Using biomedical signals with the help of fragmentary-wavelets on digital processing

Urakov Sh. U
Samarkand State Medical University, Associate Professor of Informatics and Information Technology, Samarkand, 140100, Uzbekistan

Juraev J. U
Samarkand State University, PhD candidate, Samarkand, 140100, Uzbekistan

D. K. Kholmurodova
Doctor of sciences in technics. Head of department of medical chemistry. Samarkand state medical university, Samarkand, 140100, Uzbekistan

Raxmanova F. E
Assistant of department of medical chemistry. Samarkand state medical university, Samarkand, 140100, Uzbekistan

Tohirova F. O.
Samarkand State Medical University, assistant of department of Informatics and Information Technology, Samarkand, 140100, Uzbekistan

Abstract---This article is devoted to rebuilt for an important fragmentary of wavelet models of Biomedical Signal Processing. These models were built using Haar’s fragmentary-unchanged wavelets as well as Doubuchi wavelets. The Haar’s fragmentary-unchanged wavelets models has a high accuracy for Biomedical signals on digital work, and this provides doctors for making any useful decisions about the patients diseases. For example, the first signal of Gastroenterology experimental information was took, and there were built on the basis of this information the of fragmentary-unchanged and Doubuchi wavelet models and evaluated their errors. It is known that modification of signals using fragmentary wavelets results in formation of orthonormal wavelets, due to there will be sharp increase in errors along the signal graph, thus in order to reduce errors Doubuchi wavelets were used and achieved the goal.

Keywords---Doubuchi wavelets, conversion wavelets, digital processing error, relative error, orthonormal wavelets.
Introduction

There are many types of wavelets available today. The most common is the Haar's wavelets. The Haar’s wavelets is expressed in an iterative form. The main disadvantage of the Haar’s wavelets is, it has lack of analytical views of function and also increase errors in the restoration of compression coefficients. It should be noted that any type of wavelet advantage of the signal depends on analysis[2], because scaling functions of wavelets apparently interpreted to the different types analyses.

In matters of image recognition from wavelets, during the processing and synthesis of various signals, such as speech, in the analysis of various images in nature (color of the retina, radiography of the kidney, studying the surface properties of crystals and nanoobjects, satellite images of clouds or planetary surfaces, etc. can be used in the study of the properties of vortex fields and in other cases [14].

The wave lines of extend along the signal graph with the time of axis. In most cases, the Haar’s-wavelets graph closer along the form of a one-way wave lines signal, which some signals compress a handful of good results. Its mathematical interpretation allows the analysis of wave states at different frequencies. The amplitude of the graph of the Haar’s-wavelet function decreases to zero and produces vibration waves[1].

The Main Part

Construction of the Haar's-Wavelet

The Haar's wavelet has fast-conversion algorithms, and its orthogonal wavelest is widely used in solving practical problems. In the Haar's bases [2] it is considered as a wavelet. The Haar’s wavelets attract the attention of experts for two reasons:

1. To reduce the number of coefficient relative to the total number of binary segments (giving for accuracy approach).
2. Absence of “long” operations in the calculation of coefficients. It only uses adding, scaling and conversion operations.

Deficiency of the Haar's fragmentary-unchanged wavelets is increasing errors while approaching to signal, that exceed 0.1% of the signal to ensure the accuracy of coefficients that it will be required to memorize[4].

The process of signaling the wavelets are based on the use of two types of functions[3]: the wavelet function and the scaling function, they are built by a single mother wavelet \( \psi(t) \), moving along the signal in time \( b \) and conversion the time scale \( a \):

\[
\psi_{ab}(t) = \frac{1}{\sqrt{|a|}} \psi\left(\frac{t-b}{a}\right), \quad (a, b) \in \mathbb{R}, \ \psi(t) \in L^2(\mathbb{R})
\]
In the digital signal processing, wavelet[12] functions are used to separate the details and local properties of signals, and a scaling function is used to approximate signals. In the selection of wavelet functions, special attention was paid to their characteristics such as smoothness, carrier size, and the number of zero-value cases[6].

We define $V^0$ a set of invariant functions in all $[0,1]$ intervals, that is, a set of linear vectors. In this case, the following scaling function belongs to the set-$V^0$:

$$
\phi(t) = \phi_{0,0}(t) = \begin{cases} 
1, & 0 \leq t < 1 \\
0, & \text{otherwise}
\end{cases}
$$

(1)

when scaling function is $i = 0$.

