How to Cite:
Rani, M. J., & Bhavani, R. (2022). On Micro- I *g – closed sets in micro ideal topological spaces. *International Journal of Health Sciences*, *6*(S4), 2669–2674.
https://doi.org/10.53730/ijhs.v6nS4.7792

On Micro- I *g – closed sets in micro ideal topological spaces

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***Abstract*---**The main concept of this paper is to discuss the micro topology as an unadorned extension of nanotopology. Nano topology offers a wide variety of interesting results and applications. But for some time we have been looking for extended sets in micro topological space. Then some class of Micro**-** I *gene –Closed sets is initiated and its essentials characteristics are investigated.

***Keywords*---**Mic-g*closed, Mic-$* \, gene - C(X)(closed\ sets)$, Mic- I$* \, gene -$ C(X), $Mic - {I^*}_g$ –C(X)- Mic-sgI-C(X), Mic-Is*gene-C(X) and $Mic - {I^*}_g$ – O(X)(open sets).

Introduction

S. Jaffari, N. Rajesh [1] proposed generalized closed sets for an ideal in 2011.In 1990, D. Jankovic and T. R. Hamlet, Amer [2] introduced new topologies through old-fashioned ideals. Kuratovsky [3] in 1933 and Vaidyanathaswamy [3] in 1944 proposed the concept of the ideal topological space(IDTS). In 1988 Hamlet and Jankovic [4] defined the generalization of some important features of the IDTS. O. Ravi, S. Thurmer, M.Sangeetha and J. Anthony Rex Rodrigo [5] explored * g-closed sets in IDTS in 2013. In 2019, Wade Al-Omeri and M. Abu-Saleem [6] proposed I * g-C(X) through IDTS. O. Ravi, M. Paranjothi, S. Murugesan, M. Mehr [7] initiated g * -C(X) in IDTS in 2014. Nano topology was proposed and defined by Lellis Tiwager [8] in 2013. Between the approximations of the subset of the universe and the boundary region it uses an equivalence relationship. But sometimes we want to expand some open sets in nano topology. In 2019.Introduced by S. Sakraviranan Chandrasekhar [9] on Micro topological Spaces(MTS), (2019). S. Selvaraj Ganesan [10] defined MGCS and micro

Manuscript submitted: 27 March 2022, Manuscript revised: 9 April 2022, Accepted for publication: 18 May 2022

generalized continues in MTS.S.Ganesan [11,12] established a new concept of micro topological space through micro ideals. The methodology proposed in this paper $Mic - I^*{}_g$–C(X), Mic-sgI-C(X), Mic-Is*gene-C(X) and $Mic - I^*{}_{gene}$ -O(X) are IDTS in micro and Some of their features will also be investigated.

Preliminaries

Definition 2.1

A subset A of a space $(\widetilde{\boldsymbol{U}}, \widetilde{\boldsymbol{\tau_R(X)}}, \widetilde{\boldsymbol{\mu_R}}(\boldsymbol{X}),$ **ID)** is impart to be

i) generalized –ID(ideal)-C(X) if $A^{*S} \subseteq \tilde{V}$,A⊆ $\tilde{V}$, and $\tilde{V}$ is O(X) in ($\widetilde{\boldsymbol{U}}$, τ(X)).

ii)*gene ID-C(X) if $A_* \subseteq \tilde{V}$, A ⊆ $\tilde{V}$, and $\tilde{V}$ is $\hat{g}$ –O(X) in ($\widetilde{\boldsymbol{U}}$, τ(X)).

iii) $ID *$ generalized –C(X) if $A^* \subseteq \tilde{V}$, A⊆ $\tilde{V}$,and $\tilde{V}$ is $\hat{g}$ –O(X) in ($\widetilde{\boldsymbol{U}}$, τ(X)).

iv) $ID^*{}_{gene}$-C(X) if Clr^*(A)⊂ $\tilde{V}$,A⊆ $\tilde{V}$, and $\tilde{V}$is gene-O(X) in ($\widetilde{\boldsymbol{U}}$, τ(X)).

