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An economic ordering policy to control deteriorating medicinal products of uncertain demand with trade credit for healthcare industries

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***Abstract*---**In Healthcare industries, the most challenging problem for a decision maker is to develop an optimal inventory policy to estimate medicinal products with an uncertain scenario. The proposed model has been developed in such a way that it could provide an optimal policy for all relevant factors in the healthcare industry, such as uncertainty in demand, deterioration of products, preservation technology costs, shortages of products, inflation, and trade credit. The price and supply levels of some medicinal products have a substantial impact on the rate of consumption. Therefore, in this study, the demand function is dependent on price as well as stock,

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and the retailer invests in preservation technologies to decrease the deterioration of products. This leads to increased product demand, which benefits both the retailer and the supplier. In an attempt to bring the model closer to a real world problem, the variation in cost parameters over time is included in the fuzzy. The cost minimization technique is defuzzified using the signed distance method. Furthermore, the model has been demonstrated and validated with suitable numerical examples, and a sensitivity analysis has been provided.

***Keywords*---**inventory control, fuzzy economic order quantity, deterioration, inflation, signed distance method, cost optimization.

Introduction

The highly difficult task for a decision maker is to develop inventory models for the purpose of forecasting demand for healthcare products like medicines, probiotics, vitamins, blood pressure monitors, blood glucose meters, etc., in the healthcare industry. In the largest and most dynamic sector of healthcare products, forecasting the exact annual demand rate for these products by customers over the year is difficult. With these factors in keeping in mind, many researchers have developed various inventory models with a crisp sense when consumer demand is expressed imprecisely. The nature of demand in the existing world is never deterministic (constant), linear, or non-linear. As a result, the frequency with which every product in the marketplace is in demand fluctuates. Because of the variety of products shown in the store, several options are always held. For these reasons, in the inventory system, demand might be assumed to be stock and price dependent. Therefore, demand in the marketplace is influenced not only by the price of the goods, but also by the stock in the business processes. The demand for things increases from time to time. As a result, shops sold their inventory and subsequently purchased additional assets at a lower cost. However, no one has explored a fuzzy economic order quantity model (FEOQ) for price-stock-based demand with a payment delay allowed in an inflationary environment. This study has an impact on the inventory management system by combining price stock-dependent demand with a trade credit policy that considers payment delay. As it is well known, the stock is declining due to demand and the deterioration of the economy. Deterioration is an unavoidable part of life that can't be stopped, but it can be managed with the use of advanced preservative technology. The majority of the products deteriorate with time. Almost all things, particularly healthcare products, trendy items, pharmaceuticals, and seasonal commodities, deteriorate with time. There are many goods with a high rate of deterioration in the real world. There are several things that may be done to keep things under control and slow down the rate of deterioration. To slow the rate of deterioration, technical changes and modern preservation procedures have been implemented.

Literature review

In classical inventory models, many researchers have developed several theories to obtain the optimal order policy in such a way that the relevant total cost has a minimum or maximum value. Several solutions are proposed by many researchers in which the time parameters and necessary data are already defined and known precisely. However, in the real world, these assumptions and hypotheses are unrealistic since they are often vague, unclear, and imprecise. A significant variety of literature about inventory models in fuzzy environments has been extensively investigated to overcome these hypotheses. In the second half of the twentieth century, fuzzy mathematics was developed by Zadeh (1965). This research provided multiple benefits in the arena of mathematics. Normally, most researchers assume that the wholesaler must pay off as soon as the products are received in an EOQ model. The dealer does, in fact, give the vendor a grace time period, sometimes known as a trade credit term. Wholesalers frequently use trade credit as a marketing strategy to boost sales and reduce inventory levels. The duration tied up in the vendor's capital stock is reduced after a trade credit is offered, and this reduces the vendor's holding cost of finance. Additionally, wholesalers can make cash by selling products and earning interest during the trade credit period. In fact, suppliers, particularly small enterprises with a limited number of financing options, depend on trade credit as a possible source of short-term funding. In the current world, a dealer will frequently use this policy to enhance his inventory.

Delay in payment

A fuzzy inventory model for deteriorating products has been formulated by Chen and Ouyang (2006) that allows shortfalls and permits payment delays. Fuzzy Triangle numbers are used to fuzzify carrying cost rate, interest earned rate, and interest paid rate and defuzzifier by signed distance and centroid methods. Mahata and Goswami (2007) have given major contributions to developing an EOQ model for degrading products in a fuzzy environment where wholesalers and buyers face payment delays. The authors also assumed that the supplier must provide a payment delay to the retailer, and that the retailer must provide a credit period to the client for trade. In a fuzzy environment, the total variable cost was calculated using the graded mean integration representation method (GMIR). Gani and Maheshwari (2010) studied the seller's ordering strategy in a fuzzy environment with two levels of delayed payments, using triangular fuzzy numbers to study the selling price and demand and using the GMIR method for defuzzification. Jaggi et al. (2016) presented an EOQ model in a fuzzy environment, taking into account various trade credit terms with fully backlogged shortages. At the same time, Shaikh et al. (2018) developed a fuzzy inventory model for a decaying item with allowable payment delays and determined the demand function by the selling price with an allowable partial shortfall. Kumar et al. (2020) investigated a fuzzy inventory model with decaying commodities under the influence of inflation and the time value of money, in which total demand is determined by the selling price. For a finite replenishment rate, the authors also included trade credit financing in their model. In comparison to the EPQ model, Debnath et al. (2021) focused on the FEPQ. In a fuzzy environment, a shortfall is allowed if the backlog is fully shortfall and the demand function is taken as stock-dependent. Kumar De et al. (2021) presented a carbon emission inventory model for deteriorating commodities with a trade credit policy for customers to find the optimum technique by using a fuzzy approach.

Uncertain demand

The fuzzy set theory plays a vital role in developing the inventory model. The topic of uncertainty is addressed by fuzzy set theory. Several factors, such as an increase in demand and an increase in inflation, create uncertainty. It helps in the understanding of the model as well as the parameters that influence it and cause uncertainty. At the same time, Kao and Hsu (2002) developed a model with a lot of size-reorder points and fuzziness in demand. In order to reduce costs, Lee and Yao (1998), Lin and Yao (2000) developed an EPQ with fuzzy demand. Concerning fuzzy inventory systems with fuzzy random variables H.C Chang et al. (2006) originally described a mixture inventory model in which lead time demand and total cost are fuzzified and defuzzified by using the centroid approach. Significant contributions have been made in this area by Dutta et al. (2007), Dey and Chakraborty (2009), Sarkar and Mahapatra (2015). The authors investigated a continuous and periodic review for inventory systems in a stochastic environment, where uncertain demand is addressed as a fuzzy random variable and the lead time is estimated as a triangular fuzzy number. Soni and Joshi (2014) extended the work of Dey and Chakraborty (2009) and applied fuzzy set theory with the stochastic demand function and annual demand taken as triangular fuzzy numbers. Jaggi et al. (2012) proposed a fuzzy inventory model with uncertain demand for decaying products, with the goal of determining the optimal cost of the inventory. After that, Maragatham and Lakshmidevi (2014), Roy (2015) explored a fuzzy inventory model with demand function price dependent in a fuzzy sense and the signed distance method as a defuzzifier. Indrajitsingha et al. (2018) developed a fuzzy inventory model under no shortfalls with a stock-dependent demand function for deteriorating items and used fuzzy triangular numbers to fuzzify inventory costs. Garai et al. (2019) proposed a fuzzy inventory model based on the uncertainty principle with price dependent demand and time-changing holding costs. Padiyar et al. (2021) proposed a framework for such items that have time and pricing constraints to achieve the ideal production run time by applying this fuzzy approach to increases the total turnover of firm. Poswal (2022) developed a Fuzzy EOQ model for deteriorating products having stock and price dependent demand functions under shortages. The authors considered a fuzzy trapezoidal number to fuzzify costs and a signed distance method for defuzzification.

Inflation

When analysing the inventory model, inflation plays a crucial role in determining the demand function and pricing for a specific product. Inflation nowadays has an impact on our global economy. The economic system was affected by rising prices of products and services. As a result, many developing countries' financial situations have changed. So, the effects of inflation on businesses should not be underestimated. While developing the inventory model, a number of authors examined the impact of inflation. Buzacott (1975), a newcomer, was the first to consider an EOQ model with inflation for inventory modeling. Many researchers after Buzacott, Liao et al. (2000), Mishra et al. (2012), Uthayakumar and Palanivel (2014), Hossen et al. (2016), Singh and Kumar (2018), Rajput et al. (2021) developed various inventory models in an inflationary environment. In order to fit into the current world and because of the rising complexity and unpredictability of the global economy, it is preferable to consider inflation as a fuzzy quantity. However, significant research has been completed in this area. Few scholars, like Roy et al. (2009) Kumar et al. (2020), included inflation as a fuzzy quantity in their proposed models. Recently, Rajput et al. (2021) concluded a research project. The authors developed an EOQ (Economic Order Quantity) model for deteriorating products over a finite horizon with inflation. Because of the ambiguity in the parameters, the researchers used a pentagonal fuzzy number to find the total inventory cost in a fuzzy sense. The author implemented signed distance and graded mean integration methods (GMI) to defuzzify the cost parameters. Furthermore, Vipin et al. (2021) developed an imperfect production inventory model in an inflationary and fuzzy environment where good demand is influenced by reliability and selling price.