$V^1$-set $\left[0, \frac{1}{2}\right]$ and $\left[\frac{1}{2}, 1\right]$ are the set of functions that do not change in the interval, it forms linear vectors. The scaling function belongs to the $V^1$-set and is considered as its wavelet functions[5]:

$$
\phi_{1,0}(t) = \phi(2t) = \begin{cases} 
1, & 0 \leq t < \frac{1}{2} \\
0, & \text{otherwise}
\end{cases}
$$

and

$$
\phi_{1,1}(t) = \phi(2t - 1) = \begin{cases} 
1, & \frac{1}{2} \leq t < 1 \\
0, & \text{otherwise}
\end{cases}
$$

(2)

when scaling function is $i = 1$.

This function is constant in the $[0,1]$ interval, $\left[0, \frac{1}{2}\right]$ and in the $\left[\frac{1}{2}, 1\right]$ intervals as well. Therefore, each element of the $V^0$ set is also an element of the set $V^1$, i.e. the relationship is appropriate. We define collection in a similar way of the set, $V^2$:

$$
V^2 = \left[0, \frac{1}{4}\right], \left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{3}{4}, 1\right]
$$

Collections of functions relative to interval. A $V^n$ set of similar scaling functions, i.e.

$$
\phi_{n,j}(t) = \phi(2^n t - j), \quad j = 0,1,\ldots,2^n - 1
$$

$$
0 \leq 2^n t - j < 1, \quad \frac{j}{2^n} \leq t < \frac{j+1}{2^n}
$$
\[ \phi_{n,j}(t) = \begin{cases} 1, & \frac{j}{2^n} \leq t < \frac{j+1}{2^n}, \quad j = 0,1,...,2^n - 1 \quad (3) \\ 0, \quad \text{otherwise} \end{cases} \]

Here, \( i = n \) of scaling function is \( 0 \leq 2^n t - j < 1, \quad \frac{j}{2^n} \leq t < \frac{j+1}{2^n} \)

the scaling function of conversion interval, relating the \( \phi_{n,j}(t) \) - to \( V^n \) scale functions, that includes scalar multiplication of vectors collection, thus, these collections consist of Euclidean Space. In our case, as a scalar multiplication

\[ (f, g) = \int_0^1 f(t)g(t) \, dt \quad (4) \]

we obtain the view that the coefficients are determined using this formula, in that case \( \phi_{n,j}(t) = \sqrt{2^n} \phi(2^n t - j), \quad j = 0,1,...,2^n - 1 \)

Using Figures (3) and (4), find the coefficients of the Haar's wavelet:

\[ C_n = \int_0^1 \phi_n(x)f(x) \, dx \quad (5) \]

The formula for finding the coefficients of the Haar's wavelet:

\[ f(x) \cong \sum_{n=0}^{\infty} C_n \phi_n(x) \quad (6) \]

The Haar’s scaling function draws a graph of the change in the different values of \( m \) and \( k \) (Figure 1).
Based on the model of Gastroenterology signal, in the first experimental information of invariable-piece wavelets is carried out digital processing (Figure 2).

![Figure 2. Digital processing in the analysis of Gastroenterology signal of the Haar’s scaling wavelets](image)

**Construction of Doubuchi wavelets**

Wavelet conversion is expressing giving function to the wavelet function images[8]. Wavelet is the small wave or the wave that suddenly jump. Today wavelet has been using widely for conversion to digital signal processing, image, audio compression and many other areas. In the 80 years of the last century, Grossman and Morley carried out scientific research on the theory of the wavelet conversion. Except it, there are available Furen and DKC (discret kosinus conversion) methods, the main drawback of these methods are the basic harmonic components function of wavelet conversion does not work appropriately when it is periodically sequence is not good and in the result, the opportunity to restore a part of the information is lost, for this reason, in order to eliminate these defects, the perfect way is to move on Doubuchi wavelet. In 1988 Doubuchi called the attention of specialists in the analysis orthogonal wavelet change. It should be noted that, Doubuchi wavelet is built on the basis of scaling for a limited number of coefficient indicators[7].