Definition 2.2

i)Mic-generalized- C(X) set if Mic-clr(A) ⊆ $\tilde{V}$ when so ever A⊆ $\tilde{V}$,and $\tilde{V}$ is Mic-O(X)(open) in $\widetilde{\boldsymbol{U}}$.

ii) Mic-semi-generalized-C(X) set if Mic-sclr(A) ⊆ $\tilde{V}$ when so ever A ⊆ $\tilde{V}$ and $\tilde{V}$ is Mic semi-O(X) in $\widetilde{\boldsymbol{U}}$.The supplement of Mic-semi-generalized-C(X) set is called Mic- semi-g-O(X) set.

iii) Mic-gs-C(X) set if Mic-sclr(A) □ $\tilde{V}$ whensoever A □ $\tilde{V}$ and $\tilde{V}$ is Micro-open in $\widetilde{\boldsymbol{U}}$. The completion of Mic-gene-semi-C(X) set is called Mic-gene-semi-O(X) set.

Micro- I *g -Closed Sets

Definition 3.1

A subset A of a Micro ideal(ID) space ($\tilde{U}$,$\widetilde{\tau_R(X)}$, $\widetilde{\mu_R}(X)$, ID) (ideal)) is impart to be $MIC - I^*{}_g$-closed if $MIC - Clr^*$(A)⊂ $\tilde{V}$,A ⊆ $\tilde{V}$, and $\tilde{V}$ is Mic- gene-O(X) in ($\tilde{U}$,$\widetilde{\mu_R}(X)$) isdenoted by MIC –G^*C(X).The supplement of $MIC - I^*{}_{gene}$-C(X) is reveal to be $MIC - I^*{}_{gene}$-open is denoted by MIC –G^*O(X).

Definition 3.2

A subset A of a Micro-ID- space ($\tilde{U}$,$\widetilde{\tau_R(X)}$, $\widetilde{\mu_R}(X)$, ID) is reveal to be

$i)Mic - I * gene$-C(X) if $Mic - A^* \subseteq \tilde{V}$,A⊆ $\tilde{V}$, and $\tilde{V}$ is Mic- $\hat{g}$ –O(X) in ($\widetilde{\boldsymbol{U}}$, $\widetilde{\mu_R}(X)$).The supplement of $Mic - I * gene$-C(X) set is said to be $Mic - I * gene$-O(X).

ii)$Mic - I_{s*gene}$ –C(X) if MIC- $A^* \subseteq \tilde{V}$,A ⊆ $\tilde{V}$, and $\tilde{V}$ is Mic-se-O(X) in ($\widetilde{\boldsymbol{U}}$, $\widetilde{\mu_R}(X)$).The accompaniment of $Mic - I_{s*g}$ –C(X) is said to be $Mic - I_{s*g}$ –open.

iii)Mic- sgI-C(X) if MIC- $A^{*S} \subseteq \tilde{V}$,A⊆ $\tilde{V}$, and $\tilde{V}$ is Mic- semi O(X) in ($\widetilde{\boldsymbol{U}}$, $\widetilde{\mu_R}(X)$).The supplement of Mic- sgI-C(X)set is said to be $Mic -$ sgI-O(X).

iv)MIC- g^{*S} I –C(X) if MIC- $A^{*S} \subseteq \tilde{V}$,A ⊆ $\tilde{V}$, and $\tilde{V}$ is MIC- $\hat{g}$ O(X) in ($\widetilde{\boldsymbol{U}}$, $\widetilde{\mu_R}(X)$).

Remark 3.3 Let ($\tilde{U}$,$\widetilde{\tau_R(X)}$, $\widetilde{\mu_R}(X)$, ID) be an MIDTS and A a subset of $\tilde{U}$.