In this study, we made an effort to add all of the above-mentioned components into a single problem that certifies the market's economic conditions. The manufacturer is also encouraged by the trader to pay all compensation for a reasonable delay period. The holding cost is time-dependent in trying to accommodate in the real world. To decrease the deterioration rate, a Fuzzy EOQ model is studied for uncertain demand with a trade credit policy in an inflationary environment with allowable shortages. As a result, to attain economic and financial requirements, it is necessary to concentrate more on inventories and preservation expenditure.

Table 1: Contribution of authors

Authors	Model	Objective function	Demand function	Inflation	Trade credit
Jaggi et al. (2016)	FEOQ	Profit optimization	price dependent	√	√
Shaikh et al. (2018)	FEOQ	Cost optimization	price dependent	√	√
Singha et al. (2018)	FEOQ	Cost optimization	stock dependent		√
Garai et al. (2019)	FEOQ	Cost optimization	price and time		
Kumar et al. (2020)	FEOQ	Cost optimization	price dependent	√	√
Debnath et al. (2021)	FEPQ	Cost optimization	price		and
stock	√				
Kumar et al. (2021)	FEOQ	Profit optimization	carbon mission	√	
Padiyar et al. (2021)	FEOQ	Cost optimization	price dependent		√
Rajput et al. (2021)	FEOQ	Cost optimization	uncertain		√
Vipin et.al.(2021)	FEMQ	Cost optimization	time and stock	√	
Poswalet.al.(2022)	FEOQ	Cost optimization	price and stock		√
This paper	FEOQ	Cost optimization	uncertain	√	√

Nevertheless, an extension of the crisp model of Pant et al. (2021) to a fuzzy model has been made in this paper. A fuzzy trapezoidal number to the uncertainties like purchasing cost, cost of deterioration, carrying cost, ordering cost, shortage cost, lost sale cost, gained interest rate, etc., has been applied. The total average inventory costs with trade credit policy in the fuzzy environment are derived. Further, by using trapezoidal fuzzy numbers, the cost parameters are fuzzified. The fuzzy model is defuzzified by the signed distance method.

Preliminaries

A trapezoidal fuzzy number $\tilde{A}$= (a, b, c, d). The representation of its membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a} & if \ \ a \le x \le b, \\ 0, & if \ b \le x \le c \\ \dfrac{d-x}{d-c}, & if \ c \le x \le d \\ 0, & otherwise \end{cases}$$

Signed Distance Method

Let $\tilde{A}$= (a, b, c, d) is a trapezoidal fuzzy number. After that, signed distance measured from $\tilde{A}$ to 0 is term as

d ($\tilde{A}$, 0) = $\frac{1}{2}\int_0^1 [A_L(\alpha), A_R(\alpha)]$dα

Where$A_\alpha = [A_L(\alpha), A_R(\alpha)]$, $A_L(\alpha) = a + (b-a)\alpha$, $A_R(\alpha) = d - (d-c)\alpha$, $\alpha \ \varepsilon \ [0,1]$

$$d\big(\widetilde{TAC}, 0\big) = \frac{1}{4}(a+b+c+d)$$

Notations and assumptions

During the model development, the notations and assumptions listed below are being used.

Notations

Maximum time for the product	T
Selling price	ρ
Allowed credit period.	A
Deterioration rate per unit time	χ
Constant backlogging rate	ψ
Purchasing cost	C_p
Cost of deterioration per unit	C_d
Carrying cost	C_c
Ordering cost	C_o
Shortage cost	C_s
Preservation technology cost, $\phi \ge 0$	ϕ
Reduced deterioration rate	$\Omega(\lambda)$
Resultant deterioration	x t-$\Omega(\lambda)$
Lost sale cost	L
Time at which the stock level reaches zero	u
Inflation rate	i
Gained interest rate	C_{GIR}
Charged interest rate	C_{CIR}
Preservation technology cost	C_{PTC}
Total average cost	TAC
Fuzzy purchasing cost	$\tilde{c_p}$
Fuzzy shortage cost	$\tilde{c_s}$

Fuzzy carrying cost	$\tilde{c_c}$
Fuzzy ordering cost	$\tilde{c_o}$
Fuzzy deterioration cost	$\widetilde{c_d}$
Fuzzy lost sale cost	$\tilde{L}$
Fuzzy gained interest cost	$c_{\widetilde{GIR}}$
Fuzzy total average cost	$\widetilde{\text{TAC}}$
Defuzzify value of $\widetilde{\text{TAC}}$	TAC_g^*

Assumptions

1. The developed model was assessed for a fixed period of time.
2. This model has been formed for linearly deteriorating healthcare products with an expiration date.
3. The rate of inflation is taken into consideration.
4. $S(\rho, q(t)) = S(\rho) + \epsilon[q(t)]$, where $\epsilon, (0 < \epsilon << 1)$is stock dependent consumption rate and $S(\rho) = \theta(e - j\rho) + (1 - \theta)Q\rho^{-\beta}$ is a hybrid price combines linear and non-linear pricing with $0 \leq \theta \leq 1$, $e > 0$, $q > 0$, $\beta > 1$, $\frac{e}{j} \geq \rho$ fulfilled
5. A shortfall is acceptable and is assumed to be partially backlogged.
6. The trader is granting a specific period to compensate for the purchase rate, which doesn't include any interest.
7. No claim will be made during the trade credit cap period, but after that period, certain interest will be charged on the unpaid balance.

Mathematical formulation

In this analysis, a mathematical model is formed for those commodities (medicines, probiotics, vitamins etc.) that decline with time, and a mixture of price-stock-centered for decaying products is analysed. Consider the uncertainties in this model, such as the time-varying cost of carrying and inflation. A lot of commodities have an expiration date, and after that, these goods may harm the environment. As a result, the developed model proposes preservation financing. Our inventory's behavior was visualized in Fig 1. In the period [0, u], the initial size of stock that is depleted due to the impact of demand and deterioration. At t = u, the stock value is zero. Following that, there is a shortfall throughout the interval [u, T]. Then, the decreased deterioration rate is assumed when the wholesaler expends Φ for the preservation equipment in Fig.1.

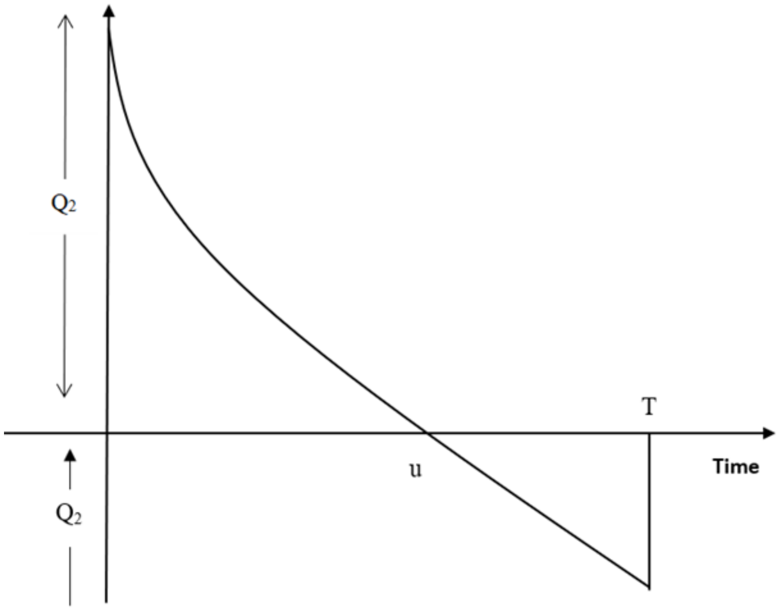

Figure 1: Graphical representation of inventory time graph

Let, $q_1(t)$is the stock's quantity at time $t = 0$ to $t = u$, and $q_2(t)$ is the stock's quantity at time $t = u$ to $t = T$. The differential equations of the system are as follows:

$$\frac{dq_1(t)}{dt} = (\Omega(\lambda) \;\; - \chi\, t)q_1(t) \; - \left(S(\rho) + \epsilon q_1(t)\right)\left(\frac{T-t}{T}\right), 0 \le t \le u \tag{1}$$

$$\frac{dq_2(t)}{dt} = -S(\rho)\left(\frac{T-t}{T}\right), u \le t \le T \tag{2}$$

With boundary constraints: $q_1(t) = q_2(t) \; = 0$

From equation (1) the stock level $q_1(0)$ is given by:

$$Q_1 = S(\rho)\left\{u + \frac{\epsilon u^2}{2} + \frac{\chi u^3}{6} - \frac{\epsilon u^3}{2T} - \frac{\chi b\Phi u^3}{6} - \frac{u^2}{2T} - \frac{\chi u^4}{8T} + \frac{\epsilon u^4}{8T^2} + \frac{\chi b\Phi u^4}{8T}\right\} \tag{3}$$

Due to a stock out, the following amount will be back-ordered:

$$Q_2 = \int_u^T S(\rho)\left(\frac{T-t}{T}\right)\Psi dt \tag{4}$$

Cost associated: These costs shown below are for the established model.

