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A mathematical model for prolactin secretion in healthy adults using lognormal distribution

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***Abstract*---**The concentration of prolactin in healthy persons is the focus of this investigation. The concentration of serum PRL in blood is determined by gender based. Traditional employment, sleeping patterns, psychological stress, and medicine are maintained for both genders. As a result of the research, the concentrations of PRL in men and women are linked. Log-Normal distributions were employed in the Mathematical Model. The medical results were analysed using the mathematical model that corresponded to them.

***Keywords*---**Log – Normal Distribution, Psychological Stress, traditional employment.

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Introduction

The probability distribution of a random variable whose logarithm follows a normal distribution is known as the log-normal distribution. It simulates phenomena whose relative growth rate is independent of size, as is the case with most natural phenomena such as tissue size and blood pressure, wealth distribution, and even chess game duration[1].The distribution may also be seen in seemingly odd places, such as the number of moves it takes to conclude a chess game[2].The lognormal distribution takes on numerous forms depending on the values of its parameters, including a bell-curve similar to the normal distribution[3].The lognormal distribution may also be used to describe data that would otherwise be labeled normally distributed but is more or less skewed[4].

Mathematical Model

The lognormal distribution is important in the description of natural phenomena[5]. Leema, Manickam and Mamtasay that the log – normal distribution is often a plausible and empirically adequate model[6].A positive random variable Y is log – normally distributed, if the natural logarithm of Y is normally distributed with mean π and variance δ^2.[7]If $Y = \log(H)$ has a normal distribution with probability density function given by

$$f_Y(y) = \frac{1}{(\delta^2 2\mu)^{1/2}} \exp\left[-\frac{(y-\pi)^2}{2\delta^2}\right] \text{ where } -\infty < y < +\infty$$

That is if $H \sim normal(\pi, \delta^2)$ then $H = e^Y$ has a lognormal distribution with pdf

$$f_H(h) = \frac{1}{h(\delta^2 2\mu)^{1/2}} \exp\left[-\frac{(logh - \pi)^2}{2\delta^2}\right]$$

Therefore $Y \sim lognormal(\pi, \delta^2)$

Stochastic differential Equations (SDE):

An SDE model for the growth of a single human can be written as

$$H(t)' = H(t)g\big(H(t)\big)t' + \delta H(t)W(t)'$$

The quantity $H(t)$ becomes a diffusion process having continuous sample paths with probability one[8]. Two functions infinitesimal mean and variance are defined respectively as

$$m_H(h) = \lim_{p\to 0}\frac{1}{p}E[\Delta H \mid H(t) = h]$$

$$v_H(h) = \lim_{p\to 0}\frac{1}{p}E[(\Delta H)^2 \mid H(t) = h]$$

Where $\Delta H = H(t+p) - H(t)$.
Therefore

$$m_H(h) = hg(h) + \omega h$$

$$v_H(h) = \delta^2 h^2$$

Here ω is an indicate variable which depends on the type of stochastic integral[9].

One of the most useful features of diffusion processes is the transformation property. If $H(t)$ is a diffusion, then $Y(t) = p\big(H(t)\big)$ is also a diffusion process, provided p is a continuous, strictly increasing function[10]. The infinitesimal mean and variance are given by

$$m_Y(y) = \frac{v_H(h)p''(h)}{2} + m_H(h)p'(h)$$
$$v_Y(y) = v_H(h)[p'(h)]^2$$

This characteristic frequently allows a novel diffusion process to be transformed into a recognised process with well-studied statistical aspects[11].
A well – known result gives a normal transition distribution for $Y(t)$:

$$Y(t){\sim}normal(y_0 + \left(r + \omega - \frac{\delta^2}{2}\right)t, \delta^2 t)$$

Equivalently, the distribution of $H(t)$ becomes lognormal

$$H(t){\sim}\ lognormal(yh_0 + \left(r + \omega - \frac{\delta^2}{2}\right)t, \delta^2 t)$$

