Eccentric domination path decomposition polynomial of path and cycle

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Abstract—A Decomposition($G_1,G_2, ..., G_n$) of $G$ is said to be Eccentric Domination Path Decomposition (EDPD) if i) $G$ admits EDD ii) Each $G_i$ is a path$(1 \leq i \leq n)$ iii) $q(G_1) = 1$ and $q(G_2) = 2$ or 3 iv) $q(G_i) = 3i - 5$ or $3i - 4$ or $3i - 3$, $i = 3, 4, ..., n$. In this paper we establish Eccentric Domination Path Decomposition polynomial of a $G$. In a particular, we investigate Eccentric domination Path Decomposition of Path and Cycle.

Keywords---Domination, Decomposition, Eccentric Domination Decomposition, Eccentric Domination Path Decomposition.

Introduction

In the article, all the terminologies from the graph theory are used in the case of Frank Haray. A simple Undirected graph without loops or Multiple edges are Considered here. As usual $p, q$ denote the number of vertices and edges of a graph $G$ respectively. A path on $p$ vertices is denoted by $P_p$.

Definition 1.1
A closed walk in which no vertices, except the end vertices, are repeated is called the cycle and the number of edges in a cycle is called its length.
**Definition 1.2**
A set $D \subseteq V(G)$ of vertices in a graph $G$ is a dominating set if every vertex $v$ in $V - D$ is adjacent to a vertex in $D$. The Minimum Cardinality of a dominating set of $G$ is called the domination number of $G$ and is denoted by $\gamma(G)$.

**Definition 1.3**
A set $D \subseteq V(G)$ is an eccentric dominating set if $D$ is a dominating set of $G$ and for every $v \in V - D$, there exists at least one eccentric point of $v$ in $D$.
If $D$ is an eccentric dominating set, then every superset $D' \supseteq D$ is also an eccentric dominating set. A subset $D'' \subseteq D$ is not necessarily an eccentric dominating set.

**Definition 1.4**
The eccentric domination number $\gamma_{ed}(G)$ of a graph $G$ equals the minimum Cardinality of an eccentric dominating set, That is, $\gamma_{ed}(G) = \min |D|$ where, the minimum is taken over $D$ in $D$, Where $D$ is the set of all Minimal eccentric dominating sets of $G$.

**Definition 1.5**
Let $G = (V,E)$ be a simple connected graph with $P$ vertices and $q$ edges. If $G_1, G_2, ..., G_n$ are connected edge disjoint subgraphs of $G$ with $E(G_1) \cup E(G_2) ... \cup E(G_n)$ then $(G_1, G_2, ..., G_n)$ is said to be a Decomposition of $G$.

**Definition 1.6**
A decomposition $(G_1, G_2, ..., G_n)$ of $G$ is said to be Eccentric Domination Decomposition if

1. $E(G) = E(G_1) \cup E(G_2) ... \cup E(G_n)$
2. Each $G_i$ is connected
3. $\gamma_{ed}(G_i) = i$, $i = 1, 2, ..., n$.

**Definition 1.7**
A Decomposition$(G_1, G_2, ..., G_n)$ of $G$ is said to be Eccentric Domination Path Decomposition (EDPD) if

1. $G$ admits EDD.
2. Each $G_i$ is a path($1 \leq i \leq n$)
3. $q(G_1) = 1$ and $q(G_2) = 2$ or 3
4. $q(G_i) = 3i - 5$ or $3i - 4$ or $3i - 3$, $i = 3, 4, ..., n$.

**Eccentric Domination Path Decomposition Polynomial**

**Definition 2.1**
Let $G$ be a graph which admits, EDPD$(G_1, G_2, ..., G_n)$ and let $M(G, q(G_k))$ be a family of subgraphs,(That is, Path) with size $q(G_k)$ and $m(G, q(G_k)) = |M(G, q(G_k))|$. Then the Eccentric Domination path Decomposition (EDPD) polynomial of a graph $G$ is defined as
$$M(G, x) = \sum_{k=1}^{n} m(G, q(G_k)) x^{q(G_k)}.$$
Eccentric Domination Path Decomposition Polynomial Of Path

**Theorem 3.1**

If $p, p = \frac{3j^2-j+6}{2}, j \in N$ admits EDPD $(P_2, P_3, P_5, ..., P_{3m+2})$, then $M(P_p, x) = (p-1)x + (p-1)x^2 + \sum_{k=1}^{c} (p - (3k + 1))x^{3k+1}$.

**Proof:**

Let $\{u_1, u_2, ..., u_p\}$ be the vertices of $P_p$. Assume that $G_1 = P_2, G_2 = P_3, G_3 = P_5, ..., G_{m+2} = P_{3m+2}$. Then $M(P_p, x) = \sum_{k=1}^{m+2} m(P_p, q(G_k))x^{q(G_k)}$.

Let $F_1, F_2, ..., F_p$ be the subgraphs of $G$ and each $F_s$ starts with $v_s$, where $s = 1, 2, ..., p$.

If we consider the size of each $F_s$ as 1, then $E(F_1) = \{v_1v_2\}, E(F_2) = \{v_2v_3\} ... E(F_p) = \{v_{p-1}v_p\}$. Therefore, $|M(P_p, q(G_1))| = p - 1$. That is, $m(P_p, q(G_1)) = p - 1$.