Purchasing cost

If the primary stock level is q_1 and the entire back-ordering amount isq_2..Then the purchase expense is calculated with the following formula:

$$PC = C_P(Q_1 + Q_2)e^{-iA}$$

$$= \left[C_P S(\rho) \left\{ \begin{matrix} \left(u + \frac{\epsilon u^2}{2} + \frac{\chi u^3}{6} - \frac{\epsilon u^3}{2T} - \frac{\chi b \Phi u^3}{6} - \frac{u^2}{2T} - \frac{\chi u^4}{8T} + \frac{\epsilon u^4}{8T^2} + \frac{\chi b \Phi u^4}{8T} \right) \\ + \Psi \left(\frac{T}{2} - u + \frac{u^2}{2T} \right) \end{matrix} \right\} (1 - iA) \right]$$

(5)

Carrying cost

Whenever the inventory is stored for a period of time by the system, it imposes a carrying expense which is calculated in the following way:

Carrying cost $= C_C \int_0^u q_1(t) t e^{-it} dt$ (6)

Cost of deterioration

The cost of deterioration is the price of the system's decaying goods. The following formula can be used to calculate $C_D$per-unit cost of deterioration:

$$C_D = C_d \int_0^u \left(q_1(0) - \left(S(\rho) + \in q_1(t) \right) \left(\frac{T-t}{T} \right) e^{-it} \right) dt \quad (7)$$

Cost of ordering

The following is an estimate of the order cost:$OC = C_O$

Shortage cost

When the stock level reaches zero at time t = u, the stock out situation arises. At level t = T, the shortages are at its peak. The following is an estimate of the shortage cost

$$SC = C_S \int_u^T \left(S(\rho) \left(\frac{T-t}{T} \right) e^{-i(u+t)} \right) dt \quad (8)$$

Lost sale cost

Over the time period$[u, T]$, backlogging occurs at a rate of Ψ. As a result, the lost sale cost can be calculated as follows:

$$LSC = L(1 - \Psi) \int_u^T \left(S(\rho) \left(\frac{T-t}{T} \right) e^{-i(u+t)} \right) dt \quad (9)$$

Permissible delay

The provider gave a grace period during which the money for the acquired products could be returned without interest. During this time, the buyer sells the commodity and uses the earnings to earn interest at a rate C_{GIR}. After this time period, the buyer returns the cost of the item to the suppliers. If somehow the buyer failed to repay the refund during this time period, the buyer must face penalties in the form of interest at a rate of two possibilities arise as a consequence of the trade credit period.

Case 1 When the allowable delay period is higher than and equal to the time at which the stock reaches zero, i.e., $A \geq u$. In this case, the trader sold his entire inventory before receiving payment. As a result, the firm has sufficient funds to pay all of the debts. There will be no interest in these circumstances. Interest will be collected during the period [0, A]. Figure 2 illustrates a graphic representation of this state.

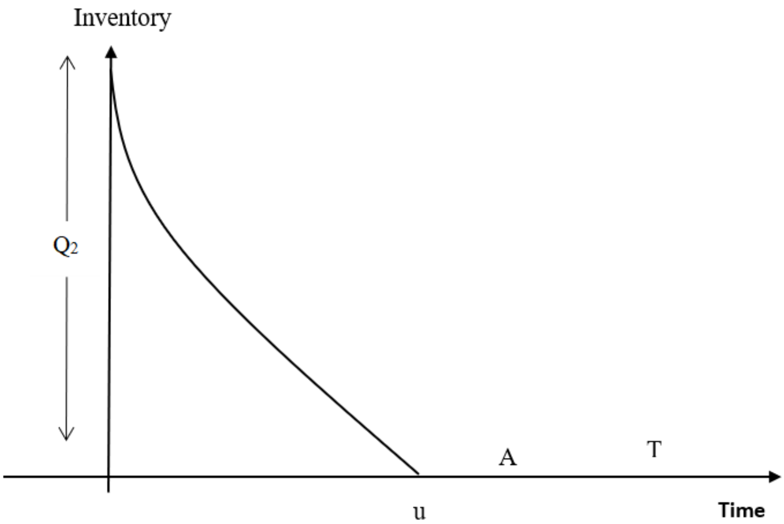

Figure 2: Graphical representation of inventory time graph

$$\mathrm{IG}_1 = \left[\rho C_{GIR} \begin{Bmatrix} \int_0^u \left(\{S(\rho) + \epsilon q_1(t)\} \left(\frac{T-t}{T}\right) te^{-it} \right) dt \\ +(A-u) \int_0^u \left(\{S(\rho) + \epsilon q_1(t)\} \left(\frac{T-t}{T}\right) e^{-it} \right) dt \end{Bmatrix}\right] \quad (10)$$

Case 2: When the acceptable delay time is less than the replenish time, i.e., *in* this case, the maximum allowable delay is less than the time it took for all of the goods to sell out. Figure 3 shows the graphical representation. Two scenarios are illustrated below, based on the quantity available at the moment of payment.

Case 2.(a) Businesses in this circumstance have a balance that is more than or equal to the amount due at the time of payment. As a result, interest charges would be zero. During the interval [0, A], there would be an increased interest.

$$IG_{2.1} = \left[\rho C_{GIR} \left\{ \int_0^A \left(S(\rho) \left(\frac{T-t}{T}\right) te^{-it} \right) dt \right\}\right] \quad (11)$$

Case 2.(b) In this case, the retailer's balance at the time of payment would be insufficient to fulfil all of his obligations, so extra charges on the unpaid amount would be imposed, and interest would be earned throughout [0, A].

$$IG_{2.2} = \left[\rho C_{GIR} \left\{ \int_0^A \left(S(\rho) \left(\frac{T-t}{T}\right) te^{-it} \right) dt \right\}\right] \quad (12)$$

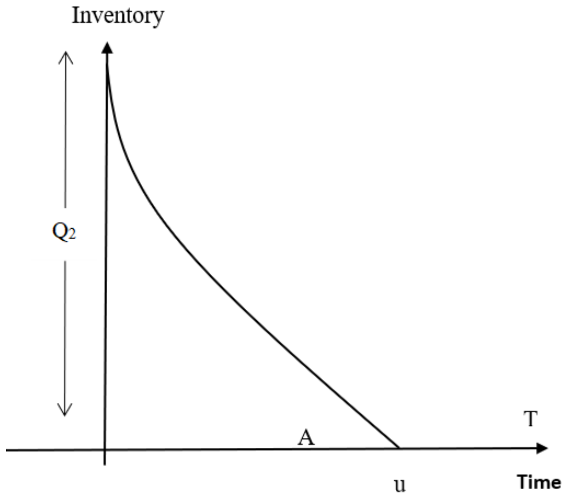

Figure 3: A graphical illustration of the inventory time graph is shown above.

Total inventory cost

$$\mathrm{TAC} = \frac{1}{\mathrm{T}}[\text{purchasing cost} + \text{carring cost} + \text{deterioration Cost} + \text{ordering cost} + \text{shortage cost} + \text{lost sale cost} + \text{interest cost} - \text{interest gained}] \quad (13)$$

Fuzzy Inventory Model

The inventory parameter must be considered in a fuzzy sense in order to approach this problem in a fuzzy context. To reflect the data's uncertainty, the regular trapezoidal membership function was used.

We consider $\widetilde{c_p} = (c_{p_1}, c_{p_2}, c_{p_3}, c_{p_4})$, $\widetilde{c_s} = (c_{s_1}, c_{s_2}, c_{s_3}, c_{s_4})$,
$\widetilde{c_d} = (c_{d_1}, c_{d_2}, c_{d_3}, c_{d_4})$, $\widetilde{c_o} = (c_{o_1}, c_{o_2}, c_{o_3}, c_{o_4})$,
$\widetilde{c_c} = (c_{c_1}, c_{c_2}, c_{c_3}, c_{c_4})$, $\tilde{L} = (L_1, L_2, L_3, L_4)$
$\widetilde{c_{GIR}} = (c_{GIR_1}, c_{GIR_2}, c_{GIR_3}, c_{GIR_4})$ are as trapezoidal number
Hence the total average cost in fuzzy sense is given by

$$\widetilde{\mathrm{TAC}} = \frac{1}{\mathrm{T}}[\text{fuzzy purchasing cost} + \text{fuzzy carring cost} + \text{fuzzy deterioration cost} + \text{fuzzy ordering cost} + \text{fuzzy lost sale cost} + \text{interest cost} - \text{interest gained}] \quad (14)$$

$$\widetilde{TAC} = \frac{1}{T}\left[\widetilde{C_P}S(\rho)\left\{\begin{matrix}\left(u+\frac{\epsilon u^2}{2}+\frac{\chi u^3}{6}-\frac{\epsilon u^3}{2T}-\frac{\chi b\Phi u^3}{6}-\frac{u^2}{2T}-\frac{\chi u^4}{8T}+\frac{\epsilon u^4}{8T^2}+\frac{\chi b\Phi u^4}{8T}\right)\\ +\Psi\left(\frac{T}{2}-u+\frac{u^2}{2T}\right)\end{matrix}\right\}(1-iA)\right.$$