Comparing various measures of tendency with ODE $\frac{(z(t))'}{t'} = rh(t)$is given by

$$h(t) = \ h_0 e^{rt}$$

the mean, Geometric mean and Hormonic mean of $H(t)$ are given respectively

$$E[H(t)] = \ h_0 e^{(r+\omega)t}$$
$$exp\{E[\log H(t)]\} = \ h_0 e^{(r+\omega-(\delta^2/2))t}$$
$$1/E\left[\frac{1}{N(t)}\right] = h_0 e^{(r+\omega-\delta^2)t}$$

The expectations are conditioned on $H(t) = h_0$. Also the median and mode of the distribution of $H(t)$ are:

$$median\big(H(t)\big) = \ h_0 e^{(r+\omega-(\delta^2/2))t}$$
$$mode\big(H(t)\big) = \ h_0 e^{(r+\omega-(3\delta^2/2))t}$$

For lesser values$r$, certain central tendency measurements rise exponentially, while others decay to zero[12]. Obviously if $r < (\delta^2/2) - \omega$, the probability that $H(t)$ is arbitrarily close to zero and approaches 1 as t becomes large[13].

$$P[0 < H(t) \le \varepsilon] = P[-\infty < Y(t) \le log\varepsilon]$$
$$= P\left\{-\infty < \frac{Y(t) - E[Y(t)]}{[var(Y(t))]^{1/2}} \le \frac{log\varepsilon - E[Y(t)]}{[var(Y(t))]^{1/2}}\right\} \to 1$$

As $t \to \infty$. It is the probability that a standard normal random variable is less than or equal to $[log\varepsilon - y_0]/\big(\delta\sqrt{t}\big) - [r + \omega - (\delta^2/2)]/\delta$, which increases without limit as t increases.

The central tendency measurements of the two variants have a cascade of interrelationships[14].Let $H_I(t)$ and $H_S(t)$ be the Ito and Stratonovich versions respectively of $H(T)$. then,

$$E[H_I(t)] = \exp\{E[\log H_S(t)]\} = h(t)$$
$$exp\{E[\log H_S(t)]\} = 1/E[1/H_S(t)]$$
$$1/E[1/H_I(t)] \ = mode(H_S(t))$$

The θth moment about the origin if $H(t)$ is

$$E\left[\left(H(t)\right)^{\theta}\right] = h_0^{\theta} e^{[r+\omega-\delta^2/2]\theta t+(\delta^2/2)\theta^2 t}$$

The generalised mean of $H(t)$ is:

$$\left\{E\left[\left(H(t)\right)^{\theta}\right]\right\}^{1/\theta} = h_0 e^{[r+\omega+(\theta-1)](\delta^2/2)\theta^2 t}$$

The Ito – Stratonovich interrelationship becomes

$$\left\{E\left[\left(H_I(t)\right)^{\theta}\right]\right\}^{1/\theta} = \left\{E[\left(H_S(t)\right)^{\theta-1}\right\}^{1/(\theta-1)}$$

The Stratonovich transition lognormal distribution is a weighted Ito transition lognormal distribution, which is derived from a generic feature of the exponential growth SDE[15].
The Ito – Stratonovich lognormal transition pdfs are related by

$$p_S(h,t|\, h_0) = \frac{h^{1/2} p_I(h,t|\, h_0)}{E[\left(H_I(t)\right)^{1/2}]}$$

Application

PRL is a type of protein released by the pituitary gland that has a variety of functions, the most well-known of which is the lactotrophic activity. The hormone has a variety of impacts on animals, including reproduction, metabolism, and tumorigenicity. In the meantime, little is known about this hormone's function[16]. PRL secretion alternates between pulsatile and basal modes of release. PRL secretion is principally regulated by hypothalamic dopamine and ultra-short auto feedback[17]. However, the physiological relevance of different hormones in humans remains unknown.The challenge of defining a normative range of PRL secretion is a little more challenging[17]. Gender, age, sex-steroid concentrations, body mass, core temperature, diet, stress, exercise, medication, and renal illness are all biological or clinical variables that may influence the difficulties[18]. The function of hypothalamic dopamine regulates prolactin at first. In males, several hormones generated by various bodily roles are not secreted[19]. In comparison to men, women have a greater PRL level.