(1) If I=P($\tilde{U}$), then MIC- Clr^*(A)=A for all A ⊂ $\tilde{U}$ and hence, each subset of $\tilde{U}$ is $MIC - I^*{}_g -$ C(X)

(2) Since $MIC - A^{*=} \varphi$ for every member of I is MIC- $I^*{}_g$ –C(X) .

(3) Every MIC - I *gene –C(X) set is MIC- I$*$ $gene$-C(X) set .

Theorem: 3.4

Each MIC - $*$C(X) (closed) is a MIC -I *gene – C(X) set .

Proof:
Enable A be a MIC-⋆-C(X),then MIC-A* $\subseteq$ A. Then A $\subset$ $\tilde{V}$ where $\tilde{V}$ is MIC-O(X).We have MIC-$Clr^{*}(A) \subseteq \tilde{V}$ whensoever A $\subseteq$ $\tilde{V}$ and $\tilde{V}$ is MIC –gene- O(X).

Example: 3.5
Let $\widetilde{U}$={ l,m,n,o} with $\frac{\widetilde{U}}{R}$(X)={{ l,n},{q},{o}} and X={m,o}⊂ $\widetilde{U}$
Then $\widetilde{\tau_R(X)}$={ φ, $\widetilde{U}$,{m,o}} and μ={ l }.Micro topology $\widetilde{\mu_R}(X)$={φ, $\widetilde{U}$,{ l},{m,o},{ l,m,o}} and I={ $\emptyset,\{n\}$}. Let S={m,n},{n,o},{ l,m,n}and{ l,n,o} is MIC -I *g –C(X)set but S is not
MIC-∗C(X).
Theorem: 3.6
Enable ($\widetilde{U}$,$\widetilde{\tau_R(X)}$, $\widetilde{\mu_R}(X)$, ID) be a Micro –ID - topological space (MIDTS).Then Every
MIC -I^{*}gene-C(X) set is a MIC –gene-C(X) set.
Proof:
Let A be a MIC-I^{*}gene- C(X) set.Then MIC-Clr^{*}(A) ⊆ $\tilde{V}$ and A $\subset$ $\tilde{V}$ *where* $\tilde{V}$ is MIC-gene- O(X) .We have MIC – Clr(A) ⊆ $\tilde{V}$ when so ever A ⊆ $\tilde{V}$ and $\tilde{V}$ is MIC –O(X) in
($\widetilde{U}$, $\widetilde{\mu_R}(X)$).
Example: 3.7
Enable $\widetilde{U}$={$\tilde{e}$, $\tilde{f}$, $\tilde{g}$, $\tilde{h}$} with $\frac{\widetilde{U}}{R}$(X) ={{$\tilde{e}$},{$\tilde{f}$,$\tilde{h}$},{$\tilde{g}$}}and X={$\tilde{e}$, $\tilde{f}$}.Then
$\widetilde{\tau_R(X)}$={$\emptyset$, $\widetilde{U}$,{ $\tilde{e}$ },{$\tilde{f}$, $\tilde{h}$},{$\tilde{e}$, $\tilde{f}$, $\tilde{h}$},μ={$\tilde{g}$}, $\widetilde{\mu_R}(X)$,={φ, $\widetilde{U}$,{ $\tilde{e}$},{$\tilde{g}$},{$\tilde{e}$, $\tilde{g}$}{$\tilde{e}$,$\tilde{f}$, $\tilde{h}$}{$\tilde{f}$, $\tilde{h}$}, {$\tilde{f}$, $\tilde{g}$, $\tilde{h}$} and I={ φ,{ $\tilde{f}$, $\tilde{h}$}}.Let S={$\tilde{e}$, $\tilde{f}$},{$\tilde{e}$, $\tilde{f}$, $\tilde{g}$},{$\tilde{e}$, $\tilde{g}$, $\tilde{h}$} and S= {$\tilde{f}$, $\tilde{g}$} is MIC-gene-
C(X) yet S is not MIC -I^{*}gene- C(X) set.
Theorem: 3.8
Let ($\widetilde{U}$,$\widetilde{\tau_R(X)}$, $\widetilde{\mu_R}(X)$, ID(ideal)) be a MIDTS. Then Each MIC –g*-C(X) is a MIC -I^{*}gene-
C(X) set yet not conversely.
Proof:
Enable A be a MIC –g* -C(X) .Then MIC – Clr(A) ⊆ $\tilde{V}$, $\tilde{V}$ is MIC – gene –O(X) We have
MIC-Clr^{*}(A) ⊆ $\tilde{V}$, A ⊆ $\tilde{V}$ and $\tilde{V}$ is MIC-gene-O(X) in ($\widetilde{U}$, , $\widetilde{\mu_R}(X)$).
Example: 3.9
Let $\widetilde{U}$={i,j,k, l} with $\frac{\widetilde{U}}{R}$(X) ={{i},{j, l},{k}}and X={i, l}.Then μ={k},
$\widetilde{\mu_R}(X)$={φ, $\widetilde{U}$,{i},{k},{i,k}{i,j, l}{j, l},{j,k, l}}and I={ φ,{j, l }}.Let S={j},{ l} is
MIC -I^{*}gene- C(X) but S is not MIC-g*-C(X).