$$+\frac{\widetilde{C_C}S(\rho)u^3}{40320P^3}\left\{\begin{matrix}315u^5\epsilon^2+30T\epsilon u^3\{28+u(-32i-68\epsilon+21\chi u(b\Phi-1))\}+6720T^3\\ +8uT^3\begin{pmatrix}210\epsilon-336\epsilon^2u+7\chi u(35\epsilon u-18)(b\Phi-1)-45\chi^2u^3(b\Phi-1)^2\\ +28i(5\chi u^2(b\Phi-1)-12\epsilon u-15)\end{pmatrix}\\ -5040T^2u+3T^2u^2\left\{-784\epsilon+5u\begin{pmatrix}280\epsilon^2-8\chi(20\epsilon u-7)(b\Phi-1)\\ +21\chi^2u^2(b\Phi-1)^2\\ +32i\left(28+5u(7\epsilon-2\chi u(b\Phi-1))\right)\end{pmatrix}\right\}\end{matrix}\right\}$$

$$+\widetilde{C_O}+\widetilde{C_S}S(\rho)\left\{\frac{T}{2}-\frac{iuT}{2}-\frac{iT^2}{6}-u-\frac{\epsilon T^2}{6}+3\left(\frac{iu^2}{2}\right)+\frac{u^2}{2T}-5\left(\frac{iu^3}{6T}\right)\right\}$$

$$+\widetilde{C_d}S(\rho)\left(-u+u^2+\frac{iu^2}{2}+\frac{u^2}{2T}-\frac{2iu^3}{3}-\frac{u^3}{2T}+\frac{iu^4}{4T}+\frac{u^4\chi}{6}-\frac{iu^3\chi}{12}-\frac{u^5\chi}{8T}+\frac{iu^6\chi}{16T}-\frac{\epsilon u^2}{2}+\frac{\epsilon u^3}{2}+\frac{i\epsilon u^3}{6}+\right.$$
$$\frac{\epsilon u^3}{2T}-\frac{i\epsilon u^4}{4}-\frac{\epsilon u^4}{8T^2}-\frac{\epsilon u^4}{2T}-\frac{5i\epsilon u^4}{24T}+\frac{\epsilon u^5}{8T^2}+\frac{i\epsilon u^5}{15T^2}+\frac{i\epsilon u^5}{4T}-\frac{i\epsilon u^6}{16T^2}-\frac{\epsilon u^4\chi}{12}+\frac{i\epsilon u^5\chi}{20}+\frac{11\epsilon u^5\chi}{120T}-\frac{\epsilon u^6\chi}{48T^2}-\frac{5i\epsilon u^6\chi}{72T}+$$
$$\frac{iu^7\chi\epsilon}{42T^2}+\frac{1}{72}u^6\chi^2\epsilon-\frac{u^7\chi^2\epsilon}{48T}+\frac{u^8\chi^2\epsilon}{128T^2}-\frac{u^3\epsilon^2}{6}+\frac{1}{8}iu^4\epsilon^2+\frac{u^4\epsilon^2}{4T}-\frac{u^5\epsilon^2}{8T^2}-\frac{13iu^5\epsilon^2}{60T}+\frac{u^6\epsilon^2}{48T^3}+\frac{iu^6\epsilon^2}{8T^2}-\frac{iu^7\epsilon^2}{42T^3}+$$
$$\frac{1}{12}u^5\chi\epsilon^2-\frac{7u^6\chi\epsilon^2}{48T}+\frac{u^7\chi\epsilon^2}{12T^2}-\frac{u^8\chi\epsilon^2}{64T^3}+\frac{u^4\epsilon^3}{8}-\frac{u^5\epsilon^3}{4T}+\frac{3u^6\epsilon^3}{16T^2}-\frac{u^7\epsilon^3}{16T^3}+\frac{u^8\epsilon^3}{128T^4}-\frac{1}{6}bu^4\chi\phi+\frac{1}{12}biu^5\chi\phi$$
$$+\frac{bu^5\chi\phi}{8T}-\frac{biu^6\chi\phi}{16T}+\frac{1}{12}bu^4\chi\epsilon\phi-\frac{1}{20}biu^5\chi\epsilon\phi-\frac{11bu^5\chi\epsilon\phi}{120T}+\frac{bu^6\chi\epsilon\phi}{48T^2}+\frac{5biu^6\chi\epsilon\phi}{72T}+\frac{bu^7\chi^2\epsilon\phi}{24T}-$$
$$\frac{bu^8\chi^2\epsilon\phi}{64T^2}-\frac{1}{12}bu^5\chi\epsilon^2\phi+\frac{7bu^6\chi\epsilon^2\phi}{48T}-\frac{biu^7\chi\epsilon\phi}{42T^2}-\frac{1}{36}bu^6\chi^2\epsilon\phi-\frac{bu^7\chi\epsilon^2\phi}{12T^2}+\frac{bu^8\chi\epsilon^2\phi}{64T^3}+\frac{1}{72}b^2u^6\chi^2\epsilon\phi^2-$$
$$\left.\frac{b^2u^7\chi^2\epsilon\phi^2}{48T}+\frac{b^2u^8\chi^2\epsilon\phi^2}{128T^2}\right)-\rho\frac{\widetilde{C_{\mathrm{GIR}}\ u}}{40320T^4}\left(u(91u^7\epsilon^3+Tu^5\epsilon^2(456-330iu-819u\epsilon+182u^2\chi(-1+\right.$$
$$\left.\left.b\phi)\right)+T^3u(6720+11760u\epsilon+1008u^2\epsilon(7\epsilon-6b\chi\phi)\right]\tag{15}$$

The signed distance method used to defuzzify the value, we get

$TAC_g^* = \frac{1}{4T}[TAC_{g1}+ TAC_{g2} +TAC_{g3} +TAC_{g4}]$

$$= \frac{1}{4T}\left[\sum_{j=1}^{4}\left\{C_{pj}S(\rho)\begin{pmatrix}\left(u+\frac{\epsilon u^2}{2}+\frac{\chi u^3}{6}-\frac{\epsilon u^3}{2T}-\frac{\chi b\Phi u^3}{6}-\frac{u^2}{2T}-\frac{\chi u^4}{8T}+\frac{\epsilon u^4}{8T^2}+\frac{\chi b\Phi u^4}{8T}\right)\\ +\Psi\left(\frac{T}{2}-u+\frac{u^2}{2T}\right)\end{pmatrix}(1-iA)\right\}\right.$$

$$+C_{cj}\frac{S(\rho)u^3}{40320P^3}\left\{\begin{matrix}315u^5\epsilon^2+30T\epsilon u^3\{28+u(-32i-68\epsilon+21\chi u(b\Phi-1))\}\\ +6720T^3\\ +8uT^3\begin{pmatrix}210\epsilon-336\epsilon^2u+7\chi u(35\epsilon u-18)(b\Phi-1)-45\chi^2u^3(b\Phi-1)^2\\ +28i(5\chi u^2(b\Phi-1)-12\epsilon u-15)\end{pmatrix}\\ -5040T^2u+3T^2u^2\left\{-784\epsilon+5u\begin{pmatrix}280\epsilon^2-8\chi(20\epsilon u-7)(b\Phi-1)\\ +21\chi^2u^2(b\Phi-1)^2\\ +32i\left(28+5u(7\epsilon-2\chi u(b\Phi-1))\right)\end{pmatrix}\right\}\end{matrix}\right\}$$

$$+C_{0j}+C_{sj}S(\rho)\left\{\frac{T}{2}-\frac{iuT}{2}-\frac{iT^2}{6}-u-\frac{\epsilon T^2}{6}+3\left(\frac{iu^2}{2}\right)+\frac{u^2}{2T}-5\left(\frac{iu^3}{6T}\right)\right\}$$

$$+\tilde{L}(1-\Psi)S(\rho)\left\{\frac{T}{2}-\frac{iuT}{2}-\frac{iT^2}{6}-u-\frac{\epsilon T^2}{6}+3\left(\frac{iu^2}{2}\right)+\frac{u^2}{2T}-5\left(\frac{iu^3}{6T}\right)\right\}$$

