**How to Cite:**
Jinisha, K. K. S., & Gipson, K. L. (2022). Eccentric domination path decomposition polynomial of path and cycle. *International Journal of Health Sciences*, *6*(S1), 6166–6170.
https://doi.org/10.53730/ijhs.v6nS1.6272

# Eccentric domination path decomposition polynomial of path and cycle

**Jinisha Kalaiarasan K S**
Research Scholar(Fulltime), Reg No:18213112092026, PG and Research Department of Mathematics Scott Christian College (Autonomous), Nagercoil-629003, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012, Tamilnadu, India.
Email: jinishakalaiarasan@gmail.com

**DR. K. Lal Gipson**
Assistant Professor, PG and Research Department of Mathematics, Scott Christian college(Autonomous), Nagercoil-629003, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627012 Tamilnadu, India.
Email: lalgipson@yahoo.com

***Abstract*---**A Decomposition$(G_1, G_2 \dots G_n)$ of $G$ is said to be Eccentric Domination Path Decomposition (EDPD) if i) $G$ admits EDD ii) Each $G_i$ is a path$(1 \leq i \leq n)$ iii) $q(G_1) = 1$ and $q(G_2) = 2 \; or \; 3$ iv) $q(G_i) = 3i - 5 \; or \; 3i - 4 \; or \; 3i - 3, \; i = 3,4 \dots n$. In this paper we establish Eccentric Domination Path Decomposition polynomial of a $G$. In a particular, we investigate Eccentric domination Path Decomposition of Path and Cycle.

***Keywords*---**Domination, Decomposition, Eccentric Domination Decomposition, Eccentric Domination Path Decomposition.

## Introduction

In the article, all the terminologies from the graph theory are used in the case of Frank Haray. A simple Undirected graph without loops or Multiple edges are Considered here. As usual $p, q$ denote the number of vertices and edges of a graph $G$ respectively. A path on $p$ vertices is denoted by $P_p$.

### Definition 1.1

A closed walk in which no vertices, except the end vertices, are repeated is called the cycle and the number of edges in a cycle is called its length.

**Definition 1.2**
A set $D \subseteq V(G)$ of vertices in a graph $G$ is a dominating set if every vertex $v$ in $V - D$ is adjacent to a vertex in $D$. The Minimium Cardinality of a dominating set of $G$ is called the domination number of $G$ and is denoted by $\gamma(G)$.

**Definition 1.3**
A set $D \subseteq V(G)$ is an eccentric dominating set if $D$ is a dominating set of $G$ and for every $v \in V - D$, there exists atleast one eccentric point of $v$ in $D$.
If $D$ is an eccentric dominating set, then every superset $D' \supseteq D$ is also an eccentric dominating set. But $D'' \subseteq D$ is not necessarily an eccentric dominating set.
A eccentric dominating set $D$ is a minimal eccentric dominating set if no proper subset $D'' \subseteq D$ is an eccentric dominating set.

**Definition 1.4**
The eccentric domination number $\gamma_{ed}(G)$ of a graph $G$ equals the minimum Cardinality of an eccentric dominating set, That is, $\gamma_{ed}(G) = \min|D|$ where, the minimum is taken over $D$ in $D$, Where $D$ is the set of all Minimal eccentric dominating sets of $G$.

**Definition 1.5**
Let $G = (V, E)$ be a simple connected graph with $P$ vertices and $q$ edges. If $G_1, G_2 \ldots . G_n$ are connected edge disjoint subgraphs of $G$ with $E(G_1) \cup E(G_2) \ldots \cup E(G_n)$ then $(G_1, G_2 \ldots . G_n)$ is said to be a Decomposition of $G$.

