How to Cite:
Mary, K. R., & Abisha, M. J. (2022). Harary index of Lehmer three mean graphs. *International Journal of Health Sciences*, *6*(S2), 5139–5143. https://doi.org/10.53730/ijhs.v6nS2.6271

Harary index of Lehmer three mean graphs

K. Rubin Mary
Assistant professor, Department of Mathematics, St Jude's College, Thoothoor-629176, Tamil Nadu, India
Corresponding author email: rubinjude@yahoo.com

M J Abisha
Research Scholar (Full time), Reg No:19113232092003, Department of Mathematics, St Jude's College, Thoothoor-629176, Tamil Nadu, India. Affliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamil Nadu, India
Email: abisharejeesh@gmail.com

***Abstract*---**The Harary index is defined as the sum of reciprocals of distances between all pairs of vertices of a connected graph $G = (V, E)$. In this paper we investigate Harary Index of Lehmer three Mean graph for some graphs.

***Keywords*---**Harary index, Lehmer three mean graphs, some graphs.

Introduction

The Harary index of a graph G, denoted by H(G), has been introduced independently by Plavsic and by Ivanciuc in 1993. It has been named in honour of Professor Frank Harary on the occasion of his 70th birthday. The Harary index is defined as follows:

$$H(G) = \sum_{x,y\in v(G)} \frac{1}{d_G(x,y)}$$

where the summation goes over all unordered pairs of vertices of G and $d_G(x,y)$denotes the distance of the two vertices x and y in the graph G. That is the number of edges in a shortest path connecting x and y. On the basis of this work we introduce a new concept namely Harary Index of Lehmer three Mean graphs. In this paper we investigate Harary index of some graphs which admit Lehmer three Mean graphs. We will provide a brief summary of definitions and other information which are necessary for our present investigation.

Definition

Let G be a (r,s) graph.A function h is called Lehmer three mean labeling of graph G, if it is possible to label the vertices $v \in V$ with distinct labels $h(x)$ from $1,2,3,\dots\dots\dots,s+1$ in such a way that when each edge $e=xy$ is labeled with $h(e)=\left\lceil\frac{h(x)^3+h(y)^3}{h(x)^2+h(y)^2}\right\rceil$ (or) $\left\lfloor\frac{h(x)^3+h(y)^3}{h(x)^2+h(y)^2}\right\rfloor$ then the edge labels are distinct.In this case " h " is called Lehmer-3 mean labeling of G.

Definitio

Let G be a Lehmer three mean graph with vertex set $V(G)$ and edge set $E(G)$.The distance $d_G(x,y)$ between two vertices $x,y \in V(G)$ is the shortest path in G between x and y.

Main Results

Theorem

Friendship graph F_m be a Lehmer three mean graph,then harary index of F_m is $H(G)=\frac{1}{3m\times 1+\ 2m(m-1)\times 2}$

Proof

Let F_m be a Lehmer three mean graph.

$$H(G)=\sum_{x,y\in v(G)}\frac{1}{d_G(x,y)}$$

$$=\frac{1}{\begin{array}{c}d(v_1,v_2)+d(v_1,v_3)+\cdots+d(v_1,v_{2m+1})+d(v_2,v_3)+d(v_2,v_4)+\cdots+d(v_2,v_{2m+1})+\cdots\\+d(v_{2m-1},v_{2m})+d(v_{2m},v_{2m+1})\end{array}}$$

$$=\frac{1}{3m\times 1+\ 2m(m-1)\times 2}$$

$$H(G)=\frac{1}{3m\times 1+\ 2m(m-1)\times 2}$$

Example

Harary index of Friendship graph F_4 is given below

$$H(G)=\sum_{x,y\in v(G)}\frac{1}{d_G(x,y)}$$

$$=\frac{1}{\left\{\begin{array}{c}(1+1+1+1+1+1+1+1)+(1+2+2+2+2+2+2)+\\(2+2+2+2+2+2)+(1+2+2+2+2)+(2+2+2+2)+(1+2+2)+(2+2)+1\end{array}\right\}}$$

$$=\frac{1}{12\times 1+\ 24\times 2}$$

$$H(G)=\frac{1}{3m\times 1+\ 2m(m-1)\times 2}$$

Theorem

Let F-tree F(P_m) be a Lehmer three mean graph ,then harary index of F(P_m) is $H(G) = \frac{1}{\sum_{j=1}^{2}(m+1)j+m(3)+\sum_{k=2}^{m-1}\sum_{j=4}^{m}(m-k)j}$

Proof

Let $F(P_m)$ be a Lehmer three mean graph.

$$H(G) = \sum_{x,y\in v(G)} \frac{1}{d_G(x,y)}$$

$$= \frac{1}{d(v_1,v_2)+d(v_1,v_3)+\cdots+d(v_1,v_{m+2})+d(v_2,v_3)+d(v_2,v_4)+\cdots+d(v_2,v_{2m+1})+\cdots+d(v_m,v_{m+1})+d(v_{m+1},v_{m+2})+d(v_m,v_{m+2})}$$

$$= \frac{1}{\sum_{j=1}^{2}(m+1)j+m(3)+\sum_{k=2}^{m-1}\sum_{j=4}^{m}(m-k)j}$$

$$H(G) = \frac{1}{\sum_{j=1}^{2}(m+1)j+m(3)+\sum_{k=2}^{m-1}\sum_{j=4}^{m}(m-k)j}$$

