How to Cite:
AL-Ftlawy, D. M. K., & AL-Shara, I. M. T. (2022). Some chaotic properties of anosov diffeomorphisms of n-dimention tours T^n. *International Journal of Health Sciences*, *6*(S5), 4913–4921.
https://doi.org/10.53730/ijhs.v6nS5.9659

Some chaotic properties of anosov diffeomorphisms of n-dimention tours T^n

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***Abstract*---**In this search, we review the relationship of Anosov diffeomorphism with hyperbolic dynamic systems, where we present some definitions and basic observations of it. We find that the map possesses topological transitive and sensitive dependence on initial conditions and other tools of chaos, and therefore the map has chaotic properties by using many definitions of chaos.

***Keywords*---**Anosov Diffeomorphisms, Topological transitive, Sensitive dependence on initial conditions, Topological mixing.

Introduction

A diffeomorphism f of a compact differentiable manifold M is term Anosov if the tangent bundle TM admits a continuous invariant splitting $TM = E^{+} \oplus E^{-}$such that df expands$E^{+}$and contracts E^{-}exponentially. Anosov diffeomorphisms play nice and an important role in dynamical systems, where represents more perfect type of global hyperbolic behavior, it give examples structurally stable dynamical systems.

It is introduced by Dmitri Victorovich Anosov, who demonstrate that their behavior was in an appropriate sence generic. Adiffeomorphism f of a closed smooth manifold M is called Anosov if the manifold M is hyperbolic set of f see [1]. The hyperbolic automorphism of the two dimensional torus ia an example of an Anosov diffeomorphism [2], Not every manifold admits an Anosov diffeomorphism, there are no such diffeomorphisms on the sphere. The simplest examples of compact manifolds admitting them are the tori called linear Anosov diffeomorphisms. They are isomorphisms having no eigenvalue of modulus one. It

Manuscript submitted: 27 Feb 2022, Manuscript revised: 9 April 2022, Accepted for publication: 18 June 2022

was proved that any other Anosov diffeomorphism on a torus is topologically conjugate to on of this kind. One of the chaotic properties is transitivity, sufficient condition for transitivity is that all points are non-wandering, we introduce other chaotic properties is sensitive dependence on initial conditions.

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2- Basic Definition and Notation

Definition 2.1:[3] Given a smooth compact Riemannian manifold M, we define the tangent space at $x \in M, T_xM$ to be the real vector space comprising the collection of tangent vectors to M at x.
If M is embedded in R^n, this can be pictured as the hyperplane tangent to x.
Further, for a diffeomorphism$f: M \to M$, we define the differential of f at x to be the linear map
$df_x : T_xM \to T_{f(x)}M$. A diffeomorphism $f$of a compact Riemannian manifold, we called that a closed, invariant subset $\Lambda \subset M$ is hyperbolic if for every $x \in \Lambda, T_xM, T_xM$ the tangent space of M at x, can be written as the direct sum of subspacesE^s, E^u such that $df_x(E_x^s) = E_{f(x)}^s$, $df_x(E_x^u) = E_{f(x)}^u$ and there exists c > 0, $\lambda \in (0,1)$ such that for every n≥ 0, $\|df_x^n(v)\| \le c\lambda^n\|v\|$, when $\in E_x^s$, $\|df_x^n(v)\| \le c\lambda^{-n}\|v\|$, when $v \in E_x^u$,where $\|\cdot\|$ is given by the Riemannian metric on M. The subspaces E_x^s , E_x^u are called the stable and unstable subspaces at x respectively. A diffeomorphism f of a smooth smooth Riemannian manifold is called **Anosov** if M is hyperbolic.
It should be noted that since $\mathrm{Per}(f) \subset \Omega(f)$ for any diffeomorphism f, this definition entails that all periodic and fixed points of f are **hyperbolic**. We begin by presenting a
specific example of an Anosov diffeomorphism.
This definition (2.1) can also stated in terms of a splitting of the tangent manifold TM into invariant subbundles E^s and E^u .
Suppose $A \in GL_n(Z)$, where $GL_n(Z)$ is the set of all $n \times n$ invertible matrices with entries in Z. We say that A is a hyperbolic matrix if each of its eigenvalues $\lambda_i \in \mathbb{C}, i = 1, \dots, n$ satisfy $|\lambda_i| \neq 1$. We call an eigenvalue λ_i contracting if $|\lambda_i| < 1$ or expanding if $|\lambda_i| > 1$. Similiarly, a matrix A is called contraction if $|\lambda_i| < 1$ or expanding if $|\lambda_i| > 1$. Similiarly (expanding) if its eigenvalues are contacting (expanding).
Given a hyperbolic $A \in GL_n(Z)$, we can split the domain of A into the direct sum of A-invariant subspaces E^s and E^u i.e. $Z^n = E^s \oplus E^u$, where E^s and E^u are the generalized eigenspace corresponding to the contracting and expanding eigenvalues of A respectively. It follows that A restricted to one of these subspaces is contracting (E^s) or expanding (E^u). This gives us a direction in which A is expanding and another in which it's contracting. $GL_n(Z)$is diffeomorphism since $GL_n(Z) \subset GL_n(R)$ A is still a diffeomorphism fromR^n to itself. Most importantly in this case, $A(Z^n) = Z^n$. Therefore by quotientingR^n by R^n, A induces a map