**Definition 1.6**
A decomposition $(G_1, G_2 \ldots . G_n)$ of $G$ is said to be Eccentric Domination Decomposition if
i) $E(G) = E(G_1) \cup E(G_2) \ldots \cup E(G_n)$
ii) Each $G_i$ is connected
iii) $\gamma_{ed}(G_i) = i, \ i = 1,2 \ldots n.$

**Definition 1.7**
A Decomposition$(G_1, G_2 \ldots G_n)$ of $G$ is said to be Eccentric Domination Path Decomposition (EDPD) if
i) $G$ admits EDD.
ii) Each $G_i$ is a path$(1 \leq i \leq n)$
iii) $q(G_1) = 1$ and $q(G_2) = 2 \ or \ 3$
iv) $q(G_i) = 3i - 5 \ or \ 3i - 4 \ or \ 3i - 3, \ i = 3,4 \ldots n.$

**Eccentric Domination Path Decomposition Polynomial**

**Definition 2.1**
Let $G$ be a graph which admits, EDPD$(G_1, G_2, \ldots . G_n)$ and let $M(G, q(G_k))$ be a family of subgraphs,(That is, Path) with size $q(G_k)$ and $m(G, q(G_k)) = |M(G, q(G_k)|$. Then the Eccentric Domination path Decomposition (EDPD) polynomial of a graph $G$ is defined as $M(G, x) = \sum_{k=1}^{n} m(G, q(G_K)) x^{q(G_k)}$.

**Eccentric Domination Path Decomposition Polynomial Of Path**

**Theorem 3.1**

If $P_p$, $p = \frac{3j^2-j+6}{2}, j \in N$ admits EDPD $(P_2, P_3, P_5, \ldots, P_{3m+2})$, then $M(P_p, x) = (p-1)x + (p-1)x^2 + \sum_{k=1}^{t}\{p-(3k+1)\}x^{3k+1}$.

**Proof:**

Let $\{u_1, u_2, \ldots. u_p\}$ be the vertices of $P_p$. Assume that $G_1 = P_2$, $G_2 = P_3$, $G_3 = P_5 \ldots. G_{m+2} = P_{3m+2}$.

Then $M(P_p, x) = \sum_{k=1}^{m+2} m(P_p, q(G_k))x^{q(G_k)}$.

$$= m\left(P_p, q(G_1)\right)x^{q(G_1)} + m\left(P_p, q(G_2)\right)x^{q(G_2)} + \cdots m\left(P_p, q(G_{m+2})\right)x^{q(G_{m+2})}$$

Let $F_1, F_2, \ldots. F_p$ be the subgraphs of $G$ and each $F_s$ starts with $v_s$, where $s = 1,2 \ldots p$.

If we consider the size of each $F_s$ as 1, then $E(F_1) = \{v_1v_2\}$, $E(F_2) = \{v_2v_3\}, \ldots E(F_p) = \{v_{p-1}v_p\}$. Therefore, $\left|M\left(P_p, q(G_1)\right)\right| = p-1$. That is, $m\left(P_p, q(G_1)\right) = p-1$.

If we consider the size of each $F_s$ as 2, then $E(F_1) = \{v_1v_2, v_2v_3\}$, $E(F_2) = \{v_2v_3, v_3v_4\}, \ldots E(F_p) = \{v_{p-2}v_{p-1}, v_{p-1}v_p,\}$. Therefore, $\left|M\left(P_p, q(G_2)\right)\right| = p-2$. That is, $m\left(P_p, q(G_2)\right) = p-2$.

If we consider the size of each $F_s$ as 4, then $E(F_1) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5\}$, $E(F_2) = \{v_2v_3, v_3v_4, v_4v_5, v_5v_6\}, \ldots E(F_p) = \{v_{p-4}v_{p-3}, v_{p-3}v_{p-2}, v_{p-2}v_{p-1}, v_{p-1}v_p\}$. Therefore, $\left|M\left(P_p, q(G_3)\right)\right| = p-4$. That is, $m\left(P_p, q(G_3)\right) = p-4$.

Continuing in this way,If we consider the size of each $F_s$ as $3n+1$, then $E(F_1) = \{v_1v_2, v_2v_3, v_3v_4, \ldots\ldots v_{3t+1}v_{3t+2}\}$, $E(F_2) = \{v_2v_3, v_3v_4, v_4v_5, \ldots. v_{3t+2}v_{3t+3}\}, \ldots E(F_p) = \{v_{p-(3t+1)}v_{p-3t}, \ldots\ldots, v_{p-1}v_p\}$.

Therefore, $\left|M\left(P_p, q(G_{m+2})\right)\right| = p-(3t+1)$. That is, $m\left(P_p, q(G_3)\right) = p-(3t+1)$.

