m,n}$ , then G is in  $(\gamma_t \equiv \gamma_{ch}^t)$ -class

*Proof.* Let G be a complete bipartite graph with  $m+n$  vertices. Let D be the total dominating set. Let  $u$  be a vertex of  $K_{m,n}$ . Let  $x \in V(K_{m,n}) - \{u\}$  such that  $ux \in E(K_{m,n})$ . Let  $D = \{u, x\}$  and  $\chi(< D >) = \chi(G) = 2$  for every  $\gamma_t$  sets.

Thus D is chromatic total dominating set and G is in  $(\gamma_t \equiv \gamma_{ch}^t)$ -class

### Theorem 2.14

Let  $G = P_n$ . Then G belongs to  $(\gamma_t \equiv \gamma_{ch}^t)$ -class

*Proof.*

Let  $P_n = u_1, u_2, \dots, u_n$  be the path on n vertices.

**case(i):** Let  $n \equiv 0 \pmod{4}$

Let  $n = 4k$ . Let D be the total dominating set.  $\gamma_t(P_n) = n/2$  if  $n \equiv 0 \pmod{4}$  and  $\chi(< D >) = \chi(G) = 2$ . Therefore, D is the chromatic total dominating set. Hence  $P_{4k}$  belongs to  $(\gamma_t \equiv \gamma_{ch}^t)$ -class

**case(ii):** Let  $n \equiv 1, 3 \pmod{4}$

Let  $n = 4k + 1$ . Let D be the total dominating set with  $\gamma_t(P_n) = n + 1/2$  vertices if  $n \equiv 1, 3 \pmod{4}$  and  $\chi(< D >) = \chi(G) = 2$ . Therefore, D is the chromatic total dominating set. Hence  $P_{4k+1}$  belongs to  $(\gamma_t \equiv \gamma_{ch}^t)$ -class

**Case (iii):** Let  $n \equiv 2 \pmod{4}$

Let  $n = 4k + 2$ . Let D be the total dominating set with  $\gamma_t(P_n) = n/2 + 1$  vertices if  $n/2 + 1$  vertices and  $\chi(< D >) = \chi(G) = 2$ . Therefore, D is the chromatic total dominating set. Hence  $P_{4k+2}$  belongs to  $(\gamma_t \equiv \gamma_{ch}^t)$ -class.

Finally,  $G = P_n$  belongs to  $(\gamma_t \equiv \gamma_{ch}^t)$ -class

### Theorem 2.15

The cycle  $C_n$  of  $\gamma(t) \neq \gamma_{ch}^t(G)$  -class if and only if n is odd and  $C_n$  is of  $(\gamma_t \equiv \gamma_{ch}^t)$ -class if and only if n is even.

*Proof.* Let  $C_n: v_1, v_2, \dots, v_n$  be a cycle on n vertices.

**case(i)** If n is odd

Let n be odd. Let D be the minimal total dominating set of G.  $\gamma_t(C_n) = n + 1/2$  when n is odd and  $\gamma_{ch}^t(C_n) = n$ . Therefore,  $\gamma_t(C_n) = n + 1/2 < n = \gamma_{ch}^t(C_n)$  when n is odd. Thus no  $\gamma_t$  set is a chromatic total dominating set. Hence  $C_n$  of  $(\gamma_t \neq \gamma_{ch}^t)$ -class when n is odd.

**case(ii)** If n is even and  $n \equiv 0 \pmod{4}$

Let n be even and  $n = 4k$ . Let D be the minimal total dominating set in G and  $\gamma_t(C_n) = n/2$ . Also  $\chi(< D >) = 2 = \chi(G)$ . Therefore D is chromatic total dominating set of G with minimum cardinality  $\gamma_{ch}^t(G) = |D| = \gamma_t(G)$ . Hence  $C_n$  of  $(\gamma_t \equiv \gamma_{ch}^t)$ -class when n is even.

**case(iii)** If n is even and  $n \equiv 2 \pmod{4}$

Let  $n = 4k + 2$ . Let D be the minimal total dominating set in G.  $\gamma_t(C_n) = n/2 + 1$ . Also

$\chi(< D >) = \chi(G) = 2$ . Therefore,  $D$  is chromatic total dominating set of  $G$  with minimum cardinality  $\gamma_{ch}^t(G) = |D| = \gamma_t(G)$ . Hence,  $C_n$  is of  $(\gamma_t \equiv \gamma_{ch}^t)$ -class of  $G$  when  $n$  is even.