$$\widetilde{A}: x + Z^n \to A(x) + Z^n$$

on $T^n = R^n/Z^n$, the n-torus to itself. Since $T^n$is a compact manifold, we can begin to generalize the notion of hyperbolicity to diffeomorphisms on compact manifolds.

We say that $f \in$ Diff(M) and $g \in$ Diff(M′)are **topologically conjugate** if there is a homeomorphism $k: M' \to M$ such that $k \circ f = g \circ k$. Such a map h is called a conjugacy, A C^1 map $f: M \to M$ is called **structurally stable** if there exists a neighborhood U of f in the C^1 topology such that every $g \in U$ is topologically conjugate to f.

Definition 2.2:[4] We say that $A \in GL(n, Z)$ is **hyperbolic** if its eigenvalues $\{\lambda_1, \lambda_2, \ldots . \lambda_n\}$ satisfy the following condition: $\forall r \in \{1, \ldots ., c\}, \forall\, i_1, i_2, \ldots ., i_r \in \{1, \ldots, n\}$: $|\lambda_{i1}, \lambda_{i2}, \ldots . \lambda_{ir}| \neq 1$.

Given $x \in M$ and an Anosov diffeomorphism f, we define the following:

$W_\epsilon^s(x) = \{y \in M | d\big(f^n(x), f^n(y)\big) \leq \epsilon\, , for\ every\ n \geq 0\}$

$W_\epsilon^u(x) = \{y \in M | d\big(f^{-n}(x), f^{-n}(y)\big) \leq \epsilon\, , for\ every\ n \geq 0\}$

$W^s(x) = \{y \in M / \lim_{n\to\infty} d\big(f^n(x), f^n(y)\big) = 0\}$

$$W^u(x) = \{y \in M / \lim_{n\to\infty} d\big(f^{-n}(x), f^{-n}(y)\big) = 0\}$$

We call $W_\epsilon^s(x)$ and $W_\epsilon^u(x)$ the **ϵ_stable** and **ϵ_unstable** manifolds of x respectively. Similarly, $W^s(x)$ and $W^u(x)$ are called the **unstable** and **stable** manifolds.We start by describing the sets of points whose orbits stay close to the orbit of a given point .

Definition 2.3:[5] Every nonempty open sub set U of X visits every nonempty open sub set $V\ of\ X$ in the following sense $f^n(U) \cap V \neq \emptyset$ for some $n \in N$. If the system (X, f)has this property then it is called topologically transitive, simply called **transitive**.

A continuous map $f: X \to X$ is called **totally transitive** if $f^n$is topologically transitive for all $n \geq 1$.

Definition 2.4:[6]The $f: X \to X$ is called **sensitive dependence in initial condition** if there exists $\varepsilon > 0$, $\forall \delta > 0$ such that for any $x_o \in X$and any open set $U \subset X$ containing x_o there exists $y_o \in U$ and $n \in Z^+$such that $(x_o, y_o\ ) < \delta$ then $d(f^n(x_o), f^n(y_o)) > \varepsilon$

Definition 2.5:[7] Let (X, d) be metric space A point $x \in X$ is called **non-wandering point** of f if every neighborhood U of x there exists a positive integer $m > 0$ such that $f^m(U) \cap U \neq \emptyset$, we denote the set of all non-wandering point of f by Ω.

Let $q \in X$ be a periodic point for f.A **homoclinic** point to q is a point $p \neq q$ which lies in $W^s(q) \cap W^u(q)$, where W^s is stable set andW^u unstable.