Thus $M(P_p, x) = (p-1)x + (p-2)x^2 + (p-4)x^4 + \cdots + (p-\{3t+1\})x^{3t+1}$.

$M(P_p, x) = (p-1)x + (p-2)x^2 + \sum_{k=1}^{t}\{p-(3k+1)\}x^{3k+1}$.

Example:

The path $P_8$ and its EDPD polynomial $M(P_8, x) = 7x + 6x^2 + 4x^4$.

$= (p-1)x + (p-2)x^2 + (p-4)x^4$, with $m(P_8, q(G_1)) = 7$, $m(P_8, q(G_2)) = 6$, $m(P_8, q(G_2)) = 4$.

Table:

The number of subgraphs in $P_p$, $p = \frac{3j^2-j+6}{2}, n \in N$ with size $q(G_t)$, $t = 1,2 \ldots n$ and $n = 1,2 \ldots 8$ whenever $P_p$ admits EDPD$(P_2, P_3, P_5, \ldots, P_{3m+2})$ is described in the following table.

| $q(G_n)$ / n | $q(G_1)$ | $q(G_2)$ | $q(G_3)$ | $q(G_4)$ | $q(G_5)$ | $q(G_6)$ | $q(G_7)$ | $q(G_8)$ | $q(G_9)$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 2 | | | | | | | |
| 2 | 7 | 6 | 4 | | | | | | |
| 3 | 14 | 13 | 11 | 8 | | | | | |
| 4 | 24 | 23 | 21 | 18 | 15 | | | | |
| 5 | 37 | 36 | 34 | 31 | 28 | 25 | | | |
| 6 | 53 | 52 | 50 | 47 | 44 | 41 | 38 | | |
| 7 | 71 | 70 | 68 | 65 | 63 | 60 | 57 | 54 | |
| 8 | 93 | 92 | 90 | 87 | 84 | 81 | 78 | 75 | 73 |

**Eccentric Domination Path Decomposition Polynomial Of Cycle**

**Theorem 4.1**

If $C_p$, $p = \frac{3j^2-j+4}{2}$ , $j \in N$ admits EDPD $(P_2, P_3, P_5, \dots, P_{3m+2})$, then $M(C_p, x) = px + px^2 + \sum_{k=1}^{t} px^{3k+1}$.

**Proof:**

Let $\{v_1, v_2, \dots . v_p\}$ be the vertices of $C_p$. Assume that $G_1 = P_2$, $G_2 = P_3$, $G_3 = P_5 \dots . G_{m+2} = P_{3m+2}$.

Then $M(C_p, x) = \sum_{k=1}^{m+2} m(C_p, q(G_k)) x^{q(G_k)}$.

$$= m\left(C_p, q(G_1)\right) x^{q(G_1)} + m\left(C_p, q(G_2)\right) x^{q(G_2)} \dots m\left(c_p, q(G_{m+2})\right) x^{q(G_{m+2})}$$

Let $F_1, F_2, \dots . F_p$ be the subgraphs of $G$ and each $F_s$ starts with $v_s$, where $s = 1,2 \dots p$.

If we consider the size of each $F_s$ as 1, then $E(F_1) = \{v_1 v_2\}$, $E(F_2) = \{v_2 v_3\}, \dots E(F_p) = \{v_p v_1\}$. Therefore, $\left|M\left(C_p, q(G_1)\right)\right| = p$. That is, $m\left(C_p, q(G_1)\right) = p$.

If we consider the size of each $F_s$ as 2, then $E(F_1) = \{v_1 v_2, v_2 v_3\}$, $E(F_2) = \{v_2 v_3, v_3 v_4\}, \dots E(F_p) = \{v_p v_1, v_1 v_2\}$. Therefore, $\left|M\left(C_p, q(G_2)\right)\right| = p$. That is, $m\left(C_p, q(G_2)\right) = p$.

If we consider the size of each $F_s$ as 4, then $E(F_1) = \{v_1 v_2, v_2 v_3, v_3 v_4, v_4 v_5\}, E(F_2) = \{v_2 v_3, v_3 v_4, v_4 v_5, v_5 v_6\}, \dots E(F_p) = \{v_p v_1, v_1 v_2, v_2 v_3, v_3 v_4\}$.