$$
\begin{aligned}
&+L_j\,(1-\Psi)S(\rho)\left\{\frac{T}{2}-\frac{iuT}{2}-\frac{iT^2}{6}-u-\frac{\epsilon T^2}{6}+3\left(\frac{iu^2}{2}\right)+\frac{u^2}{2T}-5\left(\frac{iu^3}{6T}\right)\right\}\\
&+C_{dj}\,S(\rho)\left\{\begin{gathered}
-u+u^2+\frac{iu^2}{2}+\frac{u^2}{2T}-\frac{2iu^3}{3}-\frac{u^3}{2T}+\frac{iu^4}{4T}+\frac{u^4\chi}{6}-\frac{iu^3\chi}{12}-\frac{u^5\chi}{8T}\\
+\frac{iu^6\chi}{16T}-\frac{\epsilon u^2}{2}+\frac{\epsilon u^3}{2}+\frac{i\epsilon u^3}{6}+\frac{\epsilon u^3}{2T}-\frac{i\epsilon u^4}{4}-\frac{\epsilon u^4}{8T^2}-\frac{\epsilon u^4}{2T}-\frac{5i\epsilon u^4}{24T}+\frac{\epsilon u^5}{8T^2}+\frac{i\epsilon u^5}{15T^2}\\
+\frac{i\epsilon u^5}{4\mathrm{T}}-\frac{i\epsilon u^6}{16T^2}-\frac{\epsilon u^4\chi}{12}+\frac{i\epsilon u^5\chi}{20}+\frac{11\epsilon u^5\chi}{120T}-\frac{\epsilon u^6\chi}{48T^2}-\frac{5i\epsilon u^6\chi}{72T}+\frac{iu^7\chi\epsilon}{42T^2}+\frac{1}{72}u^6\chi^2\epsilon-\frac{u^7\chi^2\epsilon}{48T}\\
+\frac{u^8\chi^2\epsilon}{128T^2}-\frac{u^3\epsilon^2}{6}+\frac{1}{8}iu^4\epsilon^2+\frac{u^4\epsilon^2}{4T}-\frac{u^5\epsilon^2}{8T^2}-\frac{13iu^5\epsilon^2}{60T}+\frac{u^6\epsilon^2}{48T^3}+\frac{iu^6\epsilon^2}{8T^2}-\frac{iu^7\epsilon^2}{42T^3}+\frac{1}{12}u^5\chi\epsilon^2\\
-\frac{7u^6\chi\epsilon^2}{48T}+\frac{u^7\chi\epsilon^2}{12T^2}-\frac{u^8\chi\epsilon^2}{64T^3}+\frac{u^4\epsilon^3}{8}-\frac{u^5\epsilon^3}{4T}+\frac{3u^6\epsilon^3}{16T^2}-\frac{u^7\epsilon^3}{16T^3}+\frac{u^8\epsilon^3}{128T^4}-\frac{1}{6}bu^4\chi\phi\\
+\frac{1}{12}biu^5\chi\phi+\frac{bu^5\chi\phi}{8T}-\frac{biu^6\chi\phi}{16T}+\frac{1}{12}bu^4\chi\epsilon\phi-\frac{1}{20}biu^5\chi\epsilon\phi-\frac{11bu^5\chi\epsilon\phi}{120T}\\
+\frac{bu^6\chi\epsilon\phi}{48T^2}+\frac{5biu^6\chi\epsilon\phi}{72T}-\frac{biu^7\chi\epsilon\phi}{42T^2}-\frac{1}{36}bu^6\chi^2\epsilon\phi+\frac{bu^7\chi^2\epsilon\phi}{24T}-\frac{bu^8\chi^2\epsilon\phi}{64T^2}-\frac{1}{12}bu^5\chi\epsilon^2\phi\\
+\frac{7bu^6\chi\epsilon^2\phi}{48T}-\frac{bu^7\chi\epsilon^2\phi}{12T^2}+\frac{bu^8\chi\epsilon^2\phi}{64T^3}+\frac{1}{72}b^2u^6\chi^2\epsilon\phi^2-\frac{b^2u^7\chi^2\epsilon\phi^2}{48T}+\frac{b^2u^8\chi^2\epsilon\phi^2}{128T^2}
\end{gathered}\right\}\\
&-C_{GIRj}\,\rho C_{\mathrm{GIR}}\left\{\begin{gathered}
\frac{u}{40320T^4}\left\{\begin{gathered}
u(91u^7\epsilon^3+Tu^5\epsilon^2(456-330iu-819u\epsilon+182u^2\chi(-1+b\phi))\\
+T^3u(6720+11760u\epsilon+1008u^2\epsilon(7\epsilon-6b\chi\phi)-3u^5\chi^2\epsilon(91-127b\phi+36b^2\phi^2)\\
-8u^4\chi\epsilon(50b\chi\phi(-1+b\phi)-3\epsilon(-91+61b\phi))
\end{gathered}\right\}\\
+16iu\{-210-231u\epsilon+5u^3\chi\epsilon(-14+5b\phi)-7u^2\epsilon(33\epsilon-16b\chi\phi))\\
-56u^3\epsilon(75\epsilon^2+\chi(-43+b\phi-32b\epsilon\phi)\}+T^2u^3\epsilon\{-2352-3192u\epsilon\\
+3u^3\chi\epsilon(364-309b\phi)+91u^4\chi^2(-1+b\phi)^2-6iu(-168-320u\epsilon+55u^2\chi(-1+b\phi))\\
-8u^2(-348\epsilon^2+\chi(57+b(-57+50\epsilon)\phi))\}\\
-A(315u^7\epsilon^3+30Tu^5\epsilon^2(28-32iu-84u\epsilon+21u^2\chi(-1+b\phi))\\
-8T^3u\left\{\begin{gathered}
-2520-2520u\epsilon-1260u^2\epsilon(\epsilon-b\chi\phi)+15u^5\chi^2\epsilon(7-10b\phi+3b^2\phi^2)\\
+35u^4\chi\epsilon(4b\chi\phi(-1+b\phi)-3\epsilon(-7+5b\phi))\\
-14iu(-120-75u\epsilon+5u^3\chi\epsilon(-5+2b\phi)+6u^2\epsilon(-13\epsilon+6b\chi\phi))\\
-42u^3\epsilon(-30\epsilon^2+\chi(11+b\phi+12b\epsilon\phi))
\end{gathered}\right\}\\
-112T^4\left\{\begin{gathered}
3iu(-60-20u\epsilon-15u^2\epsilon^2+6u^3\chi\epsilon(-1+b\phi))\\
-5(-72-36u\epsilon-12u^2\epsilon^2-6u^4\chi\epsilon^2(-1+b\phi)+u^5\chi^2\epsilon(-1+b\phi)^2)
\end{gathered}\right\}\\
+u^3(9\epsilon^3+6\chi\epsilon(-1+b\phi))+T^2u^3\epsilon\{-48iu(-56-105u\epsilon+20u^2\chi(-1+b\phi))\\
+5\begin{pmatrix}-1008-1008u\epsilon+63u^4\chi^2(-1+b\phi)^2-96u^3\chi\epsilon(-7+6b\phi)\\ -56u^2(-27\epsilon^2+\chi(3+b(-3+4\epsilon)\phi))\end{pmatrix}
\end{gathered}\right\}
\end{aligned}
\tag{16}
$$

Optimality computational criteria

To obtain maximize profit per unit time in the fuzzy environment, firstly compute the first derivative of ∂TACg* with respect to u and Φ and then equate it to zero.∂TACg* (u, Φ) is calculated as

$$\frac{\partial \mathrm{TACg}^*(\mathrm{u},\Phi)}{\partial u}=0\quad \&\quad \frac{\partial \mathrm{TACg}^*(\mathrm{u},\Phi)}{\partial \Phi}=0$$

Also,

$$\frac{\partial^2 \mathrm{TACg}^*(\mathrm{u},\Phi)}{\partial u^2}>0\,,\qquad \frac{\partial^2 \mathrm{TACg}^*(\mathrm{u},\Phi)}{\partial \Phi^2}>0$$

And

$$\frac{\partial^2 \mathrm{TACg}^*(\mathrm{u},\Phi)}{\partial u^2}\frac{\partial^2 \mathrm{TACg}^*(\mathrm{u},\Phi)}{\partial^2 \Phi}-\left(\frac{\partial^2 \mathrm{TACg}^*(\mathrm{u},\Phi)}{\partial u\partial \Phi}\right)^2>0 \tag{17}$$

Numerical Analysis

Consider a case study of the healthcare industry which supplies pharmaceutical products like medicines, probiotics, vitamins, and supplements to a bazaar with uncertain demand. These products have an expiration date and deteriorate with time, resulting in a large volume of waste. As time passes and the product approaches its expiration date, demand steadily declines until it reaches nil. The preservation technology expense is used here with a price and stock-dependent demand and an allowable payment delay to reduce deterioration. Two different cases exist, which depend on the acceptable permissible delay time.

The following are the membership functions for each feasible cost in fuzzy context:

T=5 months, x =0.05, i = 0.08, A = 4 months (in case 1) = 0.5{case 2(a), case 2(b)},fuzzy cost of ordering, $\widetilde{c_o} = \$\,(495{,}498{,}502{,}504)$ per unit, fuzzy cost of purchasing $\widetilde{c_p} = \$\,(17{,}18{,}21{,}22)$per unit, fuzzy cost of shortage $\widetilde{c_s} = \$\,(10{,}13{,}16{,}19)$per unit, fuzzy cost of carrying$\widetilde{c_c} = \$\,(0.50{,}0.75{,}1.25{,}1.50)$per unit, lost sale cost $\tilde{L}$ = \$ (13, 15, 18,19) per unit, fuzzy cost of deterioration,$\widetilde{c_d} = \$\,(18{,}20{,}22{,}23)$ per unit, fuzzy cost of gained interest $\widetilde{C_{GIR}} = (0.07{,}0.08{,}0.10{,}0.11)$, $C_{CIR} = 0.06$, ρ = \$30 per unit, $\beta = 2, \epsilon = 0.5,\ e = 800,\ \ j = 1, q = 600, b = 0.1, \Psi = 0.4\,, \theta = 0.5$. This numerical problem is solved with the help of the optimal criteria provided in section 7 and the Mathematica 11 software. This solution procedure will help to improve the strategy for the supplier of pharmaceutical industry to minimise the total average cost and results as given in table 2.