To build the Doubuchi wavelet, we write the scaling and wave equation:

\[
\varphi(t) = \sqrt{2} \sum_{k} h_k \varphi(2t - k)
\]

\[
\psi(t) = \sqrt{2} \sum_{k} g_k \varphi(2t - k)
\]

(7)

The wave function \( \psi(t) \) of a Doubuchi wavelet is usually denoted by the letter D, and is formed by adding a number corresponding to the Doubuchi wavelet scale, i.e., D2, D4, D6.
Here are the conditions of orthogonality and smoothness of the wavelet change[10]:

\[ |m_0(\omega)|^2 + |m_0(\omega + \pi)|^2 = 1 \]  

(8)

here,

\[ |m_0(\omega)| = \sum_n h_n e^{-in\omega} / \sqrt{2} \]

\[ \frac{d^l \psi(\omega)}{d\omega} |_{\omega=0} = 0, \quad l = 0, 1, ..., N - 1 \]

(9)

\[ m_0(\omega) \propto \left( \frac{1 + e^{i\omega}}{2} \right)^N \]

\[ M_0(\omega) = \cos^{2N} \frac{\omega}{2} \cdot L(\omega) \]

here,

\[ L(\omega) = P \sin^2 \frac{\omega}{2} \]

To find the coefficients to be used, \( m_0(\omega) \) and \( P \) polynom's view will be as follows:

\[ P(y) = (1 - y)^{-N} (1 - y^N P(1 - y)) \]

The form \( h_k \) and \( g_k \) formula (7) are the coefficients of the scaling and wave equations, respectively, \( h_k \) for which \( g_k \) the following equation is appropriate according to formula (8):

\[
\begin{cases}
  h_0^2 + h_1^2 + h_2^2 + h_3^2 = 1 \\
  h_2 h_0 + h_3 h_1 = 0 \\
  h_3 - h_2 + h_1 - h_0 = 0 \\
  0h_3 - 1h_2 + 2h_1 - 3h_0 = 0
\end{cases}
\]

(10)

Solve this equation,

\[ h_0 = \frac{1 + \sqrt{3}}{4\sqrt{2}}, \quad h_1 = \frac{3 + \sqrt{3}}{4\sqrt{2}} \]

\[ h_2 = \frac{3 - \sqrt{3}}{4\sqrt{2}}, \quad h_3 = \frac{1 - \sqrt{3}}{4\sqrt{2}} \]
After the $h_k$-coefficients are determined, with the help of $h_k$ we identify $g_k$ according to the following relationship[11]:

$$g_k = (-1)^k h_{2M-k-1}$$

$$g_0 = h_3, \quad g_1 = -h_2, \quad g_2 = h_1, \quad g_3 = -h_0$$  \hspace{1cm} (9)

To change the wavelet of the function $\varphi(t)$, it is necessary to calculate the coefficients $\{a_i, d_i\}$. These coefficients will be found through the following integral [8]:

$$a_k = (f, \varphi_k) = \int_R f(x) \varphi_k(x) \, dx$$ \hspace{1cm} (10)

$$d_k = (f, \psi_k) = \int_R f(x) \psi_k(x) \, dx$$ \hspace{1cm} (11)

It should be noted that in (10) and (11) there is a problem of calculating a large number of integrals to find the coefficients $\{a_i, d_i\}$. To solve this enigma is used the rapid change wavelet method by Malla [13]. Malla’s algorithm allows the use of calculating by using algebraic operations of changed wavelet coefficients:

$$a_i = h_0 f_{2i} + h_1 f_{2i+1} + h_2 f_{2i+2} + h_3 f_{2i+3}$$

$$d_i = g_0 f_{2i} + g_1 f_{2i+1} + g_2 f_{2i+2} + g_3 f_{2i+3}$$ \hspace{1cm} (12)

$a_i$ Doubachi scaling coefficients, $d_i$ Doubachi Wavelet coefficients. These equations (12) provide fast algorithms for calculating wavelet coefficients. (12) According to the formula Doubachi wavelet changes based on the following links:

$$D(a,b) = \sum_i a_i + \sum_i d_i$$

When the fourth regularly wavelet of Doubachi changes out, for the scaling function $\varphi(t)$ two coefficients turn into a zero[7].