**Theorem 2.16**

*Let  $T$  be a tree .If there exists support vertices  $u, v$  such that  $d(u_i) = 2$  for all  $u_i$  on the path joining  $u$  and  $v$  and  $d(u, v) \equiv 0 \pmod{3}$ . Then  $u, v$  belongs to every  $\gamma_{ch}^t(G)$*

*Proof.* Let  $u, v$  be a pair of vertices in  $T$  such that  $d(u, v) \equiv 0 \pmod{3}$  and  $d(u_i) = 2$  for all  $u_i$  on the path joining  $u$  and  $v$ . Suppose there is a  $\gamma_t$  set  $D$  such that  $u \notin D$  or  $v \notin D$ . Let  $u \notin D$ . Then  $N(u)$  has exactly one pendent vertex, say  $w$ . Then  $w \in D$ . Let  $u = u_0, u_1, \dots, u_r = v$ . Then  $r \equiv 0 \pmod{3}$ . Since  $D$  is  $\gamma_t$  set and  $u_0 = u \notin D$ , Assume that  $u_2, u_5, \dots, u_{r-1} \in D$ . Let  $r = 3k$ , then  $|\{u_2, u_5, \dots, u_{r-1}\}| = k$ . If  $v \in D$ , then  $D_1 = (D - \{w, u_2, u_5, \dots, u_{r-1}\}) \cup \{u_0, u_1, \dots, u_{r-3}\}$  is a total dominating set and  $|D_1| = |D| - 1$  which contradiction. If  $v \notin D$ , then  $N(v)$  has exactly one pendant vertex  $x$  and  $x \in D$ .  $D_2 = (D - \{x, w, u_2, u_5, \dots, u_{r-1}\}) \cup \{v, u_3, u_6, \dots, u_{r-3}\}$  is a total dominating set and  $|D_2| = |D| - 1$  which is contradiction. Hence  $u \in D$  and  $v \in D$  and  $\chi(G) = \chi(< D >) = 2$  and  $D$  is also chromatic total dominating set. Then  $u, v$  belongs to every  $\gamma_{ch}^t$ -set of  $T$ .

**Proposition 2.17**

*Let  $P$  be the set of all pendant vertices of a tree  $T$ . If  $T$  has two vertices  $u, v$  such that  $|N(u) \cap p| \geq 2, |N(v) \cap p| \geq 2$  and  $uv \in E(T)$ , Then  $T$  belongs to  $(\gamma_t \equiv \gamma_{ch}^t)$ -class*

*Proof.* Let  $|N(u) \cap p| \geq 2, |N(v) \cap p| \geq 2$ . Then  $u, v$  belongs to every  $\gamma_{ch}^t$ -set  $D$  of  $T$ . since  $uv \in E(T), \chi(< D >) = 2, D$  is a  $\gamma_{ch}^t$ -set. Hence  $T$  is in  $(\gamma_t \equiv \gamma_{ch}^t)$ -class

**Theorem 2.18**

*Let  $T$  be a tree in which  $d(u, v) \equiv 0 \pmod{3}$  for some support vertices and  $d(u_i) = 2$  for all  $u_i$  on the path joining  $u$  and  $v$ . If there exists a support vertex  $w$  such that  $wv \in E(T)$  and either  $|N(w) \cap p| \geq 2$  or for some support vertex  $x, d(w, x) \equiv 0 \pmod{3}$  and  $d(x_i) = 2$  for all vertices  $x_i$  on the path joining  $w$  and  $x$ , then  $T$  belongs to  $(\gamma_t \equiv \gamma_{ch}^t)$ -class*

*Proof.* Let  $T$  be a tree in which for some support vertices  $u, v$  and  $w, d(u, v) \equiv 0 \pmod{3}$  and  $d(u_i) = 2$  for all vertices  $u_i$  on the path joining  $u, v$  and  $wv \in E(T)$ . Suppose that either  $|N(w) \cap p| \geq 2$  or there exist a support vertex  $x$  such that  $d(w, x) \equiv 0 \pmod{3}$  and  $d(x_i) = 2$  for all vertices  $x_i$  on the path joining  $w$  and  $x$ . Let  $D$  be a  $\gamma_t$ -set. By proposition 2.17,  $u, v \in D$ . If  $|N(w) \cap p| \geq 2$ , then  $w \in D$ . If there exist a support vertex  $x$  such that  $d(w, x) \equiv 0 \pmod{3}$  and  $d(x_i) = 2$  for all vertices  $x_i$  on the path joining  $w$  and  $x$ , then by proposition 2.17,  $w, x \in D$ . Thus in both cases,  $w \in D$ . Since  $wv \in E(T), \chi(< D >) = 2$ . Thus  $D$  is a  $\gamma_{ch}^t$ -set. Hence,  $T$  is in  $(\gamma_t \equiv \gamma_{ch}^t)$ -class.

**Theorem 2.19**

Let  $D$  be caterpillar.  $T$  is in  $(\gamma_t \equiv \gamma_{ch}^t)$ -class iff  $T$  has two support vertices  $u, v$  satisfying one of the following properties :

- (i)  $|N(u) \cap p| \geq 2, |N(v) \cap p| \geq 2$  and  $uv \in E(T)$
- (ii) If  $d(u, v) \equiv 0 \pmod{3}$  and  $d(u_i)=2$  for all  $u_i$  on the path joining  $u$  and  $v$ , there exists a support vertex  $w$  such that  $vw \in E(T)$  and  $|N(w) \cap p| \geq 2$  or for some support vertex  $x, d(w, x) \equiv 0 \pmod{3}$  and  $d(x_i)=2$  for all vertices  $x_i$  on the path joining  $w$  and  $x$ .