If we consider the size of each $F_s$ as 2, then $E(F_1) = \{v_1v_2, v_2v_3\}, E(F_2) = \{v_2v_3, v_3v_4\} ... E(F_p) = \{v_{p-2}v_{p-1}, v_{p-1}v_p\}$. Therefore, $|M(P_p, q(G_2))| = p - 2$. That is, $m(P_p, q(G_2)) = p - 2$.

If we consider the size of each $F_s$ as 4, then $E(F_1) = \{v_1v_2, v_2v_3, v_3v_4, v_4v_5\}, E(F_2) = \{v_2v_3, v_3v_4, v_4v_5, v_5v_6\} ... E(F_p) = \{v_{p-4}v_{p-3}, v_{p-3}v_{p-2}, v_{p-2}v_{p-1}, v_{p-1}v_p\}$. Therefore, $|M(P_p, q(G_3))| = p - 4$. That is, $m(P_p, q(G_3)) = p - 4$.

Continuing in this way, if we consider the size of each $F_s$ as $3n + 1$, then $E(F_1) = \{v_1v_2, v_2v_3, v_3v_4, ..., v_{3n+1}v_{3n+2}\}, E(F_2) = \{v_2v_3, v_3v_4, ..., v_{3n+2}v_{3n+3}\} ... E(F_p) = \{v_{p-1}v_p\}$. Therefore, $|M(P_p, q(G_{m+2}))| = p - (3t + 1)$. That is, $m(P_p, q(G_{3})) = p - (3t + 1)$.

Thus $M(P_p, x) = (p - 1)x + (p - 2)x^2 + (p - 4)x^4 + ... + (p - 3t + 1)x^{3t+1}$. $M(P_p, x) = (p - 1)x + (p - 2)x^2 + \sum_{k=1}^{c} (p - (3k + 1))x^{3k+1}$.

**Example:**

The path $P_8$ and its EDPD polynomial $M(P_8, x) = 7x + 6x^2 + 4x^4$.

$(p - 4)x^4$, with $m(P_8, q(G_1)) = 7, m(P_8, q(G_2)) = 6, m(P_8, q(G_3)) = 4$.

**Table:**

The number of subgraphs in $P_p, p = \frac{3j^2-j+6}{2}, j \in N$ with size $q(G_t), t = 1, 2, ..., n$ and $n = 1, 2, ..., 8$ whenever $P_p$ admits EDPD $(P_2, P_3, P_5, ..., P_{3m+2})$ is described in the following table.
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**Eccentric Domination Path Decomposition Polynomial Of Cycle**

**Theorem 4.1**

If \( C_p, p = \frac{3j^2-j+4}{2}, j \in N \) admits EDPD \((P_2, P_3, P_5, \ldots, P_{3m+2})\), then \( M(C_p, x) = px + px^2 + \sum_{k=1}^{m} px^{3k+1} \).

**Proof:**

Let \( \{v_1, v_2, \ldots, v_p\} \) be the vertices of \( C_p \). Assume that \( G_1 = P_2, G_2 = P_3, G_3 = P_5, \ldots, G_{m+2} = P_{3m+2} \).

Then \( M(C_p, x) = \sum_{k=1}^{m+2} m(C_p, q(G_k)) x^{q(G_k)} \).

Let \( F_1, F_2, \ldots, F_p \) be the subgraphs of \( G \) and each \( F_s \) starts with \( v_s \), where \( s = 1, 2, \ldots, p \).

If we consider the size of each \( F_s \) as 1, then \( E(F_1) = \{v_1v_2\}, E(F_2) = \{v_2v_3\}, \ldots, E(F_p) = \{v_pv_1\} \). Therefore, \( |M(C_p, q(G_1))| = p \). That is, \( m(C_p, q(G_1)) = p \).

If we consider the size of each \( F_s \) as 2, then \( E(F_1) = \{v_1v_2, v_2v_3\}, \ldots, E(F_p) = \{v_pv_1, v_1v_2\} \). Therefore, \( |M(C_p, q(G_2))| = p \). That is, \( m(C_p, q(G_2)) = p \).

If we consider the size of each \( F_s \) as 3, then \( E(F_1) = \{v_1v_2v_3, v_2v_3v_4, \ldots, v_{3t+1}v_{3t+2}\} \), \( E(F_2) = \{v_2v_3v_4, v_2v_4v_5, \ldots, v_{3t+2}v_{3t+3}\} \).

Continuing in this way, if we consider the size of each \( F_s \) as \( 3n+1 \), then \( E(F_1) = \{v_1v_2v_3, v_2v_3v_4, \ldots, v_{3t+1}v_{3t+2}\} \), \( E(F_2) = \{v_2v_3v_4, v_2v_4v_5, \ldots, v_{3t+2}v_{3t+3}\} \).

Thus \( M(C_p, x) = px + px^2 + px^4 + \ldots + px^{3t+1} \).

Example:

The Cycle \( C_{14} \) and its EDPD polynomial \( M(C_{14}, x) = 14x + 14x^2 + 14x^4 + 14x^7 \), with \( m(C_{14}, q(G_1)) = 14 \), \( m(C_{14}, q(G_2)) = 14 \), \( m(C_{14}, q(G_2)) = 14 \), \( m(C_{14}, q(G_3)) = 14 \).
The number of subgraphs in $C_p, p = \frac{3j^2-j+4}{2}, j \in N$ with size $q(G_t)$, $t = 1, 2 \ldots n$ and $n = 1, 2 \ldots 8$ whenever $C_p$ admits EDPD($P_2, P_3, P_5, \ldots, P_{3m+2}$) is described in the following table.

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