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A note on fuzzy majority domination in fuzzy graphs

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***Abstract*---**Let $G = (V, E)$ be a finite graph with n vertices and m edges. A subset $S \subseteq V(G)$ of vertices in a graph G is called a *majority dominating set* if at least half of the vertices of $V(G)$ are either in S or adjacent to vertices of S. That is $|N[S]| \geq \left\lceil \frac{p}{2} \right\rceil$. Let $G = (\sigma, \mu)$ be simple fuzzy graph. We introduce the concept of fuzzy majority dominating set for a fuzzy graph. A subset $S \subseteq V(G)$ of a vertices in *a fuzzy majority dominating set* if $|N[S]| \geq \frac{p}{2}$, where p is a order of G. The minimum cardinality of fuzzy majority dominating set is called a *fuzzy majority domination number* and is denoted by $\gamma_{fm}(G)$. We determine the fuzzy majority domination number $\gamma_{fm}(G)$, for some graph G. Also, we obtained the bounds for the fuzzy majority domination number.

***Keywords*---**fuzzy, majority domination, fuzzy graphs.

Introduction

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1995[9]. In 1995, I Broere and et al introduced the concept of majority domination in graph[3]. In 2001, Tara S Holm found the majority domination number of certain families of graphs[4]. V. Swaminathan and J. Joseline Manora introduced the concept of majority dominating set and majority domination number for standard graphs and bounds for majority domination numbers are found[5]. In order to model of

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this concept in graph theory we introduce the concept of fuzzy majority domination in fuzzy graph.

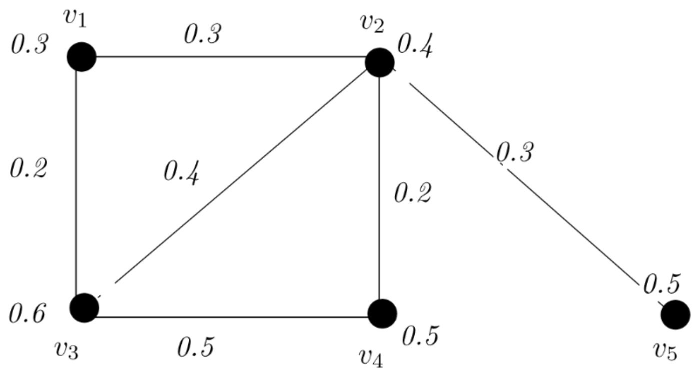
Preliminaries

Let X denote a universal set, Then the membership function μ_A by which a *fuzzy set* A is usually defined as $\mu_A: X \to [0,1]$ denote the interval of real numbers from 0 to 1 [6]. The *fuzzy cardinality* of the fuzzy set S is defined to be $\sum_{v \in V} \sigma(v)$ and it is denoted by $|S|_f$. A *fuzzy graph* $G = (\sigma, \mu)$ is a set with two functions $\sigma: V \to [0,1]$ and $\mu: E \to [0,1]$ such that $\mu(xy) \leq \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$. The *order* p and *size* q of a fuzzy graph $G = (\sigma, \mu)$ are defined to be $p = \sum_{x \in V} \sigma(x)$ and $q = \sum_{xy \in E} \mu(xy)$[8]. Let $G = (\sigma, \mu)$ be a fuzzy graph on V and $V_1 \subseteq V$. Define σ_1 on V_1 by $\sigma_1(x) = x$ for all $x \in V_1$ and μ_1 on the collection E_1 of the two element subsets of V_1 by $\mu_1(xy) = \mu(xy)$ for all $x, y \in V_1$. Then (σ_1, μ_1) is called the *fuzzy subgraph of* G induced by V_1 and is denoted by $\langle V_1 \rangle$. Let $\sigma: V \to [0,1]$ be a fuzzy subset of V. Then *the complete fuzzy graph* on σ is defined to be (σ, μ) where $\mu(xy) = \sigma(x) \wedge \sigma(y)$ for all $xy \in E$ and is denoted by K_σ. A fuzzy graph $G = (\sigma, \mu)$ is said to be bipartite if the vertex set V can be partitioned into two nonempty sets V_1 and V_2 such that $\mu(v_1 v_2) = 0$ if $v_1, v_2 \in V_1$ or $v_1, v_2 \in V_2$ further if $\mu(uv) = \sigma(u) \wedge \sigma(u)$ for all $u \in V_1$ and $v \in V_2$. Then G is called *a complete bipartite fuzzy graph* and is denoted by K_{σ_1, σ_2} where σ_1 and σ_2 are respectively. The restrictions of σ to V_1 and V_2. The *complement of a fuzzy graph* G denoted by $\bar{G}$ is defined to be $\bar{G} = (\sigma, \bar{\mu})$ where $\bar{\mu}(xy) = \sigma(x) \wedge \sigma(y) - \mu(xy)$ [10]. An edge $e = xy$ of a fuzzy graph is called an *effective edge* if $\mu(xy) = \sigma(x) \wedge \sigma(y)$. $N(x) = \{y \in V / \mu(xy) = \sigma(x) \wedge \sigma(y)\}$ is called *neighborhood of* x and $N[x] = N(x) \cup \{x\}$ is the *closed neighborhood of* x. Let $G = (V, E)$ be a finite graph with n vertices and m edges. A subset $S \subseteq V(G)$ of vertices in a graph G is called a *majority dominating set* if at least half of the vertices of $V(G)$ are either in S or adjacent to vertices of S. That is $|N[S]| \geq \left\lceil \frac{p}{2} \right\rceil$. For further details, we refer [2, 1, 7, 8].