Table 2	Inventory Parameters	Crisp value of Pant et al. (2021)	Fuzzy value of model
	$TAC_1{}^*$	\$ 3596.26	\$ 3446.30
Case 1	u	3.44547 months	3.42296 months
	Φ	355.927 per unit	358.054 per unit
	$TAC_2{}^*$	\$ 3527.39	\$ 3377.21
Case 2(a)	u	5.12515 months	5.12157 months
	Φ	959.618 per unit	974.875 per unit
	$TAC_3{}^*$	\$ 3512.64	\$ 3371.97
Case 2(a)	u	5.16463 months	5.13224 months
	Φ	1054.67 per unit	1007.25 per unit

Using the information provided above, the following optimal solution may be derived for each case:

Case 1: When the allowable delay duration is higher than or equal to the replenishment cycle time, i.e., $TAC_1{}^*$ = 3446.3, replenishment time u = 3.42296 months, and preservation technology cost = 358.054 per unit are the best values. Figure 4 illustrates the convexity of the overall cost in this situation.

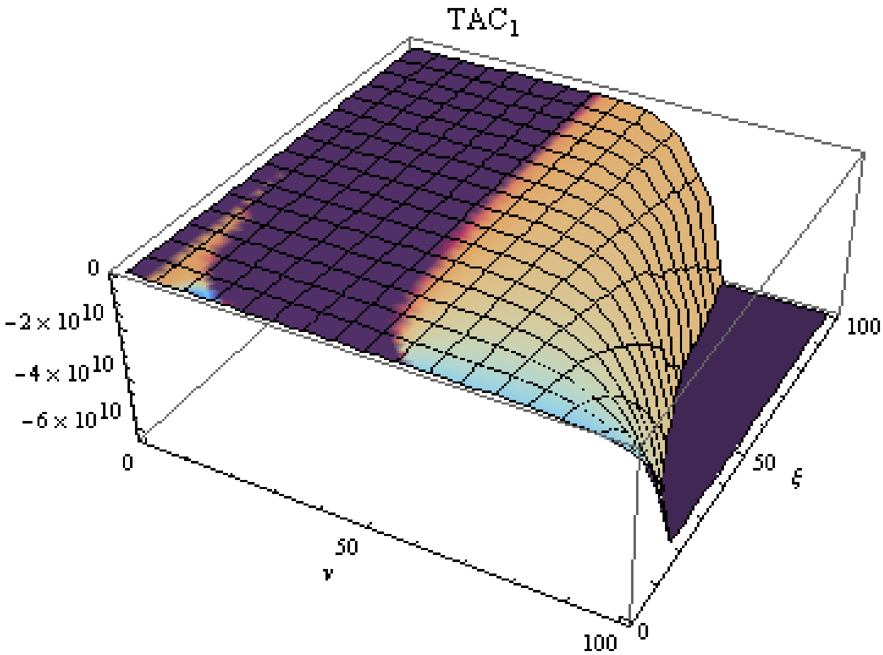

Figure 4: Convexity of the total average cost when $A \geq u$.

Case 2(a) : When the allowable delay period is shorter than the replenishing time, i.e., A < u. The optimal cost is TAC_2* = $3377.21, replenishment time u=5.12157 months, and preservation technology cost Φ = 974.875 per unit in this case. Figure 5 depicts the convexity of the overall cost in this situation.

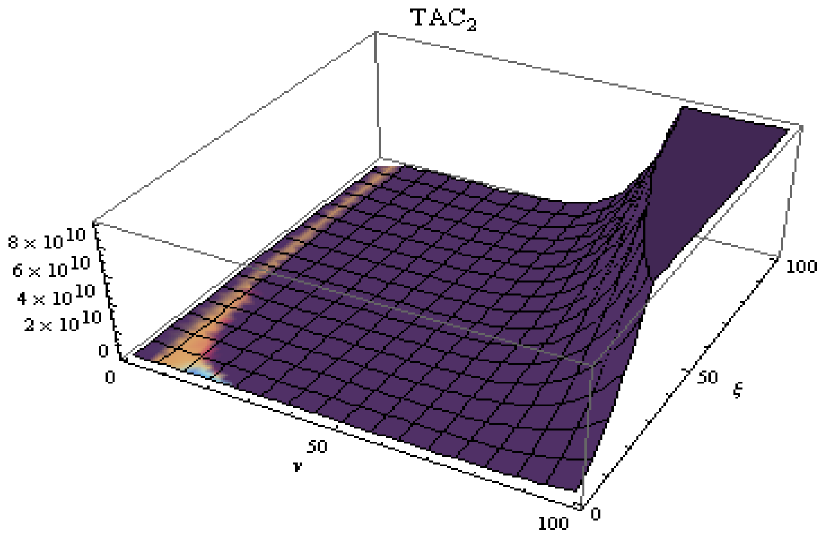

Figure 5: Convexity of the total average cost when A<u

Case 2. (b) When the allowable delay time is shorter than the replenishment time, i.e., A < u. The optimal cost is TAC_3* = $3371.97 replenishment time u = 5.13224 months, Φ =$1007.25 determined by the replacement time in months and preservation technology cost per unit in this case. Figure 6 depicts the entire cost's convexity in this situation.

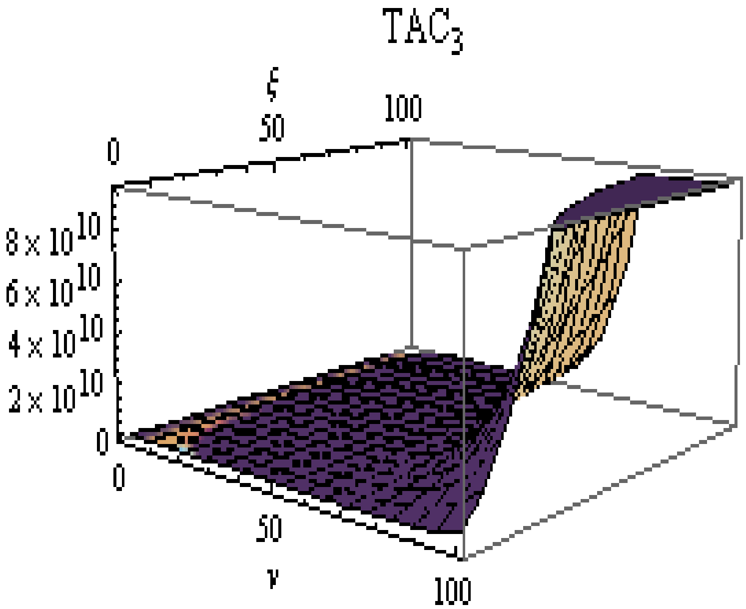

Figure 6: Convexity of the total average cost when A<u

Sensitivity Analysis

This research is required to know the nature of the parameters all through the retailer's cost function. Retailers can increase or decrease inventory system parameters to determine when they can reduce overall costs. Changing the values of some parameters has an impact on the system's overall average cost.

Case 1:

Table 3: Case-1 Sensitivity analysis with different parameter

Parameters	Change in %	TAC_1^*	u	Φ
	-50%	3696.23	3.43319	356.272
	-25%	3571.26	3.42812	357.154
Ψ	0%	3446.30	3.42296	358.054
	+25%	3321.33	3.41771	358.971
	+50%	3196.36	3.41237	359.909
	-50%	3446.24	3.45261	697.055
	-25%	3446.27	3.43763	471.068
χ	0%	3446.30	3.42296	358.054
	+25%	3446.32	3.40859	290.228
	+50%	3446.34	3.39451	244.997
	-50%	3573.75	4.18581	359.347
	-25%	3522.56	3.79904	345.127
ρ	0%	3446.30	3.42296	358.054

	+25%	3365.36	3.07732	386.561
	+50%	3283.66	2.79258	420.155
	-50%	1704.3	3.42296	358.054
	-25%	2575.3	3.42296	358.054
e	0%	3446.3	3.42296	358.054
	+25%	4317.3	3.42296	358.054
	+50%	5188.3	3.42296	358.054
	-50%	3422.58	3.94696	744.940
	-25%	3441.17	3.71838	435.180
ϵ	0%	3446.30	3.42296	358.054
	+25%	3448.03	3.04272	355.716
	+50%	3448.45	2.64615	393.120
	-50%	3243.15	3.42296	358.054
β	-25%	3414.93	3.42296	358.054
	0%	3446.3	3.42296	358.054
	+25%	3452.02	3.42296	358.054
	+50%	3453.07	3.42296	358.054
	-50%	3449.80	3.42296	358.054
q	-25%	3448.05	3.42296	358.054
	0%	3446.30	3.42296	358.054
	+25%	3444.54	3.42296	358.054
	+50%	3442.79	3.42296	358.054

Case 2 (a)