Let (X, d) be metric space A point $x \in X$ is called a **recurrent** point of f if there exists a sequence $\{n_i\}$ of positive integer with $n_i \to \infty$ such that $f^{n_i}(x) \to x$, we denoted the set of recurrent points of f by $R(f)$.

If each point is non-wandering then f is called **pointwise non-wandering** map

Definition 2.6:[8] Suppose that the orbit of p is infinite. Let $n \geq 2$ and V be any neighborhood of p. put $T_V = \{[x]: f^k(p) \in V$ for infinitely many $k \in [x] (\mathrm{mod}\ n)\}$and $T = \cap \{T_V: V$ is any neighborhood of p }.Then since $\{T_V\}$ has the finite intersection property, $T \neq \emptyset$. By methods of **Nitecki**, it is easily shown that if $[x], [y] \in T$ then $[x + y] \in T$, therefore $[0] \in T$.

Definition 2.7:[11] A map $f: X \to X$ is called **equicontinuous** if for any $\varepsilon > 0$ and any x, y there exists $\delta > 0$ with $d(x,y) \leq \delta$ then $d\big(f^n(x), f^n(y)\big) < \varepsilon, \forall n \in N$.

Definition 2.8:[6] $f: X \to X$ contiuous map of metric space X is chaotic(**Devaney**) if satisfies the following conditions

- sensitive dependence on initial conditions
- topologically transitive.
- Periodic point are dense in X

f is said to be chaotic according to Wiggins or W-chaotic if satisfies:

- topologically transitive.
- sensitive dependence on initial conditions **.**

f satisfy chaotic (**Touhey**) on X if given U and V non-empty open subsets of X then there exists a periodic point $x \in U$ and non-negative integer k such that $f^k(x) \in V$, where every pair of non-empty open subsets of X shares a periodic orbit.

Lemma 2.9: [9] Every Anosov deffeomorphism has an adapted metric $\|.\|$ such that $c = 1$ in Definition (2.1).
Proof: It is known that the Riemannian metric on M can be chosen so that $c = 1$, we assume $c = 1$ in follows the splitting is unique.

Corollary 2.10:[3]:For a diffeomorphism f, hyperbolic set Λ then

$$W^s(x) = \bigcup_{n\geq 0} f^{-n}(W^s_\epsilon(f^n(x))), and\ W^u(x) = \bigcup_{n\geq 0} f^{-n}(W^u_\epsilon(f^{-n}(x)))$$

Proposision 2.11: If $f, g: T^n \to T^n$ are two Anosov diffeomorphism which induce matrices then $f \circ g$ is Anosov diffeomorphism .
Proof: Let $x = (x_i), i = 1, \dots n \in T^n$, with out lose the generality of the theory if we take $n = 2$, $f \circ g(x_i) = f\big(B(x_i)\big) = AB(x_i)$
Since f and g are two Anosov diffeomorphism, then $A\ and\ B$are entries integer numbers so the interior of AB are integer number, since det($A.B$)=det(A).det(B) then det(AB)= ± 1.
Let the eigen values of A be$|\lambda_i| < 1, |\lambda_j| > 1, where\ i = 1, \dots, n\ and\ j = n+1, \dots, m$ and eigen values of B are$|\beta_i| < 1, |\beta_j| > 1, where\ i = 1, \dots k\ and\ j = k+1, \dots, m$ then the eigen values of $A.B$ is $|\lambda_i \beta_i| < 1$ and$|\lambda_j \beta_j| > 1$ and$|\lambda_i|.|\lambda_j|.|\beta_i|.|\beta_j| = 1$
$thus\ f \circ g$ satisfy all conditions of Anosov diffeomorphism so $f \circ g$ is Anosov diffeomorphism.

Corollary 2.12: If $f: T^n \to T^n$ is Anosov diffeomorphism, then f^n is Anosov diffeomorphism.
Proof: f is Anosov diffeomorphism, Assume f^k is Anosov diffeomorphism,
Then by induction law . by proposition(2.13)$f^{k+1} = f^k \circ f$ is Anosov diffeomorphism
f^n is Anosov diffeomorphism.

We can find general the properties

- If f and g are two Anosov diffeomorphism, then $f + g$ is not necessary Anosov diffeomorphism .

- If f is Anosov diffeomorphism and g is not Anosov diffeomorphism then $f \circ g$ is not Anosov diffeomorphism.