Fuzzy majority domination

Definition 3.1 *Let $G = (\sigma, \mu)$ be a fuzzy graph on V. A subset $S \subseteq V(G)$ of a vertices in a fuzzy graph G is called fuzzy majority dominating set if* $|N[S]|_f \geq \frac{p}{2}$. A fuzzy majority dominating set S is minimal if no proper subset of S is a fuzzy majority dominating set. The minimum fuzzy cardinality of a minimal fuzzy majority dominating set is called a fuzzy majority domination number and denoted by $\gamma_{fm}(G)$. The corresponding set is called as γ_{fm} - set G. The maximum fuzzy cardinality of minimal fuzzy dominating set is called upper fuzzy majority domination number and denoted by $\Gamma_{fm}(G)$.

Example 3.2



Let $V(G) = \{v_1, v_2, v_3, v_4, v_5\}$ be a vertex set and the membership value of v_1, v_2, v_3, v_4 and v_5 are 0.3, 0.4, 0.6, 0.5 and 0.5 respectively.
Now $p = \sum_{v\in V} \sigma(v)$= 0.3+0.4+0.6+0.5+0.5=2.3 $\Rightarrow \frac{p}{2} = \frac{2.3}{2} = 1.15$.
Therefore the fuzzy majority dominating sets of G are $\{v_2\}, \{v_3\}, \{v_4, v_5\}, \{v_2, v_5\}$ and $\{v_1, v_5\}$. Hence the fuzzy majority domination number is $\gamma_{fm}(G) = 0.6$ and hence the upper fuzzy majority domination number is $\Gamma_{fm}(G) = 1$.

Theorem 3.3 *Let K_σ be a complete fuzzy graph with n vertices.*
Then $\gamma_{fm}(K_\sigma) = \min_{v\in V(K_\sigma)} \sigma(v)$.

Proof:
Let K_σ be a complete fuzzy graph. then $D = \{v\}$ is a minimal fuzzy majority dominating set of K_σ, for each $v \in V(G)$. Therefore $\gamma_{fm}(K_\sigma) = \min_{v\in V(K_\sigma)} \sigma(v)$.

Theorem 3.4 *Let $\overline{K}_\sigma = (\sigma, \bar{\mu})$be a totally disconnected fuzzy graph.*
Let $(\sigma_1, \sigma_2, \dots, \sigma_n)$ be a non increasing sequence of membership values such that $\sigma(v_i) = \sigma_i$ for all $v_i \in V(\overline{K}_\sigma)$ and $i = 1,2,\dots,n$. $\sigma(S_k) = \sum_{i=1}^{k} \sigma_i$
where $S_k \subseteq V(\overline{K}_\sigma)$ then $\gamma_{fm}(\overline{K}_\sigma) = \min_{\sigma(S_k)\geq\frac{p}{2}} \sigma(S_k)$.

Proof:
Let $\overline{K}_\sigma = (\sigma, \bar{\mu})$ be a totally disconnected fuzzy graph with the vertex set $\{v_1, v_2, \dots, v_n\}$. Let $(\sigma_1, \sigma_2, \dots, \sigma_n)$ be a non increasing sequence of membership values such that $\sigma(v_i) = \sigma_i$ for all $v_i \in V(\overline{K}_\sigma)$. Let $\sigma(S_k) = \sum_{i=1}^{k} \sigma_i$ and $p = \sum_i \sigma_i$ for all $v_i \in V(\overline{K}_\sigma)$ then clearly $(\sigma(S_1), \sigma(S_2), \dots, \sigma(S_n))$ is a non increasing sequence. Hence there exists an integer $1 \leq i \leq k$ such that $\sigma(S_k) \geq \frac{p}{2}$. If $\sigma(S_i) > \frac{p}{2}$ for some i, then $\sigma(S_j) > \frac{p}{2}$ for all $j > i$. Therefore $\gamma_{fm}(\overline{K}_\sigma) = \min_{\sigma(S_k)\geq\frac{p}{2}} \sigma(S_k)$.