Theorem: 3.10
Each MIC-I$s^{*}gene$ – C(X) is a MIC-semi-gene-I –C(X) .
Proof:
Enable A be a MIC-I$s^{*}gene$ – C(X) .Then MIC-A*⊆ $\tilde{V}$,A ⊆ $\tilde{V}$ and $\tilde{V}$ is MIC –se - O(X). $MIC - A^{*s} \subseteq \tilde{V}$ when so ever A ⊆ $\tilde{V}$ and $\tilde{V}$ is MIC-semi –O(X).
Example: 3.11
Let $\widetilde{U}$={m,n,o,p}with $\frac{\widetilde{U}}{R}$(X)={{m,o},{n},{p}} and X={n,p}.Then $\widetilde{\tau_R(X)} = \left\{\varphi, \widetilde{U}, \{n,p\}\right\}$, μ={m}, $\widetilde{\mu_R}(X)$={ $\varphi, \widetilde{U}, \{m\}, \{n,p\}, \{m,n,p\}$} and I=($\emptyset, \{o\}$}.Let S={m} is MIC-sgI –C(X) but not MIC- I$s^{*}gene$ – C(X).

Example: 3.12

Let $\widetilde{U}$={n,o,p,q} with $\frac{\widetilde{U}}{R}$(X)={{n,p},{o},{q}}.Then Nano Topology $\widetilde{\tau_R(X)}$={$\emptyset,\widetilde{U}$,{o,q}}, μ={n},$\widetilde{\mu_R}(X)$={ $\emptyset,\widetilde{U}$,{n},{o,q},{n,o,q}},I={$\emptyset$,{p}}.Then A={n} is MIC-g^{*S}I-closed but not MIC- I*gene- C(X).Then MIC -A*={n,p},MIC - A_* ={n} where $MIC-A_*$ is contained in the MIC-gene-O(X),U={n} but MIC-A*⊈U.MIC-I*gene-C(X), since MIC-A*=$MIC-A_*$ yet not MIC-Igene-C(X) .

Theorem : 3.13

Enable ($\widetilde{U}$,$\widetilde{\tau_R(X)}$, $\widetilde{\mu_R}(X)$, ID (ideal)) be an discrete space and MIDTS, $x_0 \epsilon \widetilde{U}$, and I={$\phi$, {$x_0$}}.

Then A ⊆ $\widetilde{U}$,A≠ {x_0} if MIC-A*= $\tilde{V}$ and A={x_0} if MIC-A*= ∅.For any subset A≠ {x_0} is MIC-Igene-C(X), MIC-gI-C(X),MIC-sgI-C(X) and MIC-Is*gene – C(X) yet not MIC-I*gene- C(X).