*Proof.* Let  $T$  be a Caterpillar. Assume that  $T$  is in  $(\gamma_t \equiv \gamma_{ch}^t)$ -class. Suppose that for every pair  $u, v$  of support vertices, the given conditions fail. Let  $D$  be  $\gamma_t$  set with minimum number of edges in  $\langle D \rangle$ . Since  $D$  is in  $(\gamma_t \equiv \gamma_{ch}^t)$ -class,  $\chi(\langle D \rangle) = 2$ . Therefore, there exists  $u, v \in D$  such that  $uv \in E(T)$ . Then  $u, v \in V(T) - P$ . Since  $uv \in E(T)$  and since (i) is not true,  $|N(u) \cap p| \leq 1$ ,  $|N(v) \cap p| \leq 1$ .

**case(i)**

Let  $N(u) \cap p = \phi$ . Since  $D$  is a  $\gamma_t$ -set and  $uv \in E(T)$ ,  $u \notin P$ . Since  $T$  is caterpillar and  $N(u) \cap P = \phi, d(u)=2$ . Let  $x \in N(u) - \{v\}$ . Since  $D$  is a  $\gamma_t$ -set and  $u, v \in D$ ,  $(N(x) - \{u\}) \cap D = \phi$ . Then  $D_1 = (D - \{u\}) \cup \{x\}$  is a  $\gamma_t$  set and  $|E(\langle D_1 \rangle)| = |E(\langle D \rangle)| - 1$  which is a contradiction to the fact that  $D$  is a  $\gamma_t$  set with minimum number of edges in  $\langle D \rangle$ .

**case(ii)**

Let  $N(u) \cap P = \{x\}$  and  $N(v) \cap P = \{y\}$ . If  $(N(u) - \{v\}) \cap D \neq \phi$ , then  $D_2 = (D - \{u\}) \cup \{x\}$  is a  $\gamma_t$  set with  $|E(\langle D_2 \rangle)| = |E(\langle D \rangle)| - 1$  which is contradiction. Let  $(N(u) - \{v\}) \cap D = \phi$  and  $(N(v) - \{u\}) \cap D = \phi$ . Let  $w_1 \neq v$  be a support vertex such that  $d(u, w_1)$  is minimum and  $d(u_i)=2$  for all vertices  $u_i$  on the path joining  $u$  and  $w_1$ . Let  $w_2 \neq u$  be a support vertex such that  $d(v, w_2)$  is minimum and  $d(v_i) = 2$  for all vertices  $v_i$  on the path joining  $v$  and  $w_2$ . Let  $d(u, w_1) \equiv 0 \pmod{3}$ . Since  $uv \in E(T)$ , by our assumption  $d(v, w_2) \equiv 1 \pmod{3}$ . Let  $v = v_0, v_1, v_2, \dots, v_r = w_2$  and  $d(v_i)=2$  for all  $i=1, 2, \dots, r-1$ . Then  $r=3k+1$  or  $3k+2$ . Since  $D$  is a  $\gamma_t$  set and  $v \in D$ . We can assume that  $v_3, v_6, v_9, \dots, v_{r-1} \in D$  if  $r=3k+1$  or  $v_3, v_6, v_9, \dots, v_{r-1} \in D$  if  $r=3k+2$ . Let  $w_2 \in D$ . If  $r=3k+1$ , then  $D_3 = (D - \{v_1, v_3, v_6, \dots, v_{r-1}\}) \cup \{v_2, v_5, \dots, v_{r-2}\}$  is a  $\gamma_t$  set. But  $|E(\langle D_3 \rangle)| = |E(\langle D \rangle)| - 2$  which is a contradiction. If  $r=3k+2$ , then  $D_4 = (D - \{v_1, v_3, v_6, \dots, v_{r-2}\}) \cup \{y, v_2, v_5, \dots, v_{r-3}\}$  is a  $\gamma_t$ -set. But  $|E(\langle D_4 \rangle)| = |E(\langle D \rangle)| - 1$ , which is contradiction. If  $w_2 \neq \langle D \rangle$ , then  $N(w_2) \cap p = \{z\}$  and  $z \in D$ . If  $r=3k+1$ , then  $D_5 = (D - \{v, z, v_3, v_6, \dots, v_{r-1}\}) \cup \{y, w_2, v_2, v_5, \dots, v_{r-2}\}$  is a  $\gamma_t$ -set and  $|E(\langle D_5 \rangle)| = |E(\langle D \rangle)| - 1$  which is contradiction. If  $r=3k+2$ , then  $D_6 = (D - \{v, z, v_3, v_6, \dots, v_{r-2}\}) \cup \{y, w_2, v_2, \dots, v_{r-3}\}$  is a  $\gamma_t$  set and  $|E(\langle D_6 \rangle)| = |E(\langle D \rangle)| - 1$ , which is contradiction. Thus in all cases, we get contradictions. Hence one of the two conditions (i), (ii) holds. Conversely, Assume that  $T$  has two support vertices  $u, v$  satisfying one of the given two conditions. By proposition 2.17 and Theorem 2.18,  $T$  is in  $(\gamma_t \equiv \gamma_{ch}^t)$ -class.

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