Example 2.13: let A,B be two matrix assailed to Anosov diffeomorphism such that $A = \begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$then $A + B$ does not induce of to Anosov diffeomorphism since $\det(A + B) \neq \pm 1$.If $C = \begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix}$then $A \circ C$ also does not induce of to Anosov diffeomorphism.

Proposition 2.14: If f and g are two Anosov diffeomorphism, then $f \times g$ is Anosov diffeomorphism .

Proof: let A, B be two matrix in $GL(n, Z)$ such that $= \begin{bmatrix} a_{11} & \dots\dots\dots\dots & a_{1n} \\ \vdots & \dots\dots\dots\dots & \vdots \\ a_{n1} & \dots\dots\dots\dots & a_{nn} \end{bmatrix}$,
$B = \begin{bmatrix} b_{11} & \dots\dots\dots\dots & b_{1n} \\ \vdots & \dots\dots\dots\dots & \vdots \\ b_{n1} & \dots\dots\dots\dots & b_{nn} \end{bmatrix}$in f, g respectively, since f *and* g be two Anosov diffeomorphism so the eigen values of f and g is not equal to 1. The eigen value of A is $|\lambda_i| = \frac{1}{|\lambda_{i+1}|} > 1, where\ i = 1, \dots\dots, n$
and the eigen value of B is $|\lambda_j| = \frac{1}{|\lambda_{j+1}|} > 1, where\ j = 1, \dots\dots, n$ then the eigen value of $A \times B$ is not equal to 1 then $f \times g$ is Anosov diffeomorphism.

Note: If f is Anosov diffeomorphism then f^{-1} is also Anosov diffeomorphism.

Proposition 2.15: let f is Anosov diffeomorphism of T^n with corresponding matrix A having eigenvalues λ_i ,$i = 1, \dots\dots, n$,then the periodic points of f correspond precisely with the set of rational points of T^n.
$\{\left(\frac{p_1}{q}, \frac{p_2}{q}\right) + Z^n, p_1, p_2, q \in N\ and\ 0 \leq p_1, p_2 < q\}$

Corollary 2.16: Anosov diffeomorphism form an open set in $Diff^1(M)$.
Proof*:* see [2]

Theorem 2.17: If $f: T^n \to T^n$ is Anosov diffeomorphism of T^n,thus we call f are structural stability .
Proof: Let f: M → M be an Anosov diffeomorphism. By Corollary (2.18), ∀ $\varepsilon > 0, \exists\ \delta > 0$ such that for any g: M → M with $\mathrm{dist}_1(f, \mathrm{g}) < \delta$, we get a h: M → M with$\mathrm{dist}_0(\mathrm{h}, \mathrm{Id}) < \varepsilon$ and $\mathrm{hg} = f\mathrm{h}$, therefore, by Definition(2.1) f is structurally stable.

Remark 2.18: $F: R^n \to R^n$ induce Anosov diffeomorphism of T^n, thus we call f are structural stability.

Definition 2.19: A map $f: X \to X$ is called minimal if all points are transitive (each orbit is dense that is f has no periodic points .

3-Chaoticity Properties of Anosov Diffeomorphisms

In this section M is connected compact smooth Riemannian manifold, we prove Anosov diffeomorphism is chaotic in many different difinitions .

Theorem 3.1: If $f:\mathrm{T^n}\rightarrow\mathrm{T^n}$ is Anosov diffeomorphism of $\mathrm{T^n}$, thus we call f has sensitive dependence on initial condition.

Remark 3.2: If $F: R^n \rightarrow R^n$ is induce Anosov diffeomorphism of T^n, thus we call f has sensitive dependence on initial condition

Remark 3.3: [6] If $F: R^n \rightarrow R^n$ is induce Anosov diffeomorphism on T^n,thus we say the set of all periodic points of f is dense

Proposition 3.4 : If $f:\mathrm{T^n}\rightarrow\mathrm{T^n}$is Anosov diffeomorphism , then the homoclinic points are dense in $\mathrm{T^n}$ **.**
Proof: let W^s be the stable sub set in R^n,we claim that $W^s$is a line with irrational slope in R^n.For it this were not the case W^s would necessarily pass through a point with coordinates (K,L)for K,L$\in Z$ put then all of the iterates of (K,L) would have integer coordinates, since A is an integer matrix. But this is impossible since f^n(K,L)$\longrightarrow 0$ as $n\longrightarrow\infty$. Now let $x_N$be the x-coordinate of the point onW^s and $y=L$. Note that x_1 is the reciprocal of the slope of W^s, which is irrational. Also x_2=$2x_1$and in general$x_N = N_{x_1}$, the point (x_j, j)projects to apoint of the form $[\propto_j, 0]$ where $0 \leq\propto_j< 1$. The line $y=0$ defines a circle in T and the $\propto_j$ are the successive images of [0] under an irrational translation of this circle. By Jacobi's theorem these points are dense in T^n.