Example: 3.14

Enable $\widetilde{U}$={$\breve{a}$, $\breve{b}$, $\breve{c}$}with $\frac{\widetilde{U}}{R}$(X)={{ $\breve{a}$},{$\breve{b}$, $\breve{c}$}}.Then $\widetilde{\tau_R(X)}$={ $\emptyset,\widetilde{U}$,{ $\breve{b}$, $\breve{c}$}},μ={$\breve{a}$}∉ $\widetilde{\tau_R(X)}$ and $\widetilde{\mu_R}(X)$={ $\emptyset,\widetilde{U}$,{ $\breve{a}$},{$\breve{b}$, $\breve{c}$}}and I={∅,{$\breve{b}$, $\breve{c}$}}.Let S={$\widetilde{b}$}, {$\breve{c}$} is MIC-Igene-C(X), MIC-gI-C(X),MIC-sgI-C(X) yet not MIC-I*gene-C(X).

Theorem: 3.15

If ($\widetilde{U}$,$\widetilde{\tau_R(X)}$, $\widetilde{\mu_R}(X)$, ID (ideal)) be an MIDTS and A ⊂ $\widetilde{U}$. Then the following are true.

(a) A is MIC-ID*gene (generalized)-C(X),
(b) MIC-Clr*(A) ⊆ $\tilde{V}$ if A ⊆ $\tilde{V}$ and $\tilde{V}$ is MIC-gene-O(X) in $\widetilde{U}$,
(c) $\widetilde{U}$ ∈ MIC-Clr*(A), MIC-gClr ({$\widetilde{U}$}) ∩ A ≠ ∅,
(d)MIC- Clr*(A) - A does not contain a MIC-gene-C(x)(closed set) and
(e) MIC-A*- A does not contain a MIC- gene-C(X) set.

Proof.

(a)⟹ (b): If A is MIC-I*gene-C(X), then MIC-A*⊂ $\tilde{V}$ if A ⊆ $\tilde{V}$ and $\tilde{V}$ is MIC-gene-O(X) in $\widetilde{U}$ and so MIC-Clr*(A)=A ∪ MIC −A*⊆ $\tilde{V}$ if A ⊆ $\tilde{V}$ and $\tilde{V}$ is MIC-gene-O(X) in $\widetilde{U}$. This proves (b).

(b) ⟹ (c): Suppose $\tilde{x}$ ∈ MIC −Clr*(A).If MIC-gClr*({$\tilde{x}$}) ∩ A=∅,then A⊆ $\widetilde{U}$-MIC-gClr({$\tilde{x}$}). By(b),MIC-Clr*(A) ⊆ $\widetilde{U}$-MIC-gClr({$\tilde{x}$}),a contradiction since $\tilde{x}$ ∈ MIC −Clr*(A).

(c) ⟹ (d): Suppose A ⊆ MIC-Clr*(A)-A, A is MIC-gene-C(X) , and $\tilde{x}$ ∈A.Since A⊆ $\widetilde{U}$-MIC-Clr*(A),by (3),MIC-gClr({$\tilde{x}$})∩ A ≠ ∅.Thus,MIC-Clr*(A)-A does not contain an MIC-gene-C(X).

(d) ⟹ (e):We know MIC-Clr*(A)-A = (A∪ $MIC-A$*)-A=(A∪ $MIC-A$*)∩ A^c = (A∩ A^c) ∪ (MIC-A*∩ A^c)= MIC-A*∩ A^c= MIC-A*-A.So ,MIC-A*-A does not contains a MIC-gene-C(X) set.