There was no recent transmeridian travel, weight change, shift work, psychological stress, prescription drug usage, substance misuse, neuropsychiatric illness, or acute or chronic systemic disease reported by the sample. The results of a comprehensive medical history, physical examination, and screening biochemical tests were all within normal limits[20]. Exercising vigorously, sleeping throughout the day, eating snacks, drinking caffeinated beverages, and smoking cigarettes were all prohibited. Depending on individual sleeping habits, meals were served at 0900, 1230, and 1730 hours respectively, and room lights were shut out between 2200 and 2400 hours. For 24 hours, blood samples were taken every 10-minute intervals.

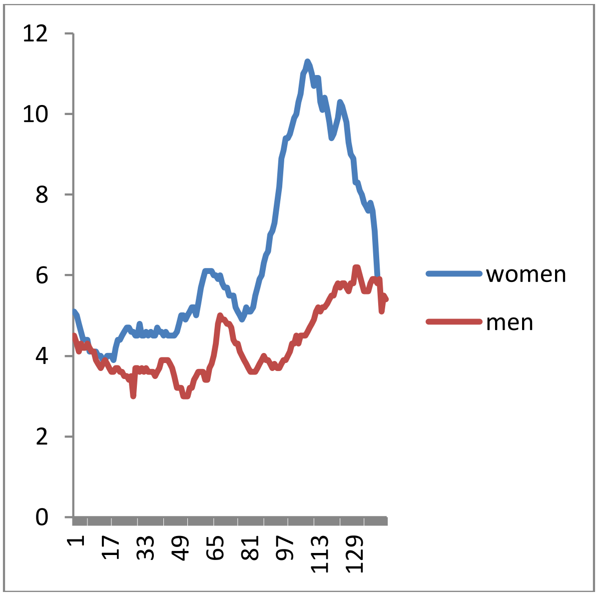

Fig.1 shows the serum PRL profiles across the 24 hour cycle

PRL concentration time series were analysed using a newly designed automated deconvolution technique, which was empirically tested utilising Hypothalamo–pituitary sampling and simulated pulsatile time series[21]. Two change point timings were determined inside each 24 hour pulse train to demarcate the start of the day and the start of the night time waveforms[22].

PRL differences between men and women are highlighted in Figure 1, notably during the light off period. In a blood sample of serum prolactin, there is a difference in women's PRL compared to men's. When postmenopausal women and premenopausal women were studied individually during the follicular phase, postmenopausal women had 40% reduced PRL production.

Mathematical Result

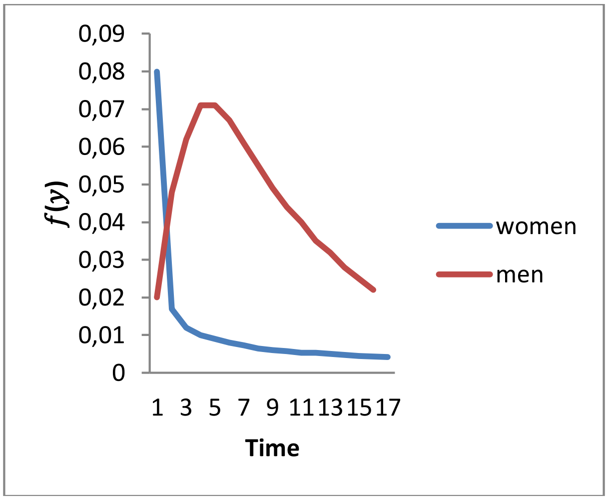

Fig. 2 Mathematical Model of Serum PRL of both Men and Women

Conclusion

PRL is a protein produced by the pituitary gland that serves a number of purposes. PRL may be a large contributor of such complexes, even the most essential single ingredient in some species.In a blood sample of serum prolactin, there is a lot of difference in women's PRL when compared to men's.The serum prolactin in the blood sample of women is decreasingly concave downwards and the serum prolactin in the blood of men is concave downwards..This study will be extremely valuable in the fields of medicine and engineering in the future.

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Author's Declaration:

There are no conflicts of interest declared by the writers. We hereby declare that all the figures in the manuscript are mine ours.

Authors' contributions statement:

M. Gayathri, accomplished data acquisition, M. Chandra Malar M.S· contributed in conception and design, A. Manickam and Pranjal Mohan Pandey analyzed the data and drafted the MS; all authors contributed interpretation, revision, and approved the final manuscript.

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