Therefore, $\left|M\left(C_p, q(G_3)\right)\right| = p$. That is, $m\left(C_p, q(G_3)\right) = p$.

Continuing in this way,If we consider the size of each $F_s$ as $3n + 1$, then $E(F_1) = \{v_1 v_2, v_2 v_3, v_3 v_4, \dots \dots v_{3t+1} v_{3t+2}\}$, $E(F_2) = \{v_2 v_3, v_3 v_4, v_4 v_5, \dots . v_{3t+2} v_{3t+3}\}, \dots E(F_p) = \{v_p v_1, v_1 v_2, \dots \dots, v_{3t} v_{3t+1}\}$. Therefore, $\left|M\left(C_p, q(G_{m+2})\right)\right| = p$. That is, $m\left(C_p, q(G_3)\right) = p$.

Thus $M(C_p, x) = px + px^2 + px^4 + \dots + px^{3t+1}$.

$$M(C_p, x) = px + px^2 + \sum_{k=1}^{t} px^{3k+1}.$$

Example:

The Cycle $C_{14}$ and its EDPD polynomial $M(C_{14}, x) = 14x + 14x^2 + 14x^4 + 14x^7$, with $m(C_{14}, q(G_1)) = 14$, $m(C_{14}, q(G_2)) = 14$, $m(C_{14}, q(G_2)) = 14$, $m(C_{14}, q(G_2)) = 14$.

Table:

The number of subgraphs in $C_p, p = \frac{3j^2-j+4}{2}$ , $j \in N$ with size $q(G_t)$, $t = 1,2 \dots n$ and $n = 1,2 \dots 8$ whenever $C_p$ admits EDPD$(P_2, P_3, P_5, \dots, P_{3m+2})$ is described in the following table.

| $q(G_n)$ / n | $q(G_1)$ | $q(G_2)$ | $q(G_3)$ | $q(G_4)$ | $q(G_5)$ | $q(G_6)$ | $q(G_7)$ | $q(G_8)$ | $q(G_9)$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 3 | | | | | | | |
| 2 | 7 | 7 | 7 | | | | | | |
| 3 | 14 | 14 | 14 | 14 | | | | | |
| 4 | 24 | 24 | 24 | 24 | 24 | | | | |
| 5 | 37 | 37 | 37 | 37 | 37 | 37 | | | |
| 6 | 53 | 53 | 53 | 53 | 53 | 53 | 53 | | |
| 7 | 72 | 72 | 72 | 72 | 72 | 72 | 72 | 72 | |
| 8 | 94 | 94 | 94 | 94 | 94 | 94 | 94 | 94 | 94 |

**References**


[1] E.Ebin Raja Merly and D. Jaya Jothi "Connected Domination Path Decomposition of Triangular Snake Graph", International Journal Of Pure and Applied Mathematics, 116(1)(2017), 105-113.

[2] F.Harary, "Graph Theory", Narosa Publishing House, New Delhi,1998.

[3] T.N.Janakiraman, M.Bhanumathi and S.Muthammai, "Eccentric Domination in Graphs", International Journal of Engineering Science, Advanced Computing and BioTechnology, 2(2000), 55-70.

[4] K.S.Jinisha kalaiarasan and K. Lal Gipson," Eccentric domination decomposition of graphs", Malaya Journal of Mathematik, 8(3)(2020), 1186-1188.

[5] Juraj Bosak, "Decomposition of Graphs", Kluwer Academic Publishers, 1990.

[6] K.Lakshmi Praha, K.Nagarajan, "Ascending Domination Decomposition of graphs", International Journal of Mathematics and Soft Computing, 4(1)(2014), 119-128.

[7] K.Lakshmi Praha, K.Nagarajan, "Ascending Domination Decomposition of subdivision of graphs", International Journal of Mathematics and Soft Computing, 4(1)(2015), 105-114.

[8] D.Jeya Jothi and E.Ebin Raja Merly, "Connected Domination Path Decomposition Polynomial of Path and Cycle", Journal of Computer and Mathematical Sciences,9(6)(2018), 619-624.