If N=4 state (D4- the fourth regularly wavelet of Doubachi changes)

The mother Doubachi[9] wavelet and scaling wavelet are shown in the in Figure 3.
Based on the model of Gastroenterological signal, the first experimental tick of information of b N= 4 case is carried out digital processing for the fourth Doubechi regularly conversion (D4) wavelet. (Figure 4).

**Error Evaluation**

Here are the digital processing errors in the Haar’s fragmentary -unchanged and Doubechi wavelets.

Revealing on \([a, b]\) continuous function \(f(x)\) in [2], segment \([a, b]\) we can divide the node into dots.

\[
a \leq x_0 < x_1 < \ldots < x_i < \ldots < x_n \leq b
\]

\[
h = x_{i+1} - x_i = \text{const} \quad (15)
\]

\(h\) - the distance between nodes.

There are formulas for determining the methodological errors of interpolation for polynomials of different levels. For example, for zero-level polynomials (for fragmentary -unchanged wavelets), the error estimation formula is:
\[ |P(x) - f(x)| \leq \frac{1}{2} \max |f'(x)| h \]

Here is an assessment of the relative errors of digital processing of the gastroenterological signal in Haar's fragmentary-unchanged wavelets:

\[ \delta_1 = \frac{|f(x_i) - \text{har}(x_i)|}{f(x_i)} \cdot 100\% = 0.087679\% \]

\( \delta_1 \) - Relative error of Haar's fragmentary-unchanged wavelets.

Here is an assessment of the relative errors of interpolation of the gastroenterological signal in Dubechi wavelets:

\[ \delta_2 = \frac{|f(x_i) - D_i|}{f(x_i)} \cdot 100\% = 0.030609\% \]

\( \delta_2 \) - The relative error of the Dubechi wavelet

Using formulas of (16) and (17), we evaluate relative errors according to the table (Table 1).

| No. | Evidence of ECG signal | Interpolation on the Haar's fragment unchanged wavelet | ECG - Haar's \( |f(x_i) - \text{har}(x_i)| \) | Relative error on the Haar's fragment-unchanged wavelet % | Interpolation on the Dubechi wavelet | Dubechi - ECG \( |f(x_i) - D_i| \) | Relative error on the Dubechi wavelet % |
|-----|------------------------|------------------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 0   | 6,370065               | 6,322574                                 | 0.047491                        | 0.087679                        | 6,370065                       | 0                               | 0.030609                        |
| 1   | 6,370065               | 6,329703                                 | 0.040362                        | 0.087679                        | 6,370065                       | 0                               | 0.030609                        |
| 2   | 5,835327               | 5,843876                                 | 0.008549                        | 0.087679                        | 5,767959                       | 0.067369                        |
| 3   | 5,30059                | 5,322287                                 | 0.021697                        | 0.087679                        | 5,467959                       | 0.167369                        |
| 4   | 4,765852               | 4,381673                                 | 0.384179                        | 0.087679                        | 4,666906                       | 0.098946                        |
| 5   | 5,567959               | 5,461979                                 | 0.10598                         | 0.087679                        | 5,466906                       | 0.101054                        |
| 6   | 6,637434               | 6,401045                                 | 0.236389                        | 0.087679                        | 6,771119                       | 0.133685                        |
| 7   | 6,904803               | 6,99168                                  | 0.086877                        | 0.087679                        | 6,771119                       | 0.133685                        |
| 8   | 6,904803               | 6,784235                                 | 0.120568                        | 0.087679                        | 6,837434                       | 0.067369                        |
| 9   | 6,370065               | 6,29328                                  | 0.076785                        | 0.087679                        | 6,374340                       | 0.004275                        |
| 10  | 6,370065               | 5,992511                                 | 0.377554                        | 0.087679                        | 6,302696                       | 0.067369                        |

**Table 1**

**Conclusion**

Using the Haar’s-wavelet modification, the gastroenterological signal was evaluated by constructing a digital operation model in the Haar’s fragmentary-unchanged and Dubechi wavelets and evaluating its errors. Based on the results of the analysis, in the process of evaluation the number of Gastroenterology signal node points for 521, the process of digital on the Haar’s fragmentary-unchanged wavelets indicated of 0.087679% relative errors, and digital processing of
Doubecchi wavelet indicated 0.030609% relative errors. Based on the results, digital processing on the Doubecchi wavelets estimated the smallest errors as well. To conclude, it is possible to present that, in the process of digital signals Doubecchi wavelets are able to make good results.

References