Table 4: Case 2 (a) Sensitivity analysis with different parameters

Parameters	Change in %	TAC_2^*	u	Φ
	-50%	3627.12	5.12157	974.876
	-25%	3502.17	5.12157	974.876
Ψ	0%	3377.21	5.12157	974.875
	+25%	3252.25	5.12158	974.875
	+50%	3127.30	5.12158	974.875
	-50%	3377.21	5.12157	1939.75
	-25%	3377.21	5.12157	1296.50
χ	0%	3377.21	5.12157	974.875
	+25%	3377.21	5.12157	781.900
	+50%	3377.21	5.12157	653.250
	-50%	3320.40	5.12157	974.875
	-25%	3403.21	5.12157	974.875
ρ	0%	3377.21	5.12157	974.875
	+25%	3320.46	5.12157	974.875
	+50%	3252.22	5.12157	974.875
	-50%	1635.21	5.12157	974.875
	-25%	2506.21	5.12157	974.875
e	0%	3377.21	5.12157	974.875
	+25%	4248.21	5.12157	974.875
	+50%	5119.21	5.12157	974.875
	-50%	3117.22	4.97877	2895.75
ϵ	-25%	3229.28	5.04129	2765.67

	0%	3377.21	5.12157	974.875
	+25%	3404.57	5.20975	616.126
	+50%	3413.93	5.30436	469.118
β	-50%	1170.61	5.12157	974.875
	-25%	3036.54	5.12157	974.875
	0%	3377.21	5.12157	974.875
	+25%	3439.41	5.12157	974.875
	+50%	3450.76	5.12157	974.875
q	-50%	3415.26	5.12157	974.875
	-25%	3396.23	5.12157	974.875
	0%	3377.21	5.12157	974.875
	+25%	3358.19	5.12157	974.875
	+50%	3339.17	5.12157	974.875

Case 2 (b)

Table 5: Case 2 (b) Sensitivity analysis with different parameters

Parameters	Change in %	TAC_3^*	u	Φ
Ψ	-50%	3621.88	5.13223	1007.25
	-25%	3496.92	5.13224	1007.25
	0%	3371.97	5.13224	1007.25
	+25%	3247.01	5.13224	1007.25
	+50%	3122.06	5.13224	1007.25
χ	-50%	3367.76	5.14207	2047.92
	-25%	3370.58	5.13557	1349.30
	0%	3371.97	5.13224	1007.25
	+25%	3372.79	5.13021	804.332
	+50%	3373.34	5.12885	670.019
ρ	-50%	3304.33	5.12707	996.429
	-25%	3394.86	5.12967	1001.84
	0%	3371.97	5.13224	1007.25
	+25%	3316.83	5.13477	1012.67
	+50%	3249.56	5.13727	1018.09
e	-50%	1629.37	5.13224	1007.25
	-25%	2500.67	5.13224	1007.25
	0%	3371.97	5.13224	1007.25
	+25%	4243.27	5.13224	1007.25
	+50%	5114.57	5.13224	1007.25
ϵ	-50%	3153.78	5. 02312	3234.23
	-25%	3213.84	5.04534	2859.01
	0%	3371.97	5.13224	1007.25
	+25%	3401.18	5.22548	635.955
	+50%	3411.13	5.32360	483.528
β	-50%	979.804	5.13224	1007.25
	-25%	3002.65	5.13224	1007.25
	0%	3371.97	5.13224	1007.25
	+25%	3439.39	5.13224	1007.25
	+50%	3451.71	5.13224	1007.25

	-50%	3413.21	5.13224	1007.25
q	-25%	3392.59	5.13224	1007.25
	0%	3371.97	5.13224	1007.25
	+25%	3351.34	5.13224	1007.25
	+50%	3330.72	5.13224	1007.25

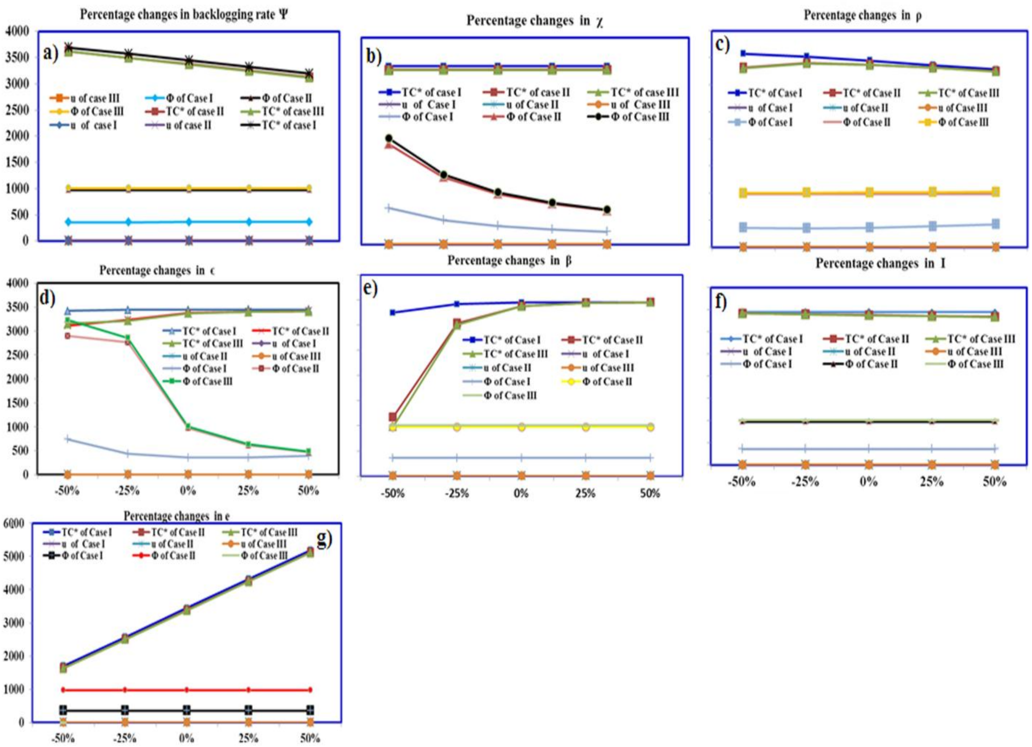

Figure7: Graphical representation of different parameters

Discussion and Observation

- As the backlogging parameter's (Ψ) value increases, across all cases, the total average cost TAC* decreases, and with the decreasing value of backlogging parameter TC of all three cases increases. Also decrement has been seen in replenishment cycle time (u) in case 1, while in case 2(a) and case 2(b) it is slightly unchanged. On the other hand, preservation technology costs (Φ) increased in case 1, and were unchanged in cases 2(a) and 2(b). While decreasing value of Ψ, preservation technology costs (Φ) deceases in case 1 but remains same in Case 2(a) and Case 2(b) which could more clarify by Fig. 7(a) and Table 3,4 and 5.
- When the value of the deterioration rate (x) increases, it is clear that, TAC* in cases 1 and case 2(a) unchanged and slightly increased in case 2(b). x(deterioration rate) decreases, TC* is unchanged in cases 1 and 2 (a), slightly decreases in case 2 (b). While on increasing or deterioration rate, there are slight decreases or increases in u in cases II and III, and no changes in u (replenishment cycle time) in case I. And the cost of preservation technology (Φ) of case 1, case II and case III increases or

decreases when value of deterioration parameter increases or decreases which could more clarify by Fig. 7(b)and Table 3,4 and 5.

- When the value of the selling price ρ rises, then it can be concluded that the total cost TAC of all three cases decreases due to the consumption of products. The replenishment cycle time and preservation investment cost increase and decrease in case1 and case 2 (b), respectively. While in case 1 and case 2 (b), preservation technology cost increase or deceases which could be more clearly seen by Figure 7(c) and Table 3,4 and 5.
- As scaling parameters (e) value increases or decreases, total cost TAC of all three cases increases or decreases respectively. While the replenishment cycle time (u) and preservation investment cost of all three cases remain fixed, Fig. 7(d)and Table 3,4 and 5.
- If the values of the scaling parameters β are increased or decreased, then the TC of all three cases increases or decreases accordingly. Total cost of all cases and all cases of replenishment cycle recorded no changes as shown in Fig 7 (e)and Table 3,4 and 5.
- If the values of the scaling parameters β are increased or decreased, then the TAC of all three cases increases or decreases accordingly. Total cost of all cases and all cases of replenishment cycle recorded no changes as shown in Fig 7 (e) and Table 3,4 and 5.
- In all cases, as the value of q increases or decreases in value, there are corresponding decreases or increases in all three total cost cases. All three cases of replenishment cycle and preservation investment remains unchanged as shown in Fig 7 (f) and Table 3,4 and 5.