Remark 3.5: If (x,y) is a periodic point for f then any homoclinic points to (x,y) tends to the orbit of (x,y) under forwared and backwared iteration of f .

Theorem **3.6**: [4] For any positive integer $n>1$, there exists an $(n-1)$ hyperbolic matrix $A\in GL(n,Z)$.

Theorem 3.7 : Let $f: T^n \rightarrow T^n$is Anosov diffeomorphism ,then f is topological transitive .
Proof: let U,V be any open sub sets of $T^n$and the set of a homoclinic points is dense then there exists $x_o$be a homoclinic point and $i>0$ such that
$f^i(u)=x_o$ where $u\in U,\ f^k(V)=x_o, k>i$.
$f^k(U)=x_o=V$ choose $m=i+k$ such that $x_o\in f^m(U)\cap V\neq\emptyset$
then f is topological transitive .

Theorem 3.8: If $f: T^n \rightarrow T^n$is Anosov diffeomorphism ,then f is totally transitive.
Proof:
Let U,V be sub sets of T^n if $U\cap V\neq\emptyset$ then it is trivial
Now if $U\cap V=\emptyset$ since f is transitive then there exists $n\in N$ such that $f^n(U)\cap V\neq\emptyset$ and $U\in T^n$ then ,f^n is Anosov diffeomorphism , it follows f^n is transitive, so f is totally transitive.

Note: that in Anosov diffeomorphism the converse also satisfy
totally transitive↔transitive

So, by is w-chaotic since by Theorem(3.7) and Theorem(3.1),thus Anosov diffeomorphism is w-chaotic

Theorem 3.9: If $f\colon T^n \to T^n$is Anosov diffeomorphism ,then f is topologically mixing.
Proof: Let f be a P-chaotic map from a continuum X to itself and let U, V be nonempty open subsets of X. There exists $x \in U, y \in V$ and $\varepsilon > 0$ such that $B(x,\varepsilon) \subset U$ and $(y,\varepsilon) \subset V$. By Corollary(2.12)
, we have $M \in N$ such that for each $k \geq M$, there exists $z_k \in$ X such that $d(x, z_k)<\varepsilon$. We see that $f^k(U) \cap V \neq \emptyset$ for all k≥ M, thus, f is mixing.

Theorem 3.10:[10] If f is pointwise nonwandering map on compact metric space to itself, then so isf^n for each $n \geq 1$.

Proposition 3.11: Let $f\colon T^n \to T^n$is Anosov diffeomorphism ,then f is not minimal.
Proof: since by Definition (2.19) f is not minimal since the set of periodic points Anosov diffeomorphism is dense.

Theorem 3.12: If $f\colon T^n \to T^n$is Anosov diffeomorphism ,then f is chaotic in sense of Touheya.

Theorem 3.13: Let $f\colon T^n \to T^n$is Anosov diffeomorphism ,then the set of all non wandering point equal to T^n
Proof: to prove $T^n \subset \Omega(f)$, let $x \in T$ then there is U open set such that $x \in U$, by theorem $f$is chaotic in sense of Touhay , thus for all $U \subset T^2$there is $k \in N$ and $p \in U$ periodic point of period n for all $f^k(p) \in U$ then $f^k(U) \cap U \neq \emptyset$ $f^{nk}(U) \cap U \neq \emptyset$ then$T^n \subset \Omega(f)$ and it is clear $\Omega(f) \subset T^n$ then $\Omega(f) = T^n$

Theorem 3.14: If f is Anosov diffeomorphism then $\Omega(f) = \Omega(f^n)$.
Proof: by Theorem (3.14) $\Omega(f) = T^n$ and f is Anosov diffeomorphism then by definition (2.5) f is pointwise nonwandering by Theorem (3.10)f^n is pointwise nonwandering then $T^n = \Omega(f^n)$ therefore $\Omega(f) = \Omega(f^n)$.

Suppose that f is a continuous map to itself then $(f) \subset AP(f) \subset R(f) \subset \Omega(f)$.