Theorem 3.5 *Let $G = (\sigma, \mu)$ be a fuzzy graph with $\mu(xy) < \sigma(x) \wedge \sigma(y)$. Let $(\sigma_1, \sigma_2, \dots, \sigma_n)$ be a non increasing sequence of membership values such that*
$\sigma(v_i) = \sigma_i$ for all $v_i \in V(G)$ and $i = 1,2,\dots,n$, $\sigma(S_k) = \sum_{i=1}^{k} \sigma_i$ where $S_k \subseteq V(G)$ then $\gamma_{fm}(\overline{K}_\sigma) = \min_{\sigma(S_k)\geq\frac{p}{2}} \sigma(S_k)$.

Proof:
Let G be a fuzzy graph with $\mu(xy) < \sigma(x) \wedge \sigma(y)$. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$. Since $\mu(xy) < \sigma(x) \wedge \sigma(y)$, no vertex v in G is dominate any other vertices of G. Let $(\sigma_1, \sigma_2, \ldots, \sigma_n)$ be a non increasing sequence of membership values such that $\sigma(v_i) = \sigma_i$ for all $v_i \in V(G)$. Let $\sigma(S_k) = \sum_{i=1}^{k} \sigma_i$ where $S_k \subseteq V(G)$ and $p = \sum_i \sigma_i$ for all $v_i \in V(G)$. then clearly $(\sigma(S_1), \sigma(S_2), \ldots, \sigma(S_n))$ is a non increasing sequence. Hence there exists an integer $1 \le i \le k$ such that $\sigma(S_k) \ge \frac{p}{2}$. If $\sigma(S_i) > \frac{p}{2}$ for some i, then $\sigma(S_j) > \frac{p}{2}$ for all $j > i$. Therefore $\gamma_{fm}(\overline{K_\sigma}) = \min_{\sigma(S_k) \ge \frac{p}{2}} \sigma(S_k)$.

Theorem 3.6 *Let $G = (\sigma, \mu)$ be a fuzzy graph then $L \le \gamma_{fm}(G) \le U$, where L is the minimum value of $\sigma(v)$ for all $v \in V(G)$ such that $N(v) \ge \frac{p}{2}$ and U is the minimum value of $\sigma(S_k)$ such that $\sigma(S_k) \ge \frac{p}{2}$, for $S_k \subseteq V(G)$. In addition, The lower bound is sharp if $G = K_\sigma$ and the upper bound is sharp if $G = \overline{K}_\sigma$.*

Proof:
Let $G = (\sigma, \mu)$ be a fuzzy graph of order p and size q. Let $(\sigma_1, \sigma_2, \ldots, \sigma_n)$ be a non increasing membership function such that $\sigma_i = \sigma(v_i)$ for $v_i \in V(G)$. Let D be a γ_{fm} -set of G then $D \ne \phi$ contains at most $\frac{n}{2} + 1$ vertices, by the definition. That is $1 \le |D| \le \frac{n}{2} + 1$. Let $S_1 = \left\{V_i : N[v_i] \ge \frac{p}{2}\right\}$. If $|D| = 1$ then $S_1 \ne \phi$. Therefore there exists a vertex v in $V(G)$ such that $N[v] \ge \frac{p}{2}$.

$$|D|_f = \min_{\sigma_i \in S_1} \sigma_i \qquad (1)$$

If $|D| > 1$ then either there is no vertex v in $V(G)$ such that $N(v) \ge \frac{p}{2}$ or there exists a vertex v in S_1 such that $\sigma(v) > \gamma_{fm}(G) = |D|_f$

$$\Rightarrow |D|_f < \min_{\sigma_i \in S} \sigma_i \qquad (2)$$

Let $\mathcal{S} = \left\{S_i : \sigma(S_i) = \sum_{v_i \in V(G)} \sigma_i \ \ and \ \ \sigma(S_i) \ge \frac{p}{2}\right\}$. Clearly, $V(G) \in \mathcal{S} \Rightarrow \mathcal{S} \ne \phi$. Hence

$$|D|_f \le \min_{\sigma(S_i) \ge \frac{p}{2}} \sigma(S_i) \qquad (3)$$

From (1), (2) and (3) we get, $L \le \gamma_{fm}(G) \le U$ where L is the minimum value of $\sigma(v)$ for all $v \in V(G)$ such that $N(v) \ge \frac{p}{2}$ and U is the minimum value of $\sigma(S_k)$ such that $\sigma(S_k) \ge \frac{p}{2}$, for $S_k \subseteq V(G)$. By theorem (3.3) and (3.4), the lower bound is sharp if $G = K_\sigma$ and the upper bound is sharp if $G = \overline{K}_\sigma$.

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