(e) ⟹ (a): Let A ⊆ $\tilde{V}$,where $\tilde{V}$ is MIC-gene-O(X) set.Thus, $\widetilde{U}$-V⊆ $\widetilde{U}$-MIC-A and is a MIC-A*∩ ($\widetilde{U}$- $\tilde{V}$) ⊆ $MIC-A^*$ ∩ ($\widetilde{U}$-A)=MIC-A*-A.So, MIC-A*∩($\widetilde{U}$- $\tilde{V}$) ⊆ MIC-A*-A. Since MIC-A* is always a C(X) ,MIC-A* is a MIC-gene-C(X) and so MIC-A* ∩ ($\widetilde{U}$-$\tilde{V}$)=∅ and ,So MIC-A*⊆ $\tilde{V}$.Therefore, A is MIC- I*gene-C(X) .

Theorem 3.16

If $(\widetilde{U},\widetilde{\tau_R(X)},\widetilde{\mu_R}(X)$, ID) is any MITS, subsequently MIC-A* is always MIC-I *gene – C(X) for every subset A of $\widetilde{U}$.

Proof:

Enable MIC-A*⊆ $\widetilde{V}$, where $\widetilde{V}$ is MIC-gene-O(X). Since (MIC-A*)*⊆ $MIC-A^*$, we have (MIC-A*)*⊆ $\widetilde{V}$ if MIC -A*⊂ $\widetilde{V}$ and $\widetilde{V}$ is MIC-gene-O(X) . Thus, MIC-A* is MIC- ${I^*}_g$– C(X) . **Theorem 3.17**

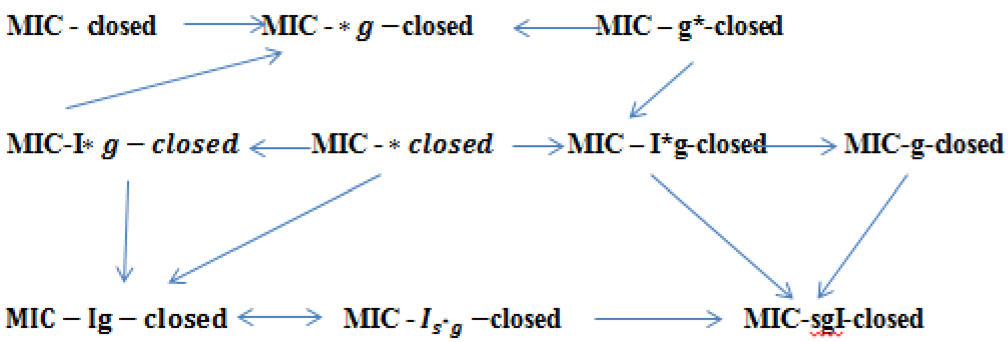
If $(\widetilde{U},\widetilde{\tau_R(X)},\widetilde{\mu_R}(X)$, ID) be an Micro-ID(ideal)-space. Afterwards each MIC -$*$-C(X) set is a MIC- ${I^*}_{gene}$ –C(X), MIC-gene – O(X) set.

Proof:

We have A is. MIC - $*$-C(X).Then MIC-Clr*(A) ⊆ V when so ever A⊆ A and A is MIC-gene -open. Thus , A is MIC- ${I^*}_{gene}$ –closed.

Lemma 3.18

Enable $(\widetilde{U},\widetilde{\tau_R(X)},\widetilde{\mu_R}(X)$, ID) be an Micro – ID - space and A ⊆ $\widetilde{U}$.If A ⊆ MIC-A*,then MIC-A*=MIC-Clr(A*)=MIC-Clr(A)=MIC-Clr*(A).



Conclusion

Here the methodology propose Micro topological spaces , We Contemplated new type of Micr-gene-O(X), Micr- gene-C(X), Micr-g*-C(X) and Micr- $*\,gene$ –C(X) in MTS and discuss about the properties and applications of its. This Paper was introduced Micr-ID*gene-O(X), Micr-ID*gene-C(X), Micr-ID $*$gene-C(X), Micr-sgI-C(X) and Micr-Is^*gene-C(X) in MIDTS and investigated certain of the basic characters in MIDTS Various interesting problems put the identified in the analyses of openness. The future Research shall be explored with respect to the applications of Micro Topological Spaces.

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