Conclusion and future work

This study contributes to combining uncertain demand and preservation technologies and establishing a Fuzzy EOQ model for deteriorating healthcare products in the pharmaceutical industry under an inflationary environment with partial shortages. Realistically, demand uncertainty occurs when a business or trade seems unable to effectively estimate customer demand for its products. Therefore, in this study, the uncertain demand factor is dependent on stock and price dependent and also the effect of inflation with preservation technology is shown. Two cases of trade credit policy exist, which are dependent on the time limit given to the buyer. Finally, a numerical example is given to validate the proposed model, and a sensitivity analysis is carried out to analyse the effect of different factors graphically. The signed distance method is used as a defuzzifier to the different fuzzy cost parameters, and a fuzzy trapezoidal number is applied to the fuzzy purchasing cost, cost of deterioration, carrying cost, shortage cost, lost sale cost, and gained interest rate. The preliminary results indicate that on increasing the fuzzy selling price of pharmaceutical items, the total cost of case 2(b) decreases with preservation technology cost and a slight decrease in the cost of replenishment cycle. This model can be further extended by considering other factors such as imperfect quality, inflation, green inventory, new demand functions, and different fuzzy numbers.

Abbreviation

Fuzzy economic production quantity model : FEOQ

Fuzzy economic production quantity : FEPQ
Economic order quantity : EPQ

References

1. Buzacott, J.A., Economic order quantities with inflation, 'Operational Research Quarterly, (1975), 26(3), 553–558. https://doi.org/10.1057/jors.1975.113
2. Chang, H-C., Yao, J-S., Ouyang, L-Y., Fuzzy mixture inventory model involving fuzzy random variable lead time demand and fuzzy total demand, European Journal of Operational Research (2006), 169(1), 65–80. https://doi.org/10.1016/j.ejor.2004.04.044
3. Chen, L.H., Ouyang, L.Y., Fuzzy inventory model for deteriorating items with permissible delay in payment, Applied Mathematics and Computation, (2006),182(1), 711–726. https://doi.org/10.1016/j.amc.2006.04.035
4. Debnath, B.K., Majumder, P., Bera, U.K., A FEPQ model of sustainable items with time and stock dependent demand under trade credit policy, International Journal of Operational Research, (2021) 41(1), 27-52.
5. Dey, O., Chakraborty, D., Fuzzy periodic review system with fuzzy random variable demand, European Journal of Operational Research, (2009), 198(2), 113-120. https://doi.org/10.1016/j.ejor.2008.07.043
6. Dutta, P., Chakraborty, D., Roy, A.R., Continuous review inventory model in mixed fuzzy and stochastic environment, Applied Mathematics and Computation, (2007) 188(1), 970–980. https://doi.org/10.1016/j.amc.2006.10.052
7. Gani. S. Maheswari., Supply chain model for the retailer's ordering policy under two levels of delay payments in fuzzy environment, Applied Mathematical Sciences, (2010) 4(22), 1155-1164.
8. Garai, T., Chakraborty, D., Roy, T.P., Fully fuzzy inventory model with price-dependent demand and time varying holding cost under fuzzy decision variables, Journal of Intelligent and Fuzzy Systems, (2019) 36(4), 3725–3738.
9. Soni, H.N., Joshi, M., A Periodic Review Inventory Model with Controllable Lead Time and Backorder Rate in Fuzzy Stochastic Environment, Fuzzy Information and Engineering, (2014) 7(1), 101-114.
10. Hossen, M.A., Hakim, M.A., Ahmed, S.S., Harifuddin, M., An inventory model with price and time dependent demand with fuzzy valued inventory costs under inflation, Annals of Pure and Applied Mathematics, (2016) ,11, 21–32.
11. Indrajitsingha, S.K.,Samanta, P.N., Misra, U.K., A fuzzy inventory model for deteriorating items with stock dependent demand rate, International Journal of Logistics Systems and Management, (2018) 30(4), 538–.555. https://doi.org/10.1504/IJLSM.2018.093588
12. Jaggi,C. K., Yadavalli,V. S. S., Sharma, A., Tiwari, S., A Fuzzy EOQ Model with Allowable Shortage under Different Trade Credit Terms, Applied Mathematics & Information Sciences, (2016) 10(2), 785-805. doi:10.18576/amis/100239
13. Jaggii, C.K., Pareek, S., Sharma, A., Nidhi. Fuzzy Inventory Model for Deteriorating Items with Time-varying Demand and Shortages, American Journal of Operational Research, (2012) 2(6),.81-92. doi: 10.5923/j.ajor.20120206.01

14. Kao, C., Hsu, W.K., Lot size-reorder point inventory model with fuzzy demand, Computers and Mathematics with Applications, (2002), 43(10), 1291–1302. https://doi.org/10.1016/S0898-1221(02)00101-3
15. Kumar, B.A., Paikray,S. K., Mishra,S., and Routray,S. A Fuzzy Inventory Model of Defective Items Under the Effect of Inflation with Trade Credit Financing, Recent Advances in Intelligent Information Systems and Applied Mathematics, (2020) 863, 804-821. https://doi.org/10.1007/978-3-030-34152-7_62
16. Kumar, S. De., Mahata, G. C., Maity, S., Carbon emission sensitive deteriorating inventory model with trade credit under volumetric fuzzy system, International Journal of Intelligent Systems, (2021), 36(12),7563-7590. https://doi.org/10.1002/int.22599
17. Kuraie, V.C., Padiyar, S.V.S., Bhagat, N., Rajput, N., Imperfect Production Inventory Model with Selling Price Dependent Demand Rate and Reliability under the Inflationary and Fuzzy Environment, International journal of Engineering Science and Computing, (2021) 11(12), 29109-29116.
18. Lee, H.M., Yao, J.S. Economic production quantity for fuzzy demand quantity and fuzzy production quantity, European Journal of Operational Research, (1998), 109(1), 203–211. https://doi.org/10.1016/S0377-2217(97)00200-2
19. Liao, H.C., Tsai, C.H., Su, C.T., An inventory model with deteriorating items under inflation when a delay in payment is permissible, International Journal of Production Economics, (2000), 63(2), 207- 214. https://doi.org/10.1016/S0925-5273(99)00015-8
20. Lin, D.C., Yao. J.S. Fuzzy economic production for production inventory, Fuzzy Sets and Systems, (2000) 111(3),465–495. https://doi.org/10.1016/S0165-0114(98)00037-2
21. Mahata, G. C., Goswami, A., An EOQ model for deteriorating items under trade credit financing in the fuzzy sense, Production Planning & Control: The Management of Operations, (2007) 18(8), 681-692. https:// doi=10.1.1.672.4646
22. Maragatham, M., Lakshmidevi, P.K., A Fuzzy Inventory Model for Deteriorating Items with Price Dependent Demand, International Journal Fuzzy Mathematical Archive (2014) ,5(1), 39-47.
23. Mishra, S., Misra, G.,Misra, U.K., Raju, L.K., An inventory model with quadratic demand pattern and deterioration with shortages under the influence of inflation, Mathematical Finance Letters, (2012), 1, 57–67. https://doi.org/10.1504/IJSAMI.2021.118124
24. Padiyar, S.V.S., Singh, S.R., Punetha, N. Inventory system with price dependent consumption for deteriorating items with shortages under fuzzy environment,' International Journal of Sustainable Agricultural Management and Informatics, (2021) 7(3), 218-231. doi.org/10.1016/j.matpr.2022.02.273.
25. Poswal, P., Chauhan, A., Boadh, B., Rajoria, Y.K., Kumar, A. and Khatak, N., Investigation and analysis of fuzzy EOQ model for price sensitive and stock dependent demand under shortages, Materials Today: Proceedings, (2022), 56(1), 542-548. doi.org/10.1016/j.matpr.2022.02.273.

26. Rajput, N., Chauhan, A.and Pandey, V. EOQ model with a discount rate of inflation and optimization with pentagonal fuzzy number, International Journal of Mathematics in Operational Research, (2021), 20(2). https://doi.org/10.1016/j.apm.2007.12.015
27. Roy, Ajanta. Fuzzy inventory model for deteriorating items with price dependent demand, International Journal of Management Science and Engineering Management, (2015) 10(4), 237–241. https://doi.org/10.1080/17509653.2014.959086
28. Roy, A.,Maiti, M.K., Kar, S., Maiti,M. An inventory model for a deteriorating item with displayed stock dependent demand under fuzzy inflation and time discounting over a random planning horizon, Applied Mathematical Modeling, (2009) ,33(2), 744–759.
29. Sarkar,B., Mahapatra, A.S. Periodic review fuzzy inventory model with variable lead time and fuzzy demand, International Transactions in Operational Research (2015) 2015,1–31. https://doi.org/10.1111/itor.12177
30. Shaikh, A.A., Bhunia, A.K., Cardenas-Barron LE., Sahoo., L. and Tiwari, S., A fuzzy inventory model for a deteriorating item with variable demand, permissible delay in payments and partial backlogging with shortage follows inventory (SFI) policy, International Journal of Fuzzy Systems, (2018),20(5), 1606–1623. https://doi.org/10.1007/s40815-018-0466-7
31. Singh, D., Kumar, A., Two-warehouses partial backlogging inventory model for deteriorating items with selling price dependent demand under trade credit in the inflationary environment, International Journal of Pure and Applied Mathematics, (2018), 118(2),1447–1457. https://doi.org/10.1016/j.fiae.2015.03.007
32. Uthayakumar, R., and Palanivel, M., An inventory model for defective items with trade credit and inflation, Production & Manufacturing Research, (2014) 2(1), 355–379. https://doi.org/10.1080/21693277.2014.893414
33. Zadeh, L. A., Fuzzy sets, Information and Control, (1965), 8(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X