Lemma 3.15:[11] On compact metric space if f is equicontinuous then $\Omega(f) = A\text{P}(f)$

Proposition 3.16: If f is Anosov diffeomorphism then $\Omega(f) = AP(f)$
Proof : By Lemma (3.15) and since f is equicontinuous on T^n
then $P(f) \subset AP(f) = \Omega(f) = T^n$

Proposition 3.17: If f is Anosov diffeomorphism then $\Omega(f) = R(f)$
Proof: By Proposition (3.16) since $\Omega(f) = T^n$ we get $R(f) = T^n$ thus $\Omega(f) = R(f)$

Proposition 3.18: If $f\colon T^n \to T^n$ is Anosov diffeomorphism then $\text{R}(f) = R(f^n)$
Proof: Let $p \in R(f)$, by Definition(2.6) there are k_i with $k_i \to \infty$ such that $f^{k_i}(p) = p$,for all but finitely manyk_i therefore p is a periodic point. Hence $p \in R(f^n)$for all $n \geq 1$. Suppose that the orbit of p is infinite by again Definition there exist $n \geq 2$ and V_i be any neighborhood of p and there exist infinitely many k such that $f^k(p) \in V$, then since $\{T_{V_i}\}$ has the finite intersection property, $T^n \neq \emptyset$ by methods of Nitecki[9], if $[x],[y] \in T^n$ then $[x+y] \in T^n$, therefore $[0] \in T^n$.

Proposition 3.19: If f is Anosov diffeomorphism then $\mathrm{AP}(f) = AP(f^n)$
Proof: $\Omega(f) = \mathrm{AP}(f) = T^n$by Theorem(3.14)and Proposition(3.16) since f is Anosov diffeomorphism. But$f^n$is continuous and by Theorem(3.14)then $\Omega(f^n) = AP(f^n) \forall n \in N$ therefore
$\Omega(f^n) = \mathrm{AP}(f) \Longrightarrow \mathrm{AP}(f) = AP(f^n)$
Theorem 3.20: If $f: T^n \to T^n$is Anosov diffeomorphism ,then $f$is a chaotic in sense of touhey.

Remark 3.21: Since Anosov diffeomorphism is transitive by Theorem(3.7), sensitive dependence on initial condition by Theorem (3.1), then the Anosov diffeomorphism satisfy Culick definition.
Since Anosov diffeomorphism is transitive by Theorem (3.7), sensitive dependence on initial condition by Theorem (3.1) and the set of all periodic point are dense then the Anosov diffeomorphism satisfy Devaney definition.
Since Anosov diffeomorphism is transitive by Theorem(3.7), sensitive dependence on initial condition by Theorem (3.1), then the Anosov diffeomorphism satisfy W-chaotic definition.
Since Anosov diffeomorphism is transitive by Theorem(3.7),sensitive dependence on initial condition by Theorem (3.1)then the Anosov diffeomorphism satisfy W-chaotic definition.

Conclusions

- If $f: T^n \to T^n$ is Anosov diffeomorphism, then f^n is Anosov diffeomorphism.
- If $f, g: T^n \to T^n$ are two Anosov diffeomorphism which induce matrices then $f \circ g$ is Anosov diffeomorphism.
- If f and g are two Anosov diffeomorphism, then $f \times g$ is Anosov diffeomorphism.
- If f and g are two Anosov diffeomorphism, then $f + g$ is not necessary Anosov diffeomorphism.
- If $f: T^n \to T^n$ is Anosov diffeomorphism of T^n,thus we call f are structural stability .
- If $f : T^n \to T^n$ is Anosov diffeomorphism of T^n, thus we call f has sensitive dependence on initial condition.
- If $f : T^n \to T^n$is Anosov diffeomorphism , then the homoclinic points are dense in T^n
- $f: T^n \to T^n$is Anosov diffeomorphism ,then f is topological transitive .
- If $f: T^n \to T^n$is Anosov diffeomorphism ,then f is topologically mixing.
- If $f: T^n \to T^n$is Anosov diffeomorphism ,then f is not minimal.
- If $f: T^n \to T^n$is Anosov diffeomorphism ,then the set of all non wandering point equal to T^n.
- If f is Anosov diffeomorphism then $AP(f) = AP(f^n) = \Omega(f) = \Omega(f^n) = R(f) = R(f^n)$.
- It was proved that any other Anosov diffeomorphism on a torus is topologically conjugate to on of this kind.
- f is said to be chaotic according to chaotic (Devaney) .
- f is said to be chaotic according to Wiggins or W-chaotic .
- f is said to be chaotic according to chaotic (Touhey).

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