Example

Harary index of Friendship graph F_7 is given below

$$H(G) = \sum_{x,y\in v(G)} \frac{1}{d_G(x,y)}$$

$$= \frac{1}{\left\{\begin{matrix}(1+2+3+4+5+6+6+7)+(1+2+3+4+5+5+6)+\\(1+2+3+4+4+5)+(1+2+3+3+4)+(1+2+2+3)+(1+1+2)+(2+1)+3\end{matrix}\right\}}$$

$$= \frac{1}{8\times 1+\ 8\times 2+7\times 3+5\times 4+4\times 5+3\times 6+1\times 7}$$

$$H(G) = \frac{1}{\sum_{j=1}^{2}(m+1)j+m(3)+\sum_{k=2}^{m-1}\sum_{j=4}^{m}(m-k)j}$$

Theorem

Let Bistar $B_{m,m}$ be a Lehmer three mean graph,then harary index of $B_{m,m}$ is $H(G) = \frac{1}{(2m+1)\times 1+\ m(m+1)\times 2+m^2\times 3}$

Proof

Let $B_{m,m}$be a Lehmer three mean graph.

$$H(G) = \sum_{x,y\in v(G)} \frac{1}{d_G(x,y)}$$

$$= \frac{1}{(d(v,u)+d(v,v_1)+d(v,u_2)\ldots+d(v,v_m)+d(u,v_m))+(d(u,u_1)+d(u,v_2)+\cdots+d(u,v_m))+\cdots+d(u_{m-1},v_m)+d(v_m,u_m)}$$

$= \frac{1}{(2m+1)\times 1+ m(m+1)\times 2+m^2\times 3}$

$$H(G) = \frac{1}{(2m+1)\times 1+ m(m+1)\times 2+m^2\times 3}$$

Example

Harary index of Friendship graph $B_{3,3}$ is given below

$$H(G) = \sum_{x,y\in v(G)} \frac{1}{d_G(x,y)}$$

$$= \frac{1}{\begin{Bmatrix} (1+1+2+1+2+1+2)+(2+1+2+1+2+1)+ \\ (3+2+3+2+3)+(3+2+3+2)+(3+2+3)+(3+2)+3 \end{Bmatrix}}$$

$$= \frac{1}{7\times 1+ 12\times 2+9\times 3}$$

$H(G) = \frac{1}{(2m+1)\times 1+ m(m+1)\times 2+m^2\times 3}$

Theorem

Let Y-tree Y_{m+1} be a Lehmer three mean graph ,then harary index of Y_{m+1} is

$H(G) = \frac{1}{\sum_{j=1}^{2}(m+1)j+\sum_{k=1}^{m-1}\sum_{j=3}^{m}(m-k)j}$

Proof:

Let Y_{m+1} be a Lehmer three mean graph.

$$H(G) = \sum_{x,y\in v(G)} \frac{1}{d_G(x,y)}$$

$$= \frac{1}{\begin{Bmatrix} (d(v_1,v_2)+d(v_1,v_3)+\cdots+d(v_1,v_m)+d(v_1,u))+d(v_2,v_3)+d(v_2,v_4)+\cdots+d(v_2,v_{2m+1})+\cdots \\ +d(v_m,v_{m+1})+d(v_{m+1},v_{m+2})+d(v_m,v_{m+2}) \end{Bmatrix}}$$

$$= \frac{1}{\sum_{j=1}^{2}(m+1)j+\sum_{k=1}^{m-1}\sum_{j=3}^{m}(m-k)j}$$

Example

Harary index of Friendship graph Y_{9+1} is given below

$$H(G) = \sum_{x,y\in v(G)} \frac{1}{d_G(x,y)}$$

$$= \frac{1}{\begin{Bmatrix} (1+2+3+4+5+6+6+7+8+9+9)+ \\ (1+2+3+4+5+5+6+7+8+8)+ \\ (1+2+3+4+4+5+6+7+7)+ \\ (1+2+3+3+4+5+6+6)+(1+2+3+4+5+5) \\ +(1+2+3+4+4)+(1+2+3+3)+(1+2+2)+(1+1)+2 \end{Bmatrix}}$$

$$= \frac{1}{10\times 1+ 10\times 2+8\times 3+7\times 4+6\times 5+5\times 6+4\times 7+3\times 8+2\times 9}$$

$$H(G) = \frac{1}{\sum_{j=1}^{2}(m+1)j+\sum_{k=1}^{m-1}\sum_{j=3}^{m}(m-k)j}$$

References

1. J.A. Gallian, “A Dynamic Survey of Graph Labeling”, The Electronic Journal of Combinatories,(2019),DS6.
2. F.Harary , “Graph theory” , Narosa publication House reading ,New Delhi (1988).
3. K. Xu, K.Ch. Das, On Harary index of graphs, Discrete Appl. Math. 159 (2011) 1631–1640.
4. G. Yu, L. Feng, On the maximal Harary index of a class of bicyclic graphs, Util. Math. 82 (2010) 285–292.
5. M.J.Abisha and K.Rubin Mary, “K-Lehmer three mean labeling of some graphs”, Malaya Journal of Mathematik, Vol 8,No.3,1